Lesson 2: *Inflation*

Elisa G. M. Ferreira

Universidade de Sao Paulo & Kavli IPMU

Early universe cosmology, USP
22/Nov/2022

Image: Nasa

Review - lesson 1

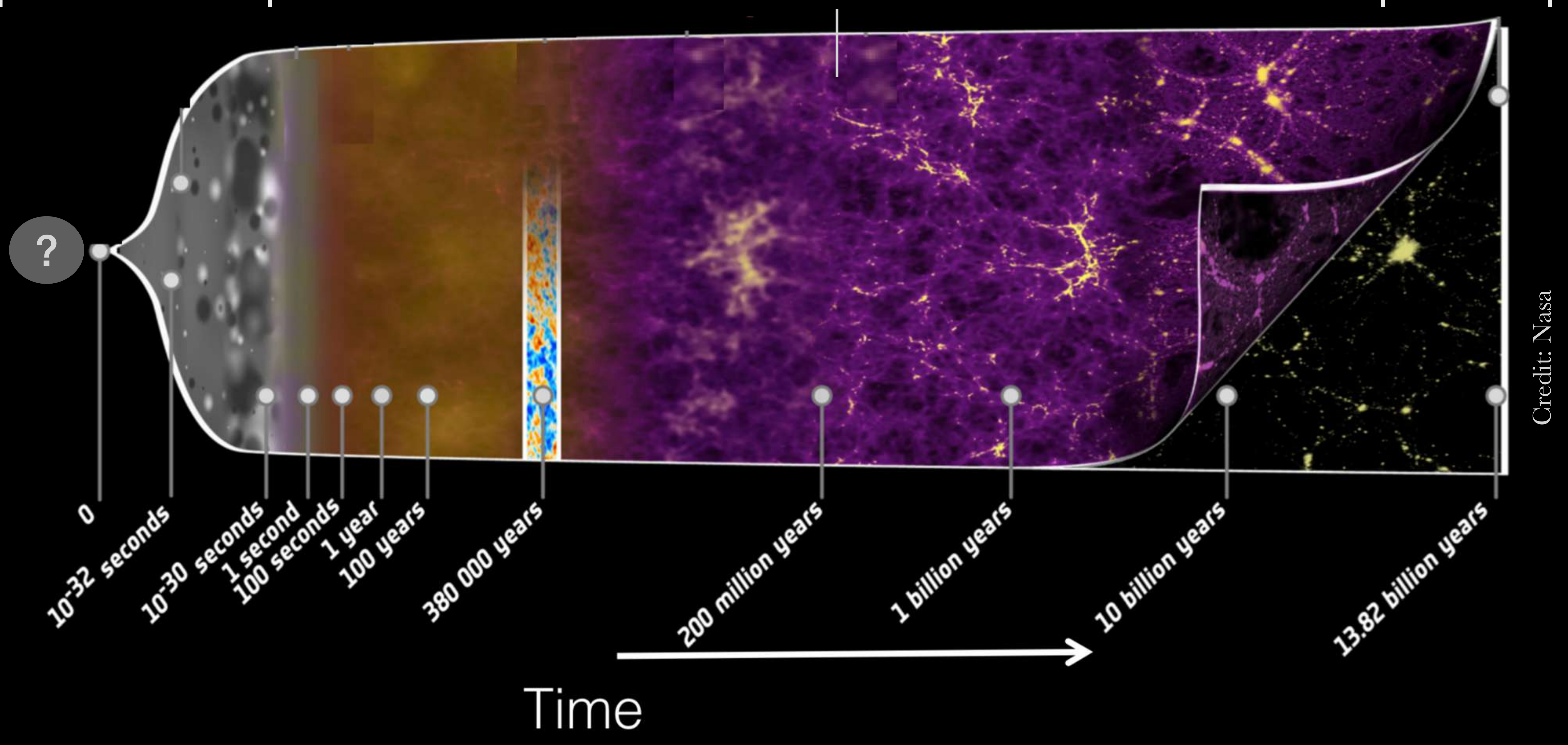
Standard cosmological model

MANY fundamental open questions

Early universe
Big Bang? What is the physics of the early universe?

Dark matter p
What is the dark matter?

Dark energy -
What is the dark energy?

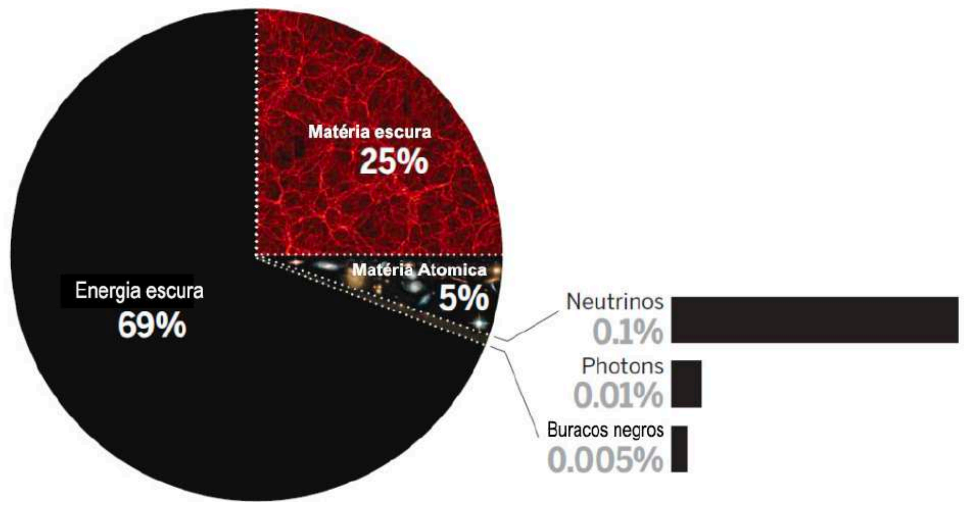


The expanding universe

Homogeneous universe (Cosm. Background) → Homogeneous and isotropic + Perfect fluid → Expansion of the universe

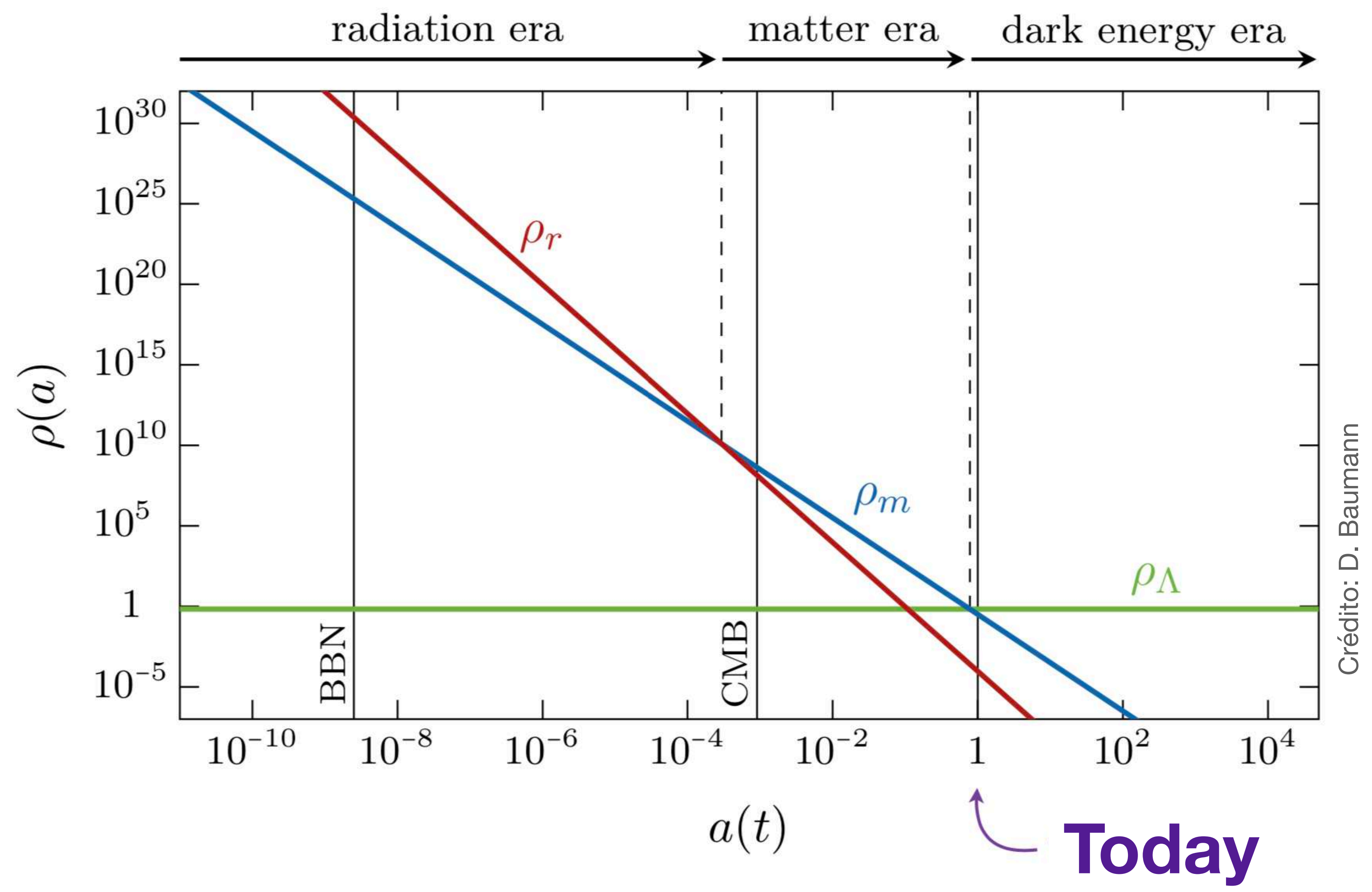
$$\Rightarrow \quad \frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a} \longrightarrow \boxed{\rho \propto a^{-3(1+w)}}$$

Components of the universe



radiation era → matter era → dark energy era

$$\Omega_{tot} = \Omega_m + \Omega_{rad} + \Omega_{de}$$



Standard cosmological model

The SCM describes the structure, evolution and composition of our universe. It also explains what we see and have in our universe today. It includes, then, the standard model of elementary particles, and explains the evolution and formation of the particles and structures we have today.

2 theoretical Pilars:

- GR
- Cosmological principle

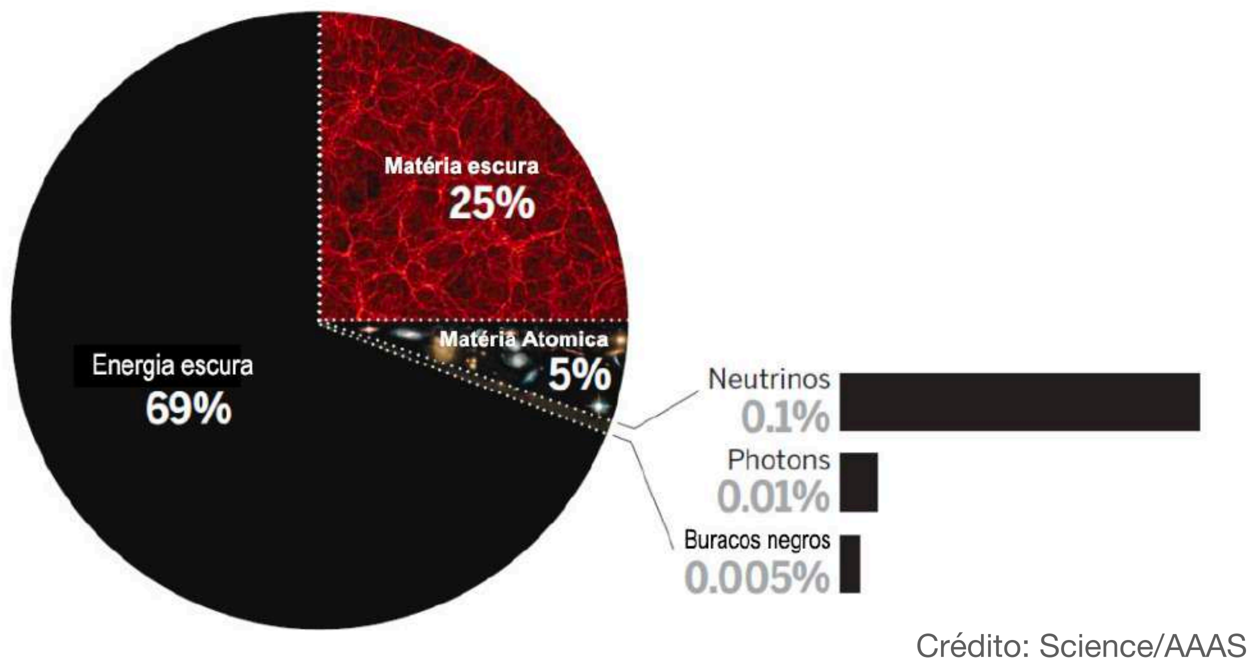
3 observational pilars:

- Hubble - Lemaître Law
- Nucleosynthesis
- Cosmic Microwave Background

\+

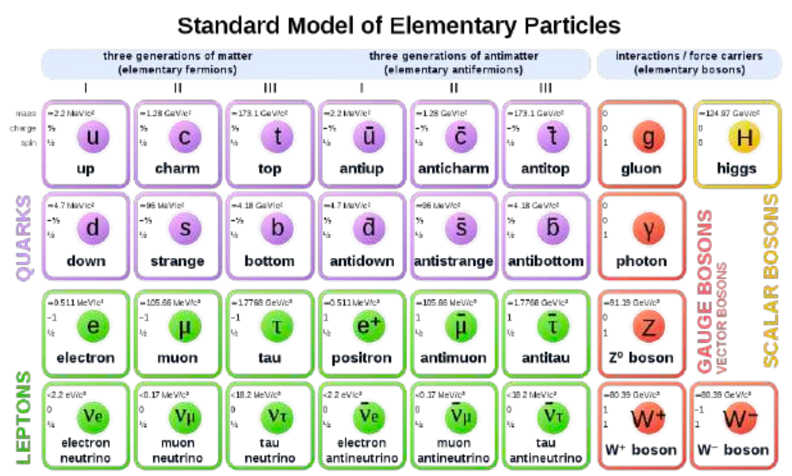
a.k.a. ΛCDM model

Parametrization: 6 parameters



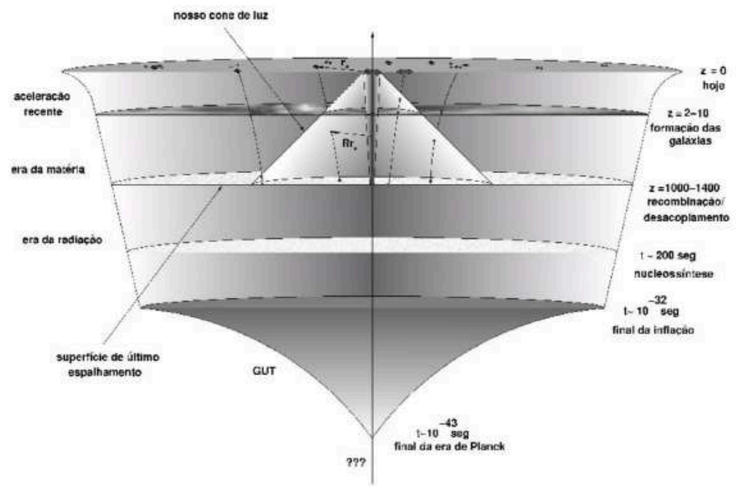
\+

Standard model of elementary particles



\+

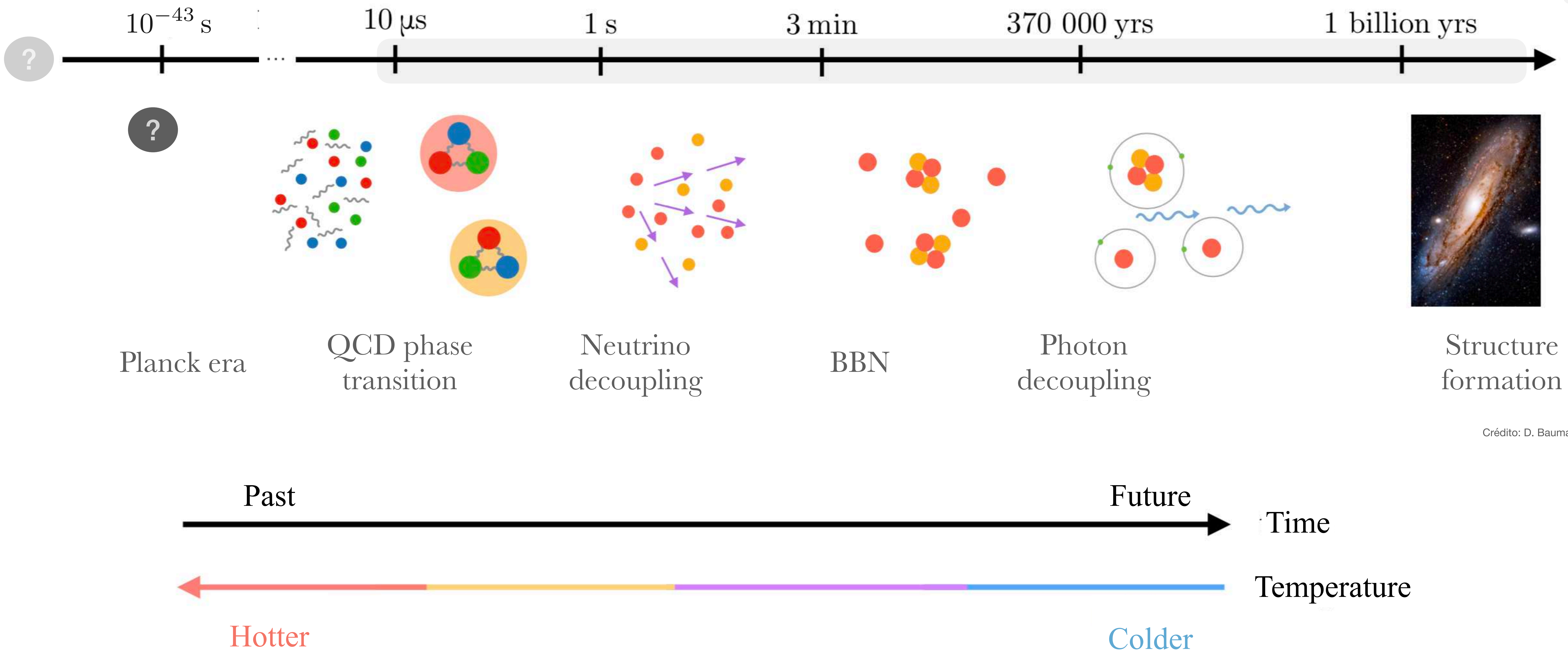
Thermal history



Thermal history of the universe

The universe“started" hot e **dense** → As it cools, the structures we know start to form

SCM

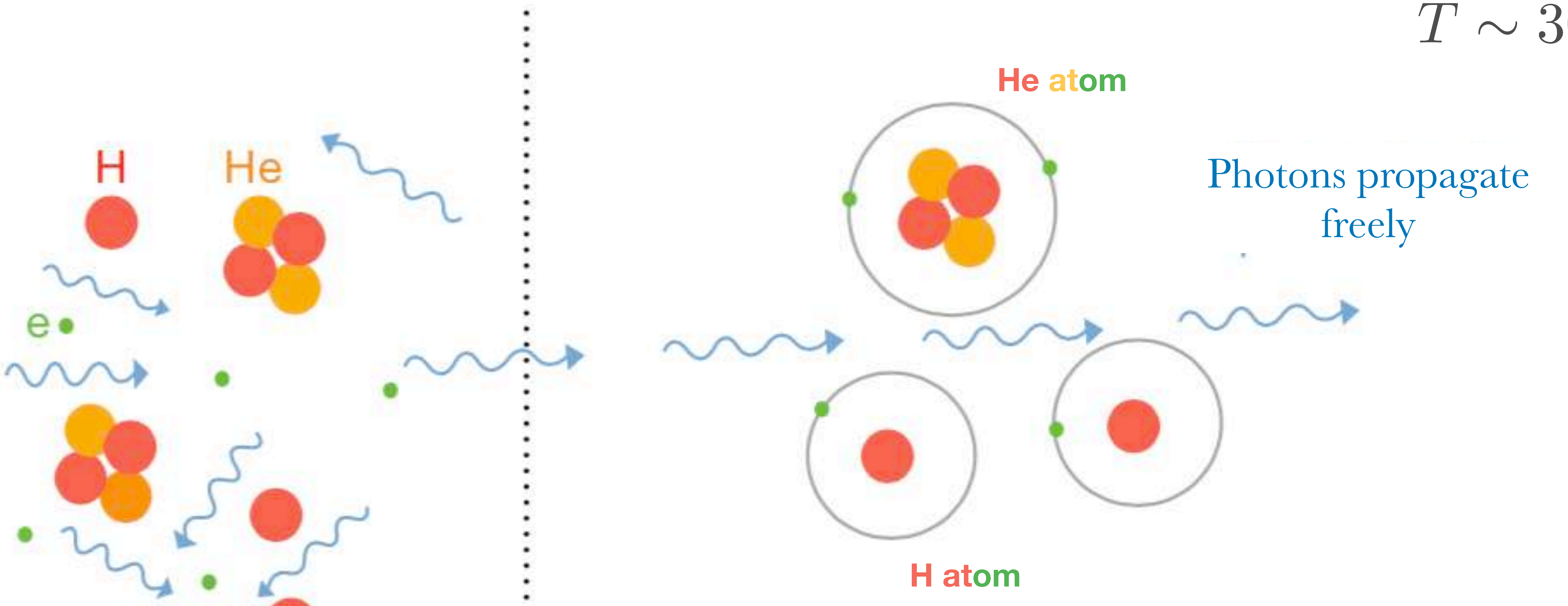


Crédito: D. Baumann

Recombination and photon decoupling

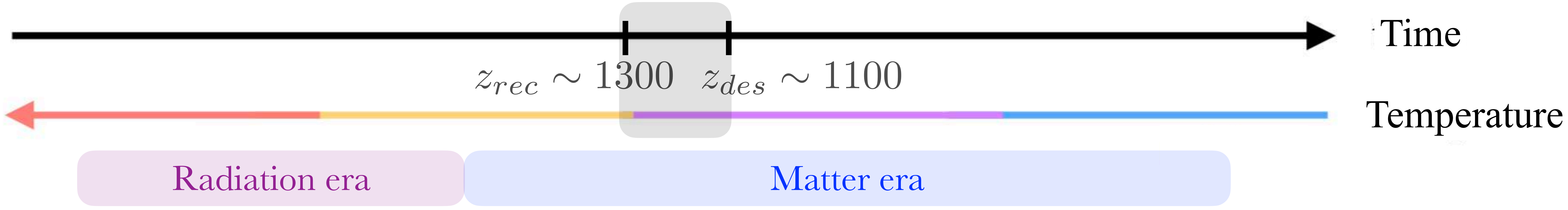
$t \sim 370000\,\mathrm{yrs}$

$T \sim 3000\,\mathrm{K}$



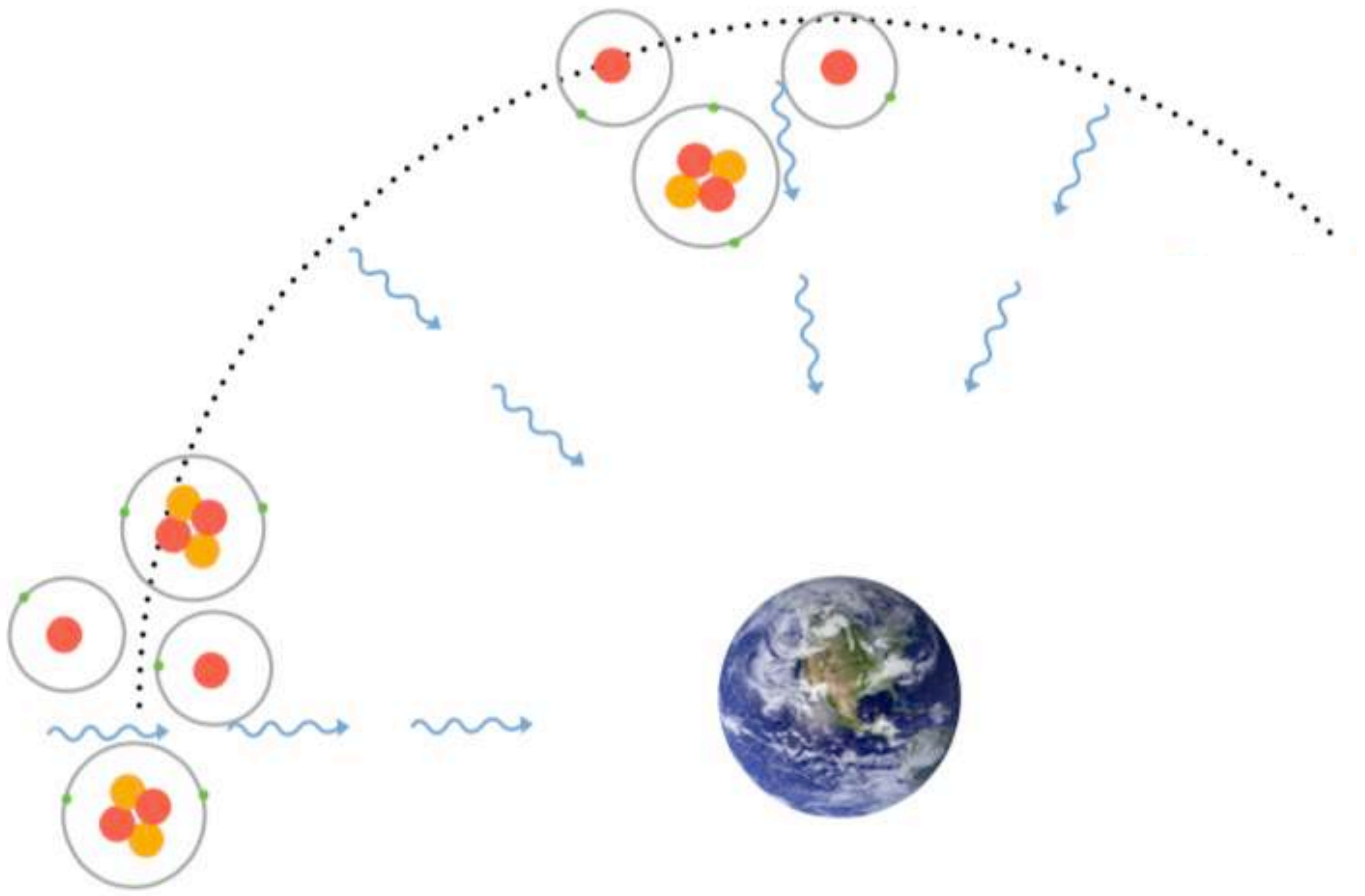
Plasma ("soup") of coupled H, He, elétrons and radiation - thermal equilibrium
- universe is opaque: radiation cannot scape!

Atoms are formed!
Charged electrons bound with n H and He nucleus



Crédito: D. Baumann

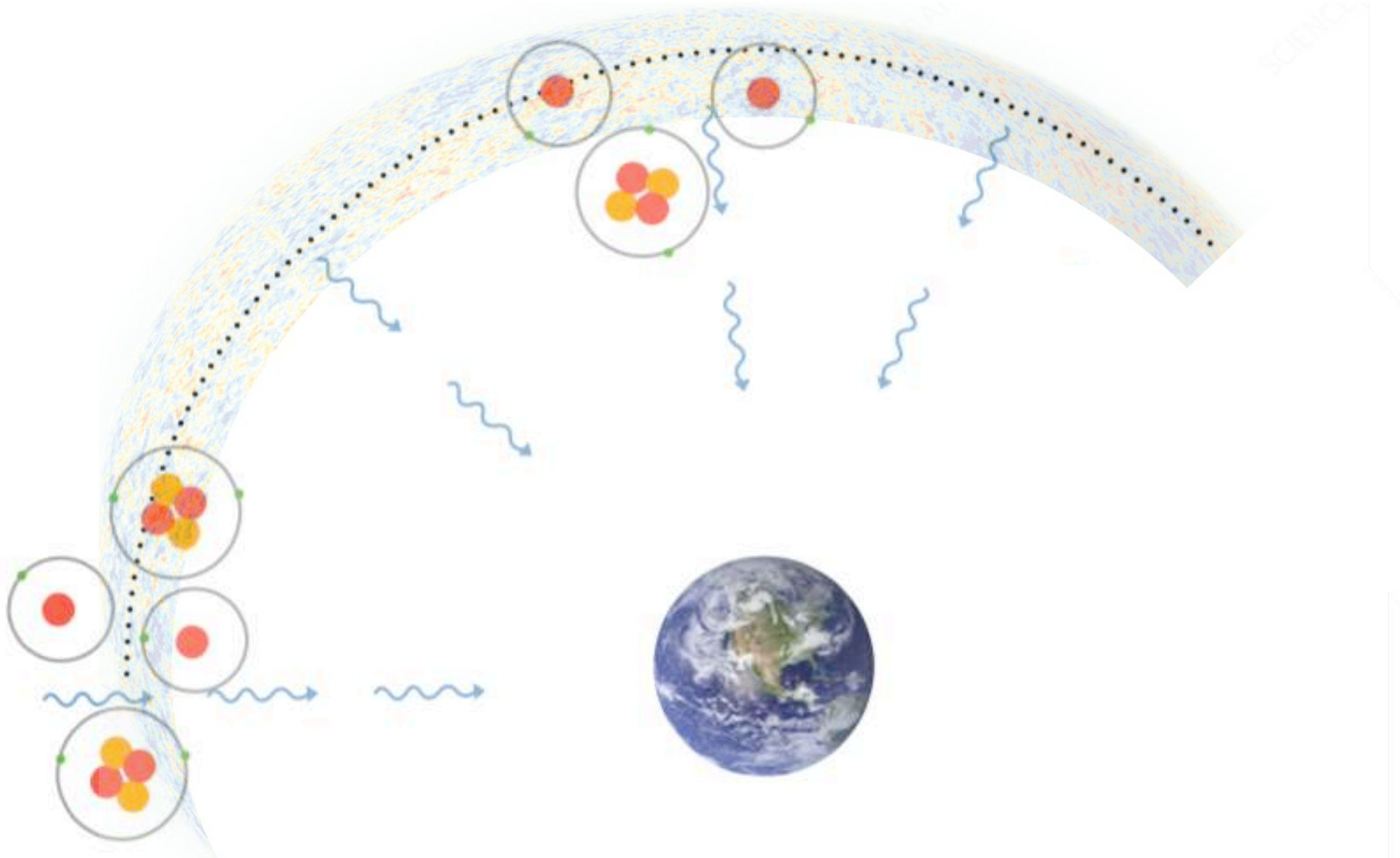
These photons are the first light of our universe…



Crédito: D. Baumann

… e tell us how the universe was at early times.

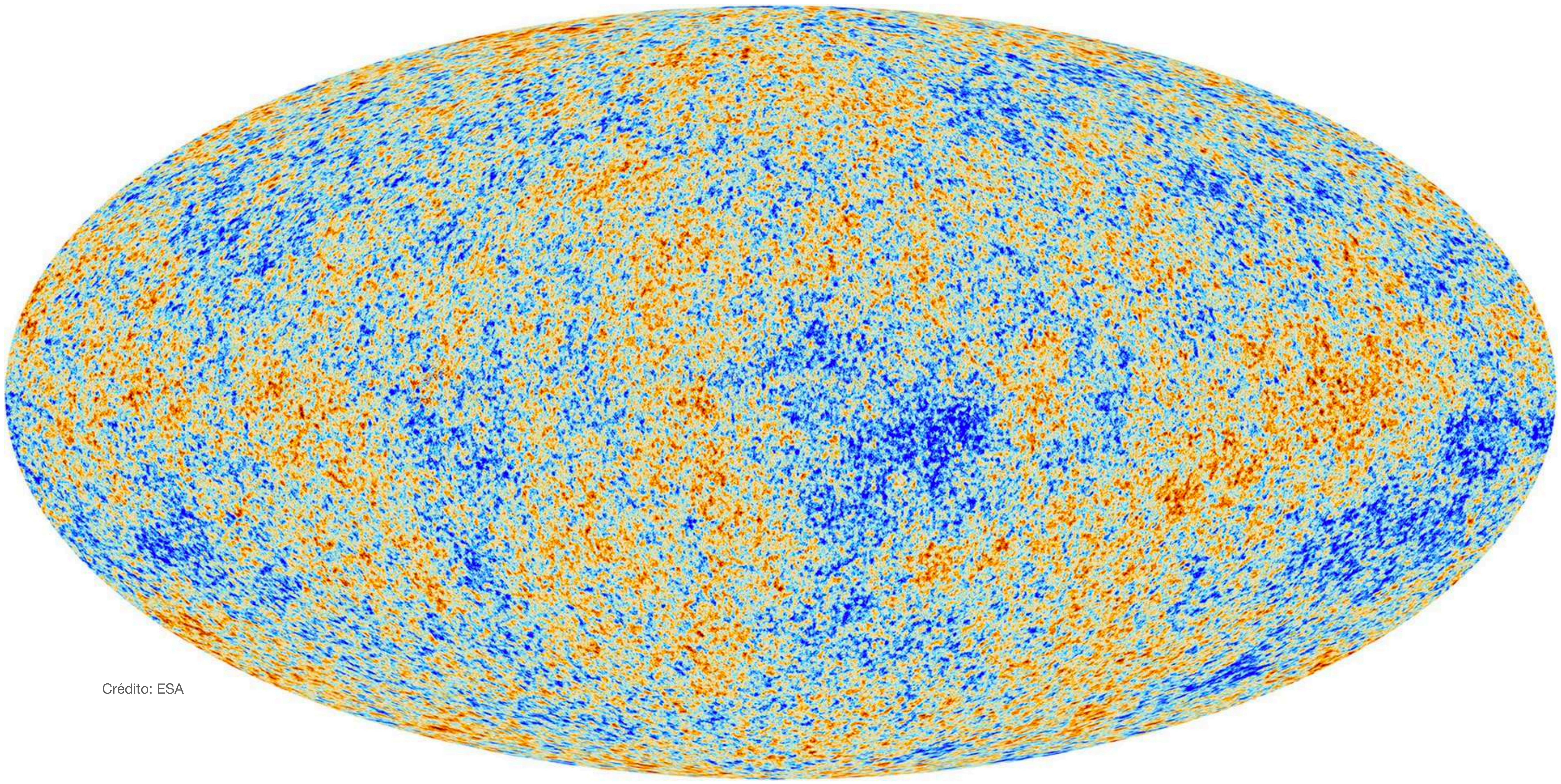
Cosmic **M**icrowave **B**ackground *(CMB)*



Crédito: D. Baumann

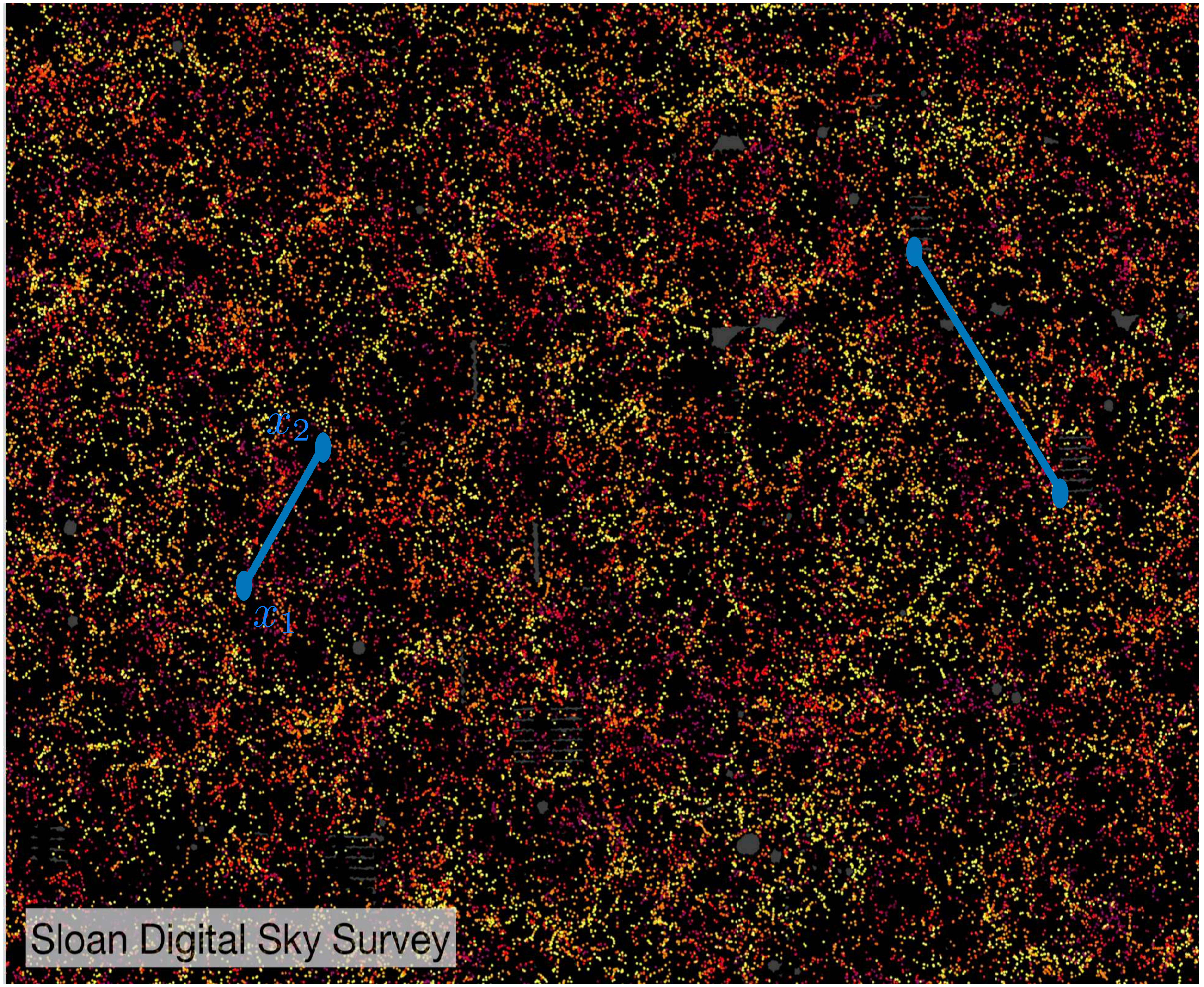
Given the expansion of the universe, we observe these photons in microwave.

Cosmic **M**icrowave **B**ackground *(CMB)*



Temperature 2.7 K. Small fluctuations - initial condition for the structures of our universe

Como medir as estruturas



Função de 2 pontos:

$$\langle \delta(x_1)\delta(x_2) \rangle$$

Se decompormos o contraste de densidade em modos de Fourier:

$$\delta(x) = \sum_k \delta_k \sin(kx + \phi_k)$$

$$k = 2\pi/\lambda$$

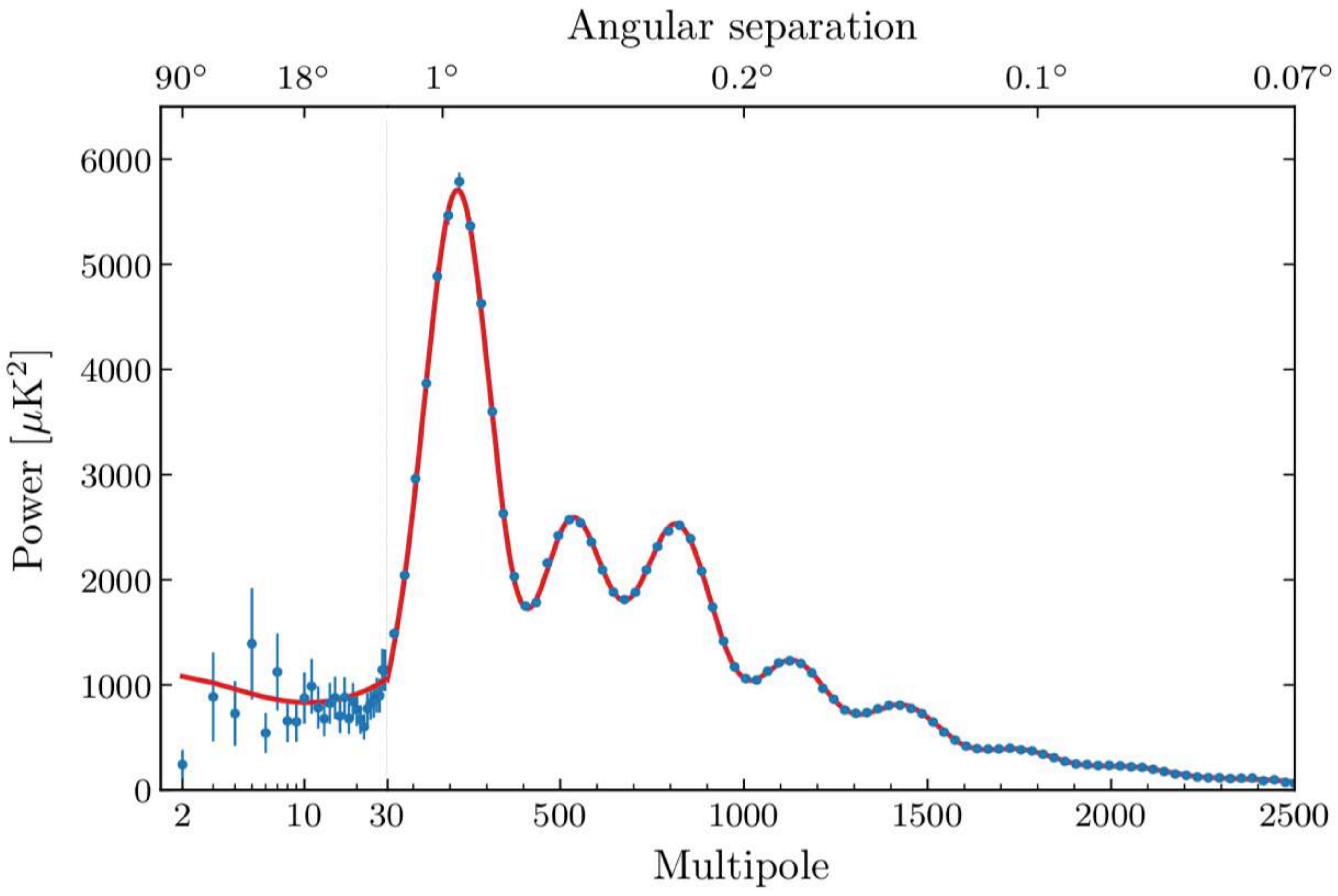
$$\Longrightarrow \quad \boxed{P(k) = |\delta_k|^2}$$

Espectro de potências

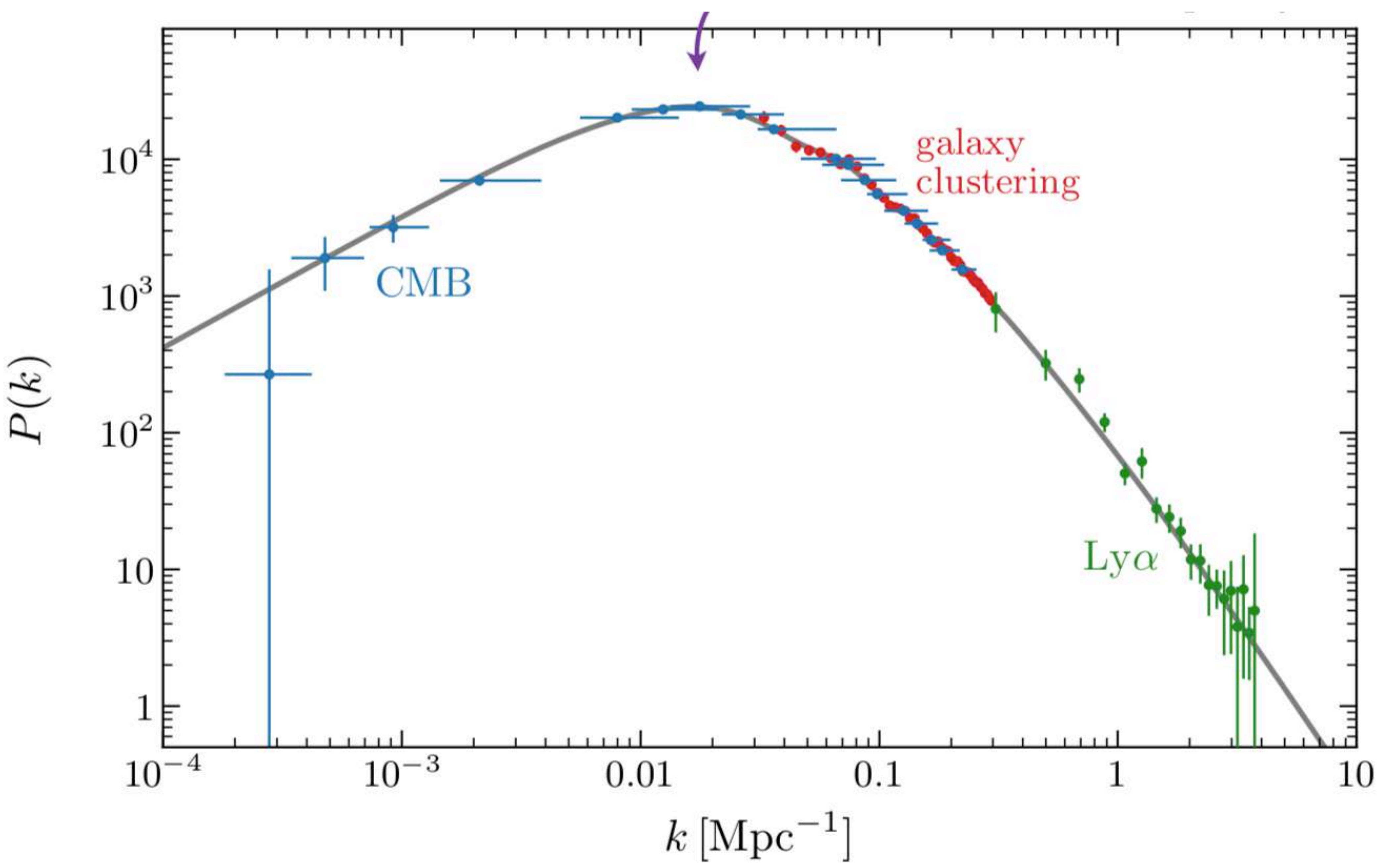
Um dos principais objetos estatísticos da cosmologia!

Espectro de potências

Radiação cósmica de fundo



Estrutura em largas escalas



Parâmetros cosmológicos

Modelo cosmológico padrão

Usando a RCF (e outras observações das estruturas em larga escala) podemos colocar vínculos nos parâmetros do MCP com incrível precisão.

Planck 2018

$\Omega_b = 0.0484 \pm 0.0003 \longrightarrow$ Quantidade de matéria visível/usual

$\Omega_m = 0.308 \pm 0.012 \longrightarrow$ Quantidade de matéria escura

$\Omega_\Lambda = 0.692 \pm 0.012 \longrightarrow$ Quantidade de energia escura

$\mathrm{n}_s = 0.9626 \pm 0.0057 \longrightarrow$ Dependência de escala das flutuações iniciais

$10^9\, A_s = 2.092 \pm 0.034 \longrightarrow$ Amplitude das flutuações iniciais

$\tau = 0.0522 \pm 0.0080 \longrightarrow$ Profundidade óptica

O quão opaco o Universo era para os fótons que viajam por ele.

Parâmetros cosmológicos

Modelo cosmológico padrão

Usando a RCF (e outras observações das estruturas em larga escala) podemos colocar vínculos nos parâmetros do MCP com incrível precisão.

Planck 2018

$$\Omega_b = 0.0484 \pm 0.0003 \quad \longrightarrow \quad \text{Quantidade de matéria visível/usual}$$
$$\Omega_m = 0.308 \pm 0.012 \quad \longrightarrow \quad \text{Quantidade de matéria escura}$$
$$\Omega_\Lambda = 0.692 \pm 0.012 \quad \longrightarrow \quad \text{Quantidade de energia escura}$$
$$\mathrm{n}_s = 0.9626 \pm 0.0057 \quad \longrightarrow \quad \text{Dependência de escala das flutuações iniciais}$$
$$10^9\, A_s = 2.092 \pm 0.034 \quad \longrightarrow \quad \text{Amplitude das flutuações iniciais}$$
$$\tau = 0.0522 \pm 0.0080 \quad \longrightarrow \quad \text{Profundidade óptica}$$

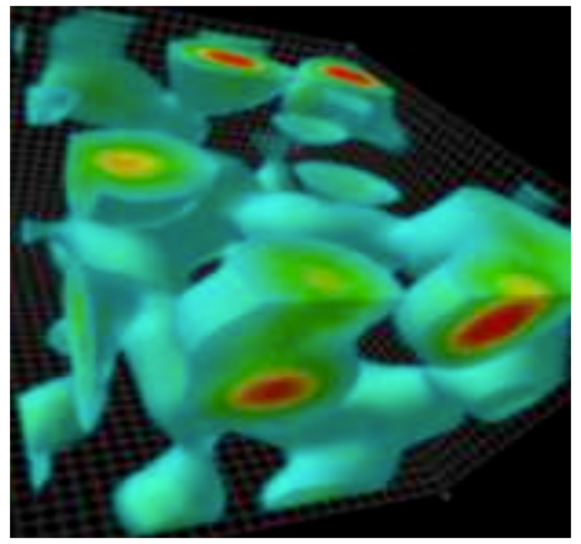
Relacionados com as perturbações iniciais - condições iniciais
Parametrizadas por:

$$P(k) = A_S \left(\frac{k}{k_*}\right)^{n_S - 1}$$

Where everything we see comes from?

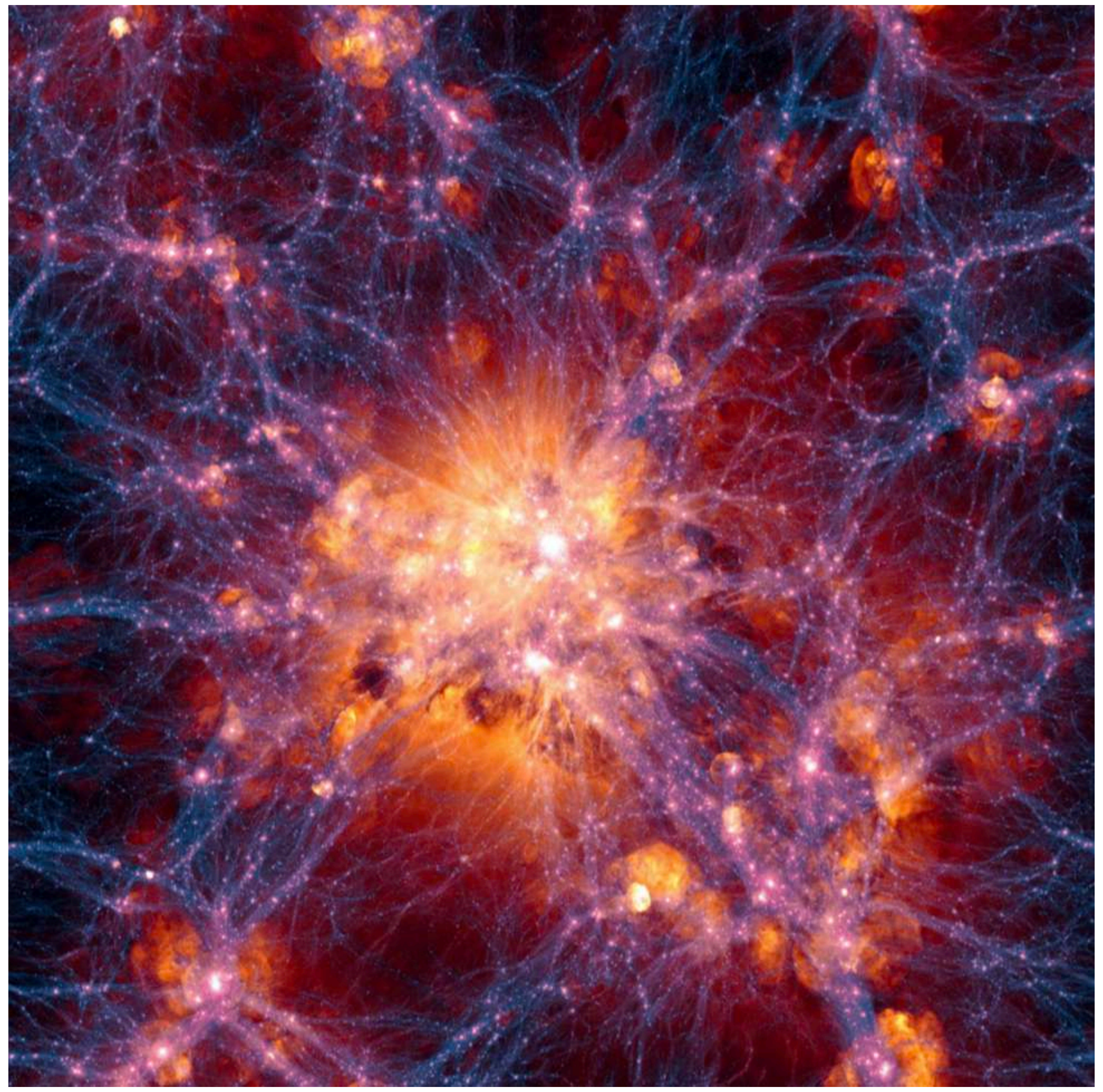
The answer comes from the interesting connection between the really small and really large…

?What is the origin of the initial density fluctuations?



Initial conditions
Initial perturbations

10^{-30} m

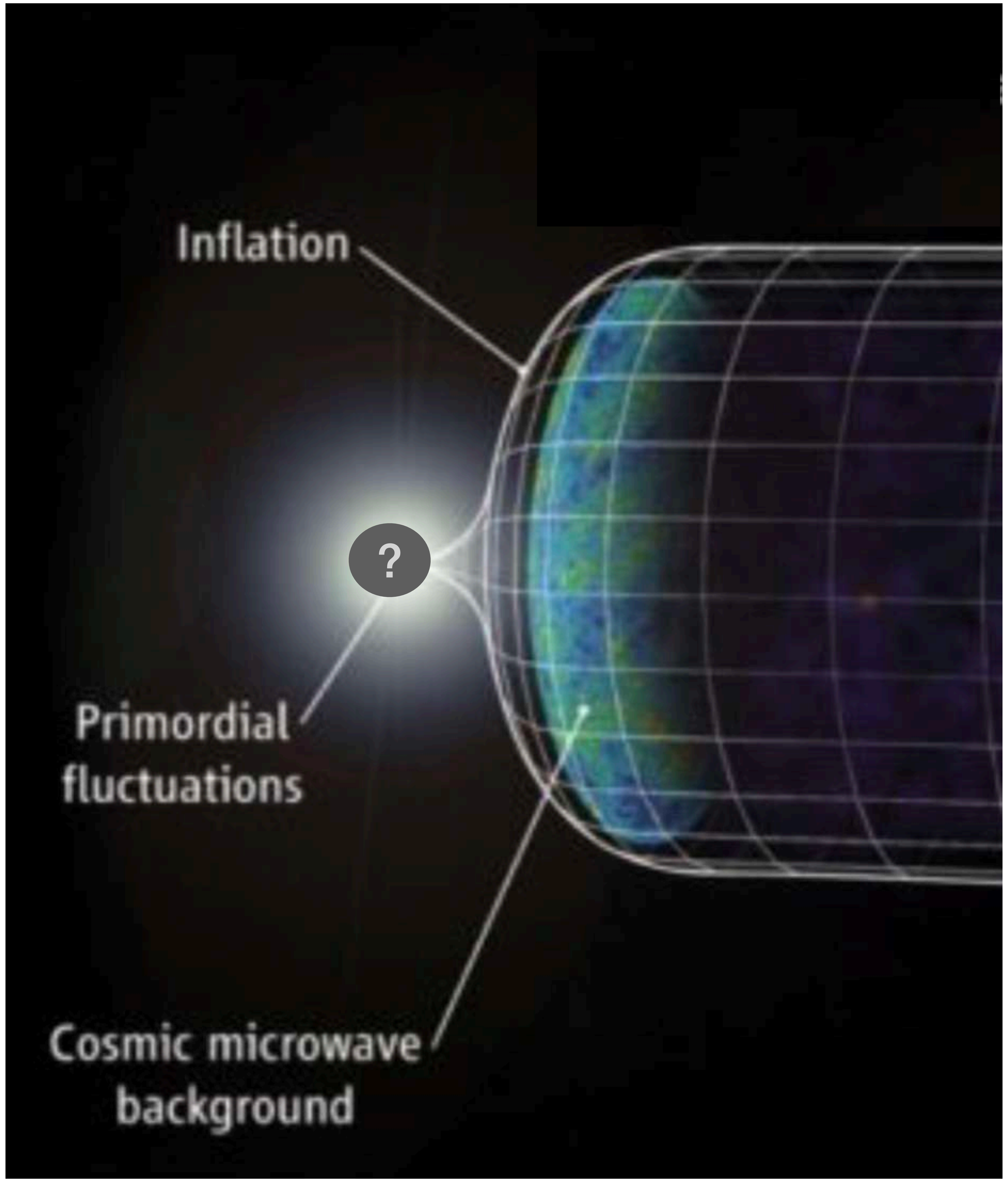


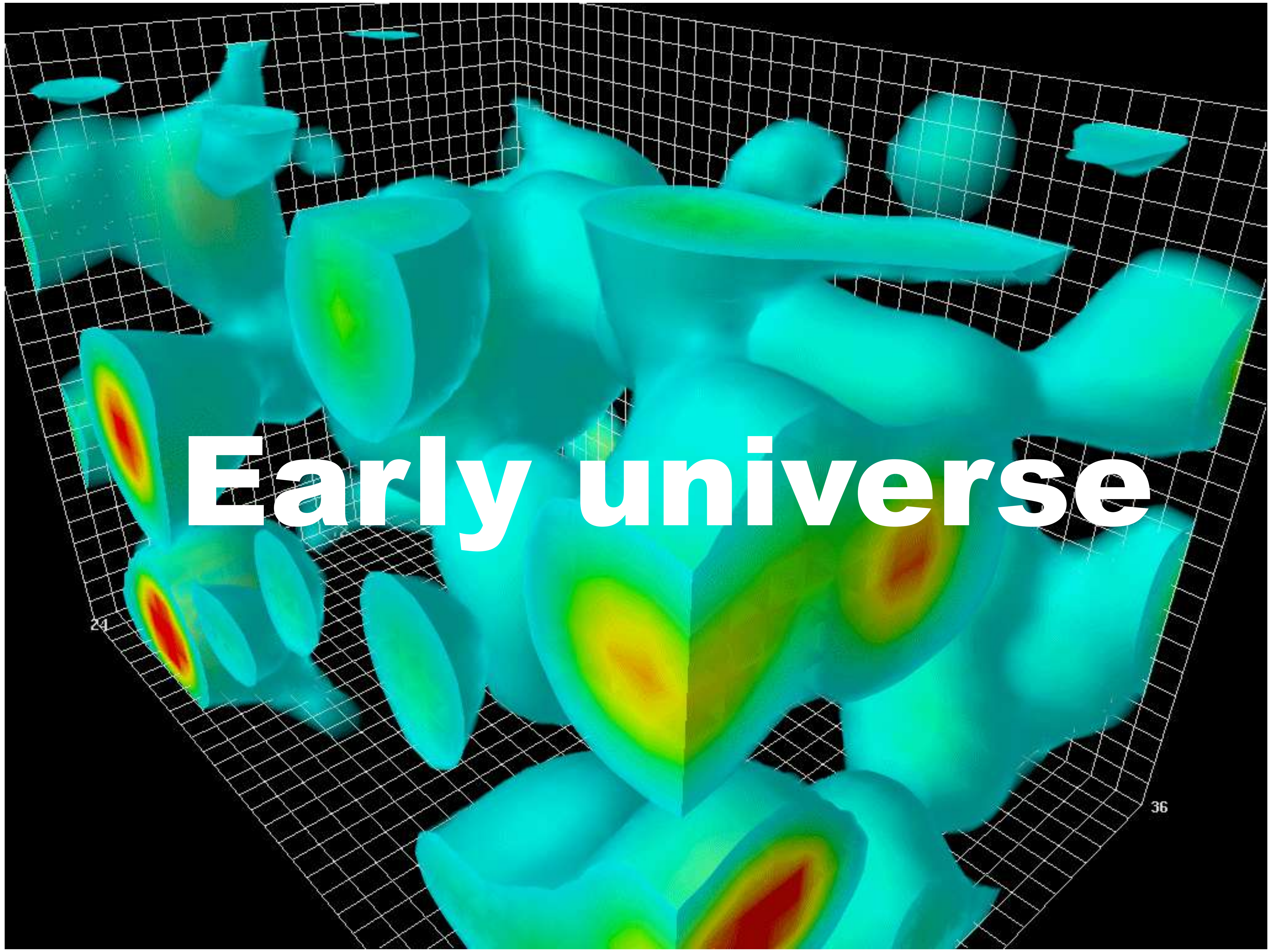
Structures of the universe

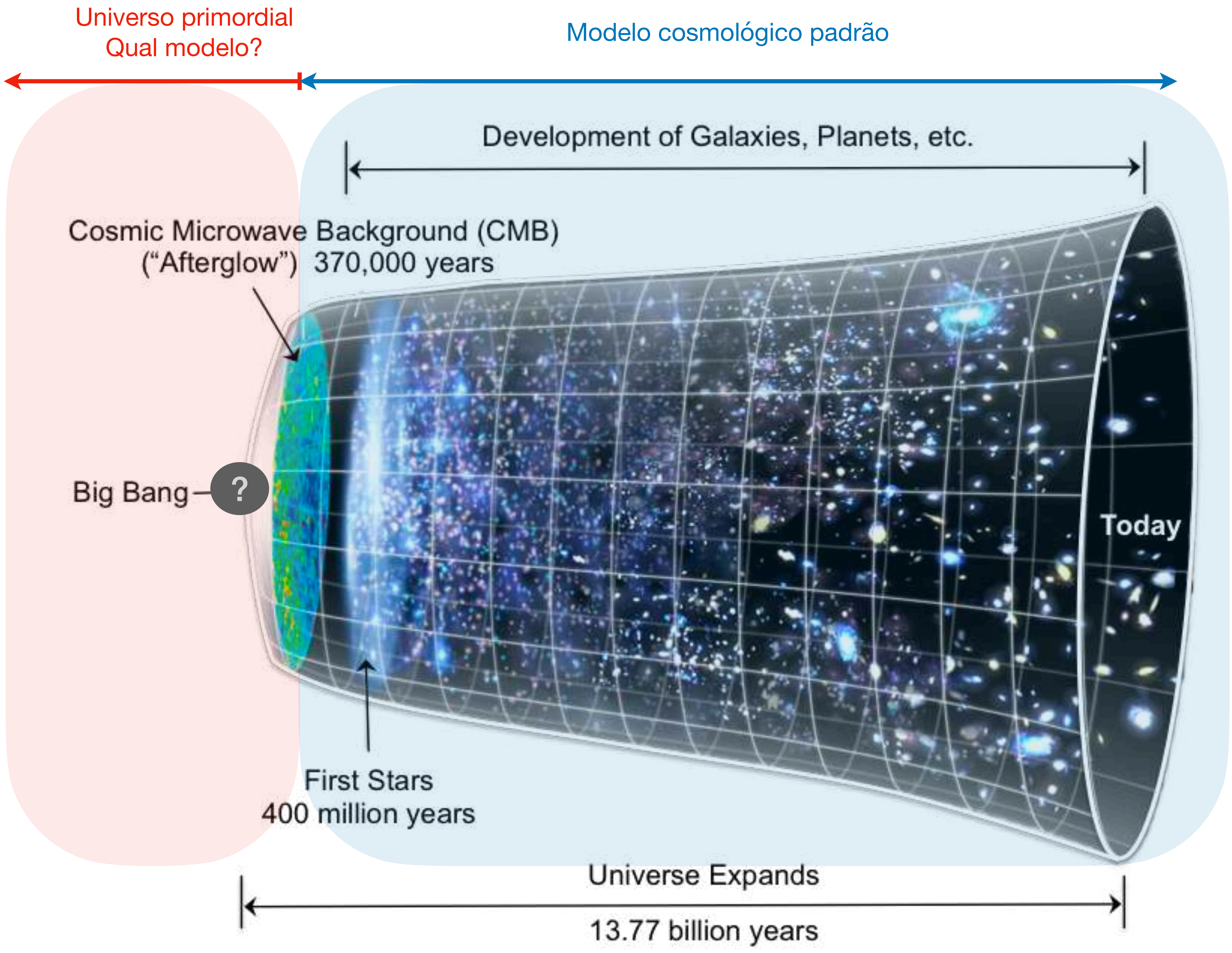
10^{25} m

Crédito: D. Baumann

This is going to depend on how was the evolution of the early universe…




Early universe
24
36



Universo primordial
Qual modelo?
Modelo cosmológico padrão
Development of Galaxies, Planets, etc.
Cosmic Microwave Background (CMB)
("Afterglow") 370,000 years
Big Bang
?
Today
First Stars
400 million years
Universe Expands
13.77 billion years

Problems of the standard cosmological model

- Horizon problem
- Problem of the origin of structures
- Flatness problem
- Problem of the magnetic monopoles
- Initial singularity
- DM and DE

Problems of the standard cosmological model

- Horizon problem
- Problem of the origin of structures
- Flatness problem
- Magnetic monopoles problem
- Initial singularity
- DM and DE

Horizon problem

Crédito: D. Baumann

Horizons in cosmology

Since the speed of light is constant and the universe is expanding, there is a limit for what is accessible to an observer in the universe.

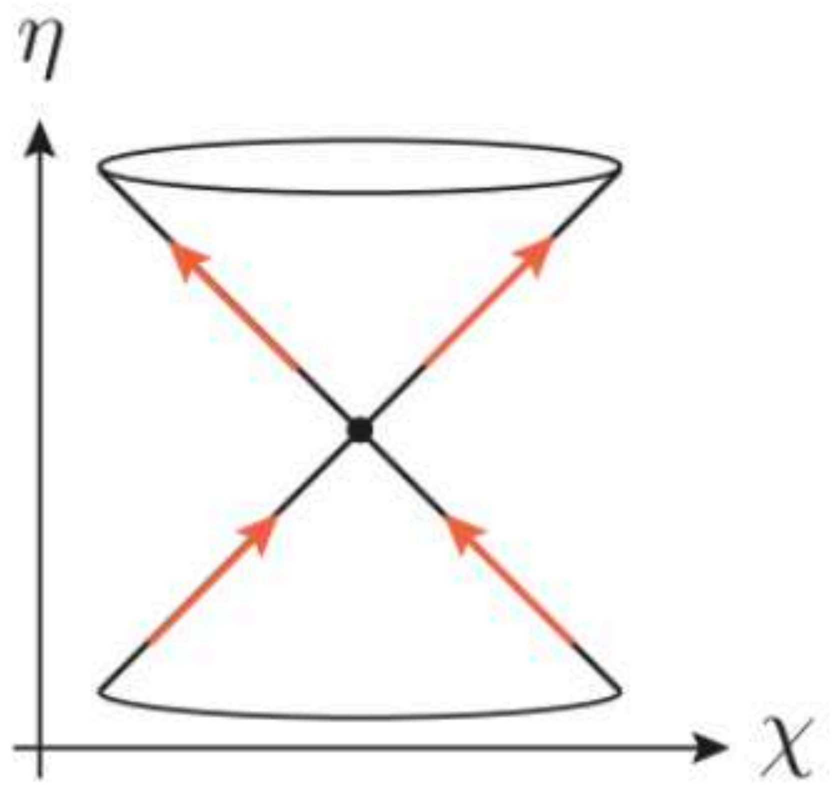
Because of isotropy, we can focus on purely radial geodesics ($d\theta = d\phi = 0$):

$$ds^2 = a^2(\eta)\left[d\eta^2 - d\chi^2\right].$$

Photons travel on null geodesics,

$$ds^2 = 0 \;\Rightarrow\; \Delta\chi = \pm\Delta\eta$$

(straight lines)



Conformal time $d\eta = dt/a(t)$

outgoing photons and the minus sign to incoming photons

* This shows the main benefit of working with conformal time: light rays correspond to straightlines at 45 degree angles in the-χ, ηcoordinates. If instead we had used physical timet, then the lightcones for curved spacetimes would be curved

Crédito: D. Baumann

Horizons in cosmology

Since the speed of light is constant and the universe is expanding, there is a limit for what is accessible to an observer in the universe.

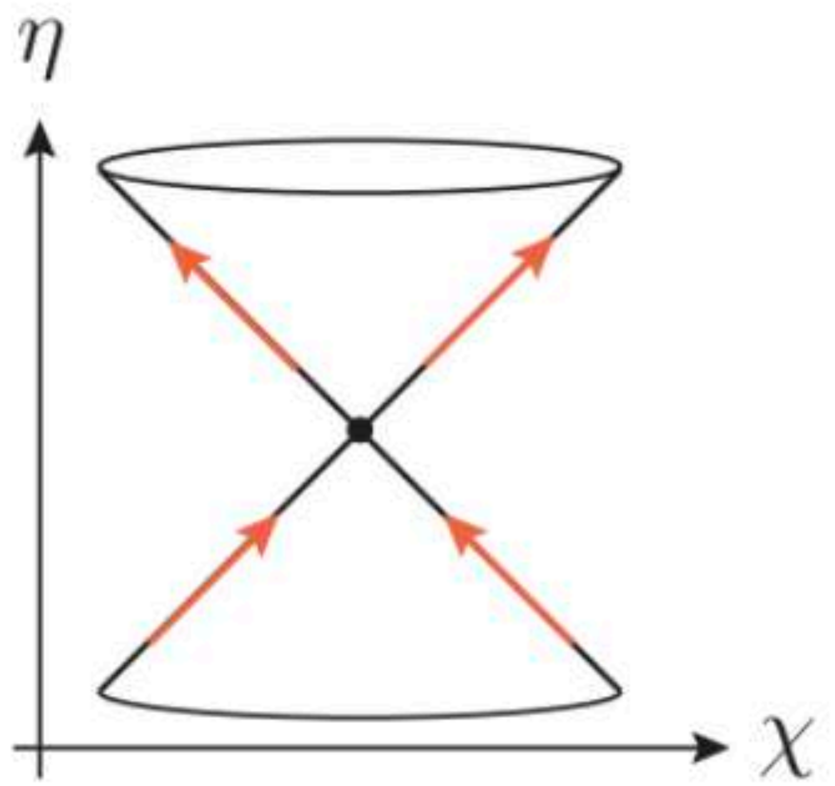
Because of isotropy, we can focus on purely radial geodesics ($d\theta = d\phi = 0$):

$$ds^2 = a^2(\eta)\left[d\eta^2 - d\chi^2\right].$$

Photons travel on null geodesics,

$$ds^2 = 0 \quad \Rightarrow \quad \Delta\chi = \pm\Delta\eta$$

(straight lines)



Conformal time $d\eta = dt/a(t)$

outgoing photons and the minus sign to incoming photons

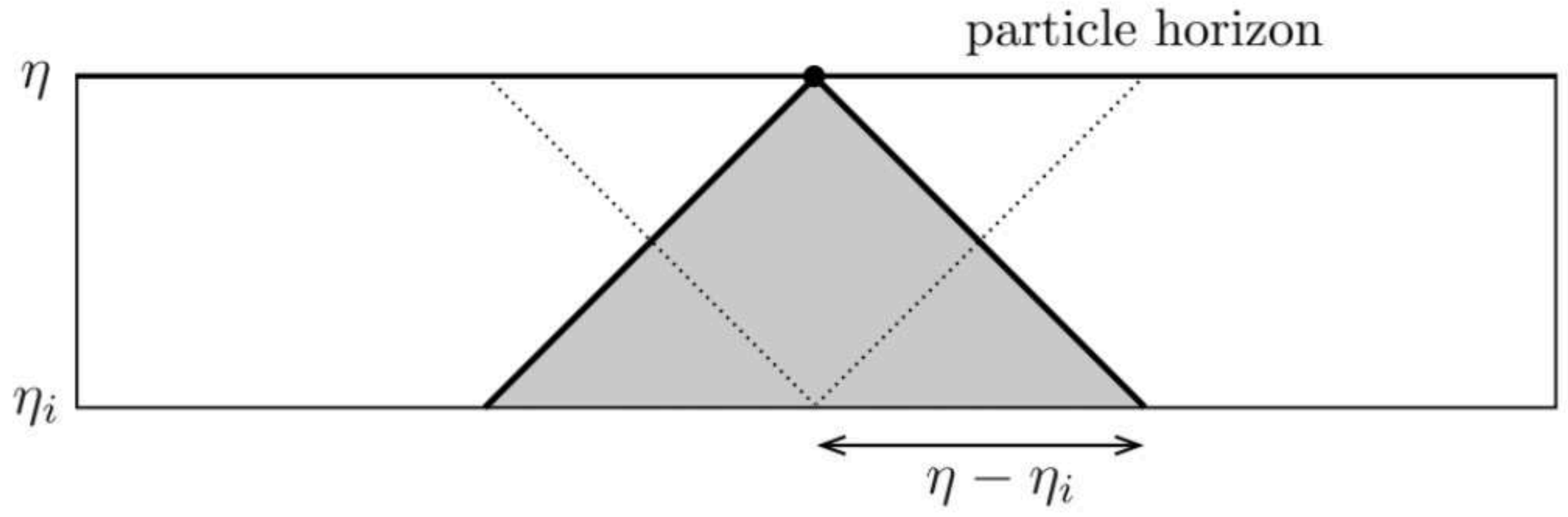
This tells us that: maximum distance light can travel between 2 times is $\Delta\eta = \eta_2 - \eta_1$ $(c = 1)$

Crédito: D. Baumann

Horizons in cosmology

Since the speed of light is constant and the universe is expanding, there is a limit for what is accessible to an observer in the universe.

Particle horizon: distance that the light travelled since the Big BangParticle horizon



$$\chi_p(\eta) = \eta - \eta_i = \int_{t_i}^{t} \frac{dt}{a(t)} = \int_{\ln a_i}^{\ln a} \frac{d\ln a}{\dot{a}} = \int_{\ln a_i}^{\ln a} (aH)^{-1}\, d\ln a$$

(comoving) Hubble radius

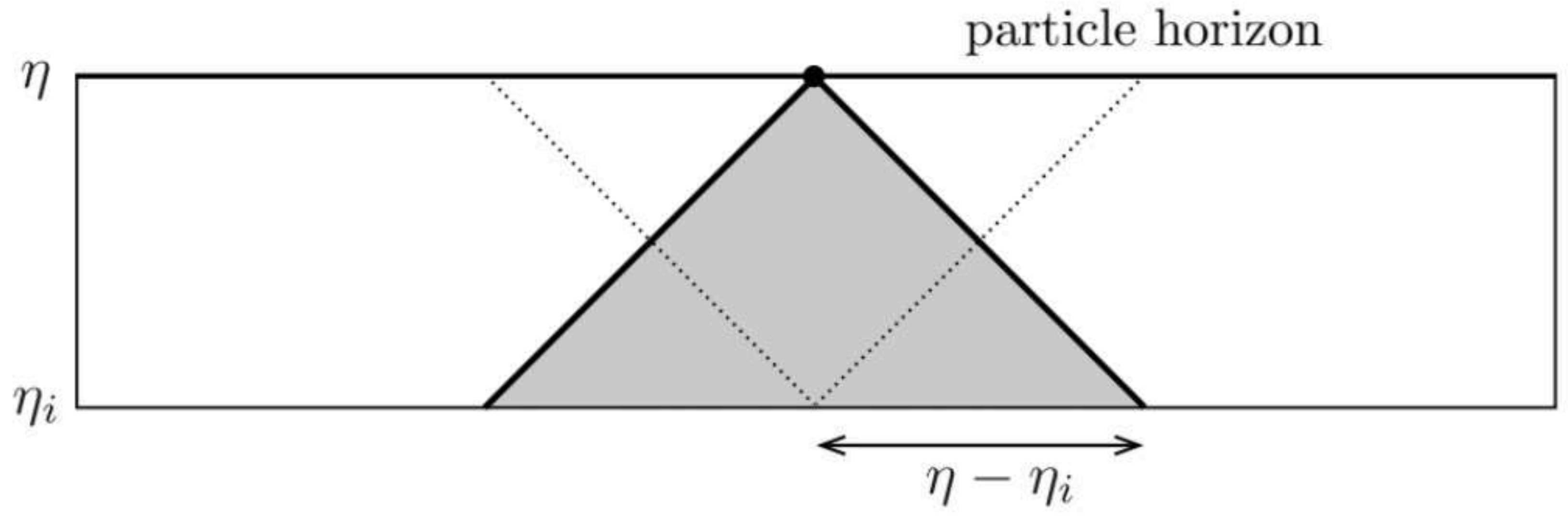
Conformal time $d\eta = dt/a(t)$

If the Big Bang started at t=0 the greatest comoving distance from which an observer at time t will be able to receive signals travelling at the speed of light is given by χ_p - comoving particle horizon

Horizons in cosmology

Since the speed of light is constant and the universe is expanding, there is a limit for what is accessible to an observer in the universe.

Particle horizon: distance that the light travelled since the Big BangParticle horizon



(comoving) Hubble radius

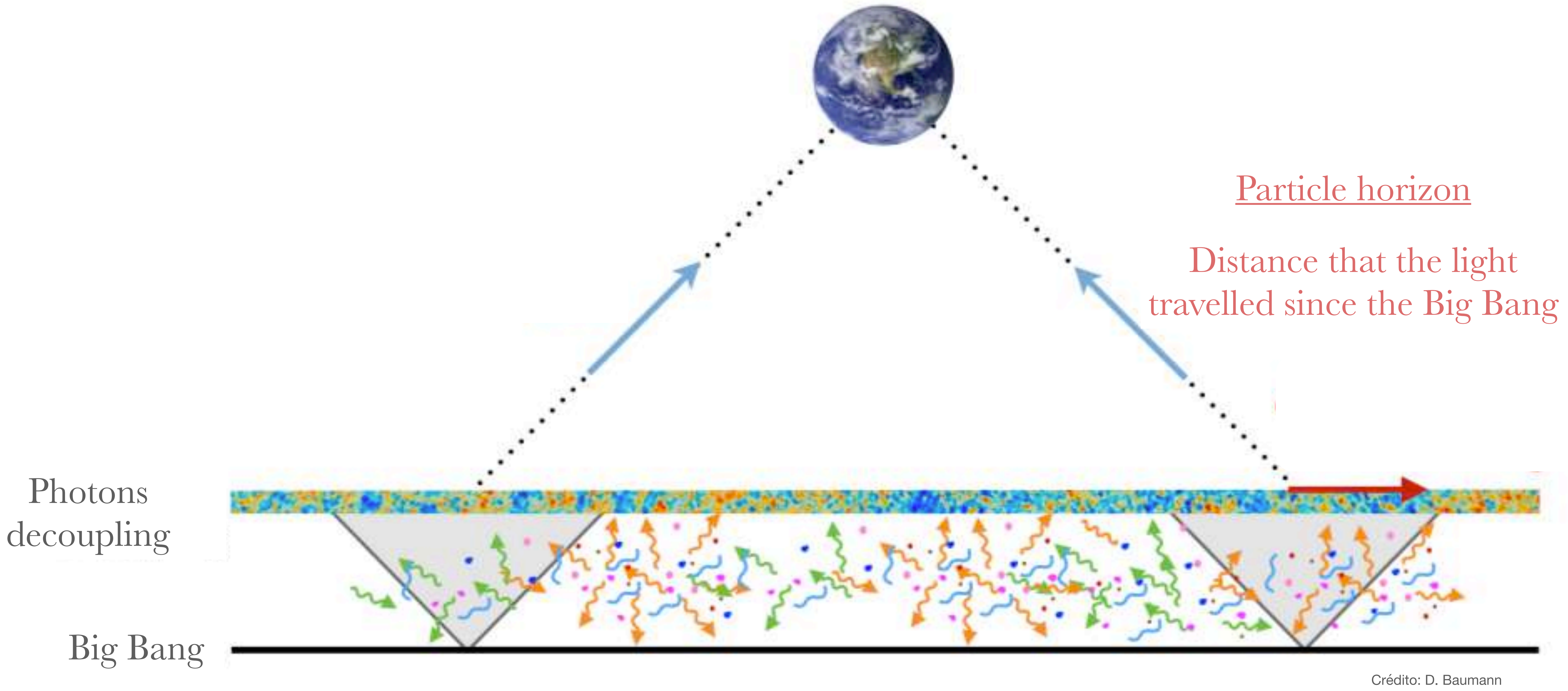
$$\chi_p(\eta) = \eta - \eta_i = \int_{t_i}^{t} \frac{dt}{a(t)} = \int_{\ln a_i}^{\ln a} \frac{d\ln a}{\dot{a}} = \int_{\ln a_i}^{\ln a} (aH)^{-1}\, d\ln a$$

Conformal time $d\eta = dt/a(t)$

The size of the particle horizon at η is the intersection of the past light cone of an observer O with the spacelike surface $\eta = \eta_i$

Horizons in cosmology

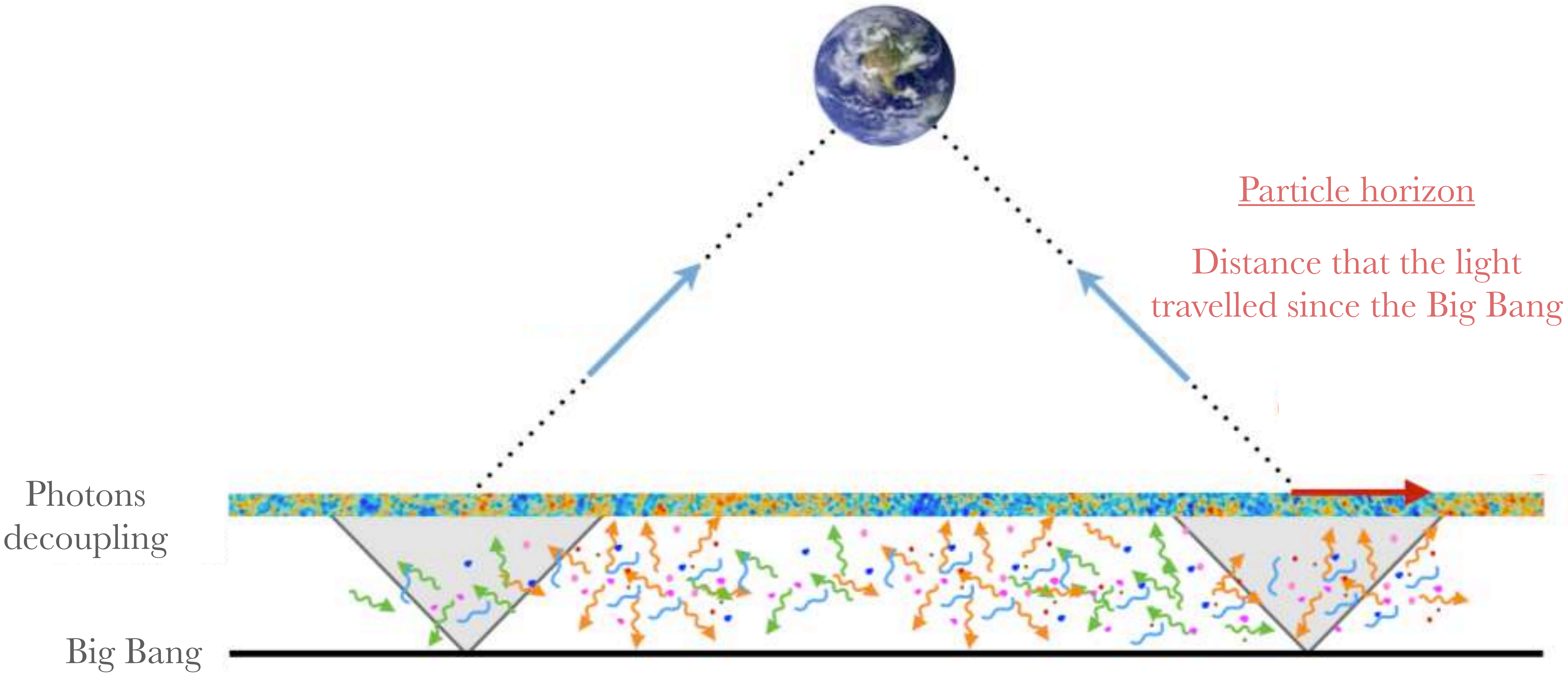
Since the speed of light is constant and the universe is expanding, there is a limit for what is accessible to an observer in the universe.



This limit of what can be observed is known as horizon.

Horizons in cosmology

Since the speed of light is constant and the universe is expanding, there is a limit for what is accessible to an observer in the universe.



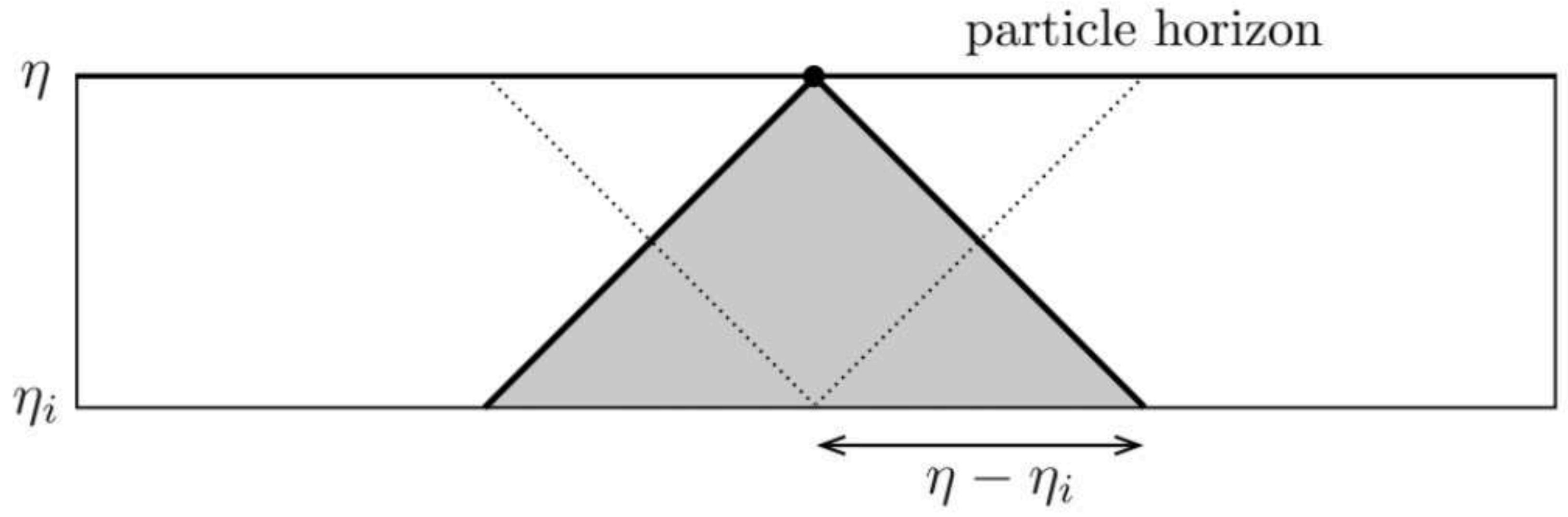
Crédito: D. Baumann

* Notice that the Big Bang singularity is a moment in time (not a point in space).
Figure: singularity described by an extended (possibly infinite) spacelike hypersurface

Horizons in cosmology

Since the speed of light is constant and the universe is expanding, there is a limit for what is accessible to an observer in the universe.

Particle horizon: distance that the light travelled since the Big BangParticle horizon



(comoving) Hubble radius

$$\chi_p(\eta) = \eta - \eta_i = \int_{t_i}^{t} \frac{dt}{a(t)} = \int_{\ln a_i}^{\ln a} \frac{d\ln a}{\dot{a}} = \int_{\ln a_i}^{\ln a} (aH)^{-1}\, d\ln a$$

Universe dominated by a fluid with $w = p/\rho$: $(aH)^{-1} = H_0^{-1} a^{(1+3w)/2}$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad (><)0$$

Strong energy condition (SEC) >0

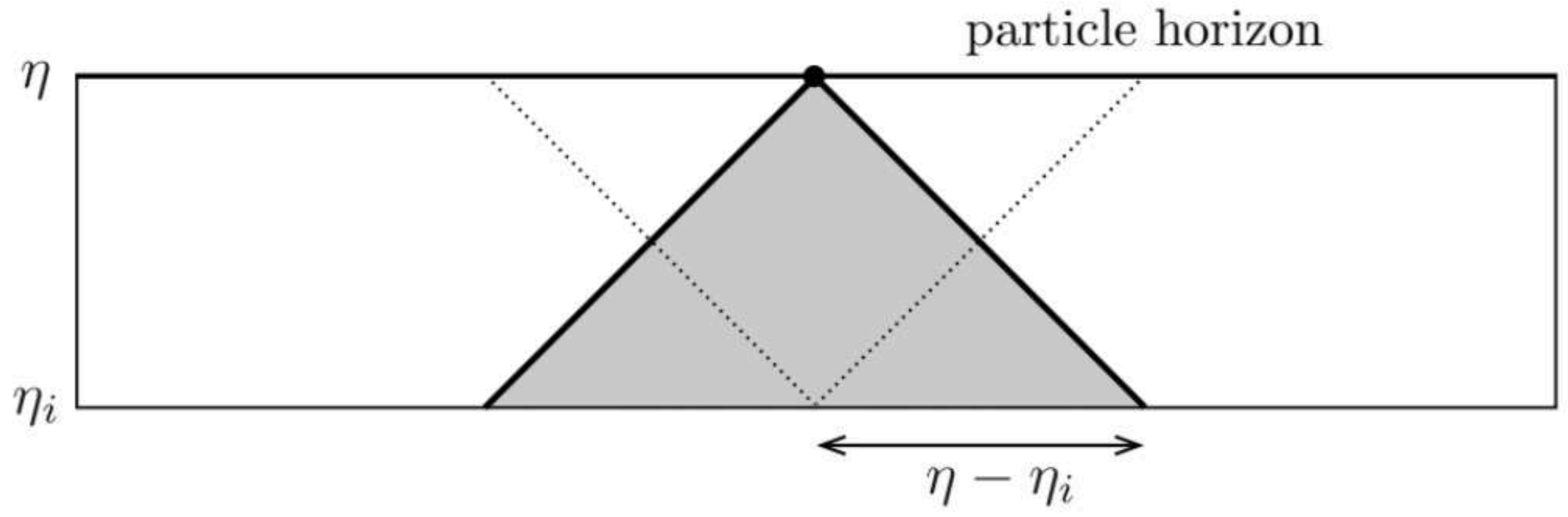
Ordinary matter $(aH)^{-1} \uparrow$

$$\chi_p = \frac{2}{1+3w}(aH)^{-1}$$

Horizons in cosmology

Since the speed of light is constant and the universe is expanding, there is a limit for what is accessible to an observer in the universe.

Particle horizon: distance that the light travelled since the Big BangParticle horizon



(comoving) Hubble radius

$$\chi_p(\eta) = \eta - \eta_i = \int_{t_i}^{t} \frac{dt}{a(t)} = \int_{\ln a_i}^{\ln a} \frac{d\ln a}{\dot{a}} = \int_{\ln a_i}^{\ln a} (aH)^{-1}\, d\ln a$$

Universe dominated by a fluid with $w = p/\rho$:

$$(aH)^{-1} = H_0^{-1} a^{(1+3w)/2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad (><)0$$

Strong energy condition (SEC) >0

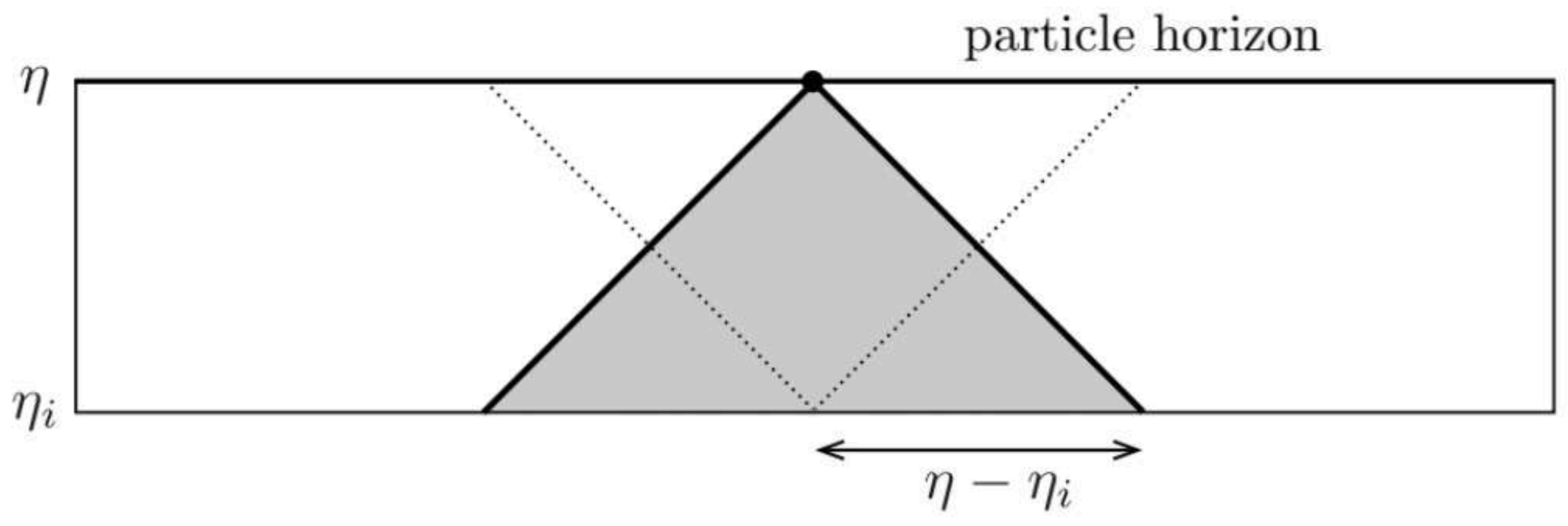
Ordinary matter $(aH)^{-1} \uparrow$

$$\chi_p = \frac{2}{1+3w}(aH)^{-1}$$

If we define
$\eta_i = 0 \Rightarrow \chi_p = \eta$
i.e. finite and dominated at late times

Horizons in *cosmology*

Particle horizon: distance that the light travelled since the Big BangParticle horizon



(comoving)
Hubble radius

$$\chi_p(\eta) = \eta - \eta_i = \int_{t_i}^{t} \frac{dt}{a(t)} = \int_{\ln a_i}^{\ln a} \frac{d\ln a}{\dot{a}} = \int_{\ln a_i}^{\ln a} (aH)^{-1}\, d\ln a$$

Universe dominated by a fluid with $w = p/\rho$:

$$(aH)^{-1} = H_0^{-1} a^{(1+3w)/2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad (><)0$$

Strong energy condition (SEC)
>0

Ordinary matter $(aH)^{-1} \uparrow$

$$\chi_p = \frac{2}{1+3w}(aH)^{-1}$$

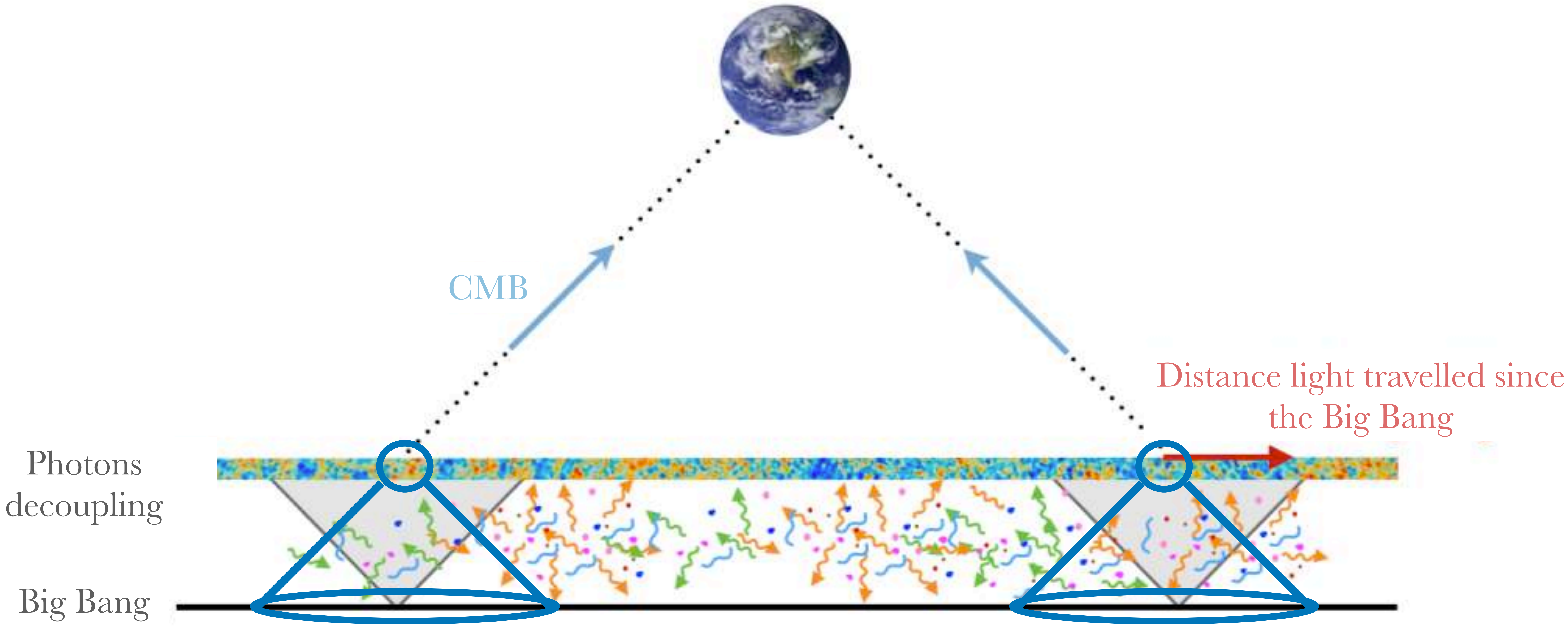
For this reason, $\chi_p \sim (aH)^{-1}$

usually people refer to the particle horizon and Hubble radius as “horizon”.

DON’T do that!

Horizon problem

As we saw, the CMB presents the same temperature in every point of the observable universe, except from small deviations

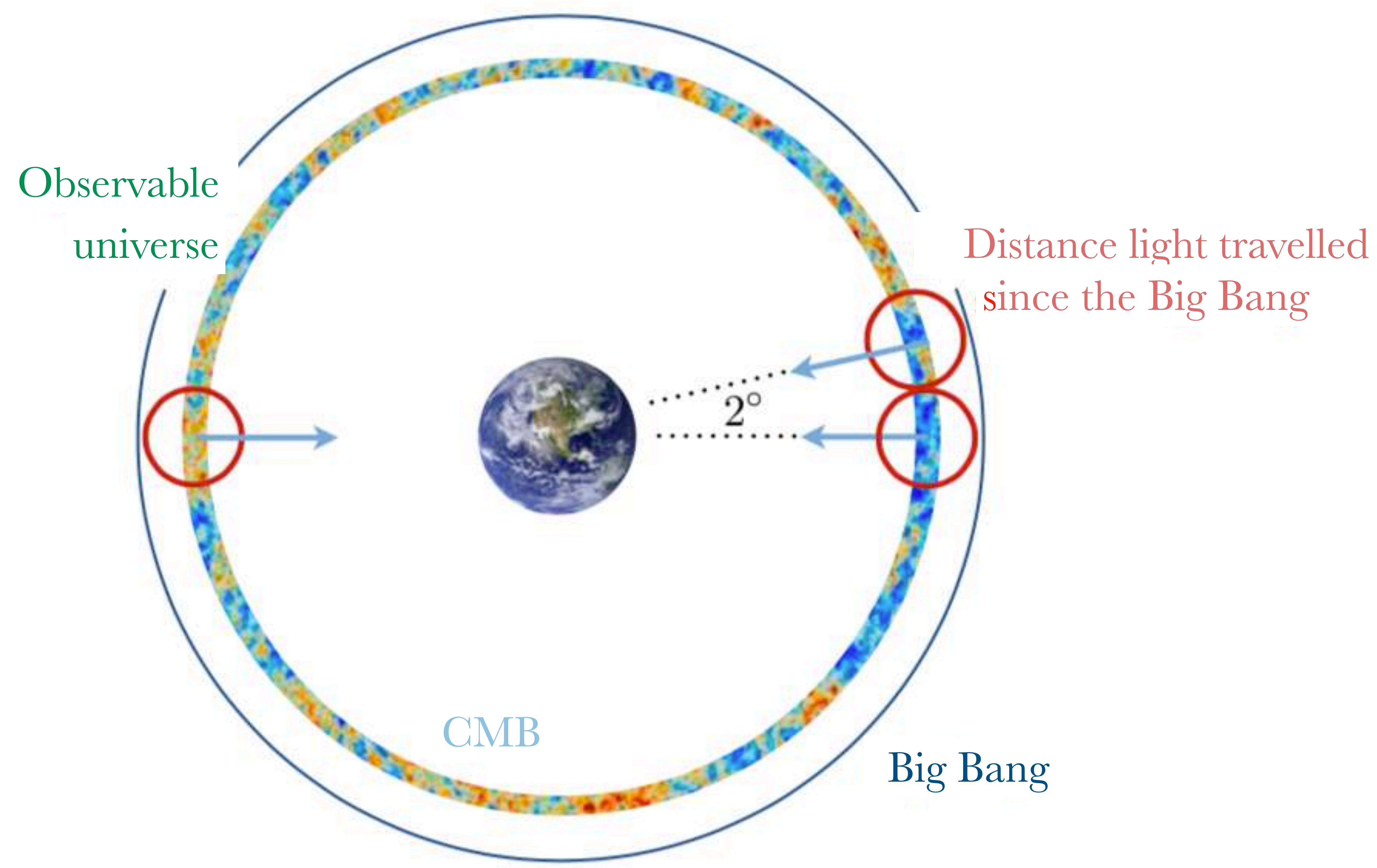


?

However, since there is a particle horizon today, HOW regions that are not in causal contact in the past can present the same characteristics?

Crédito: D. Baumann

Horizon problem

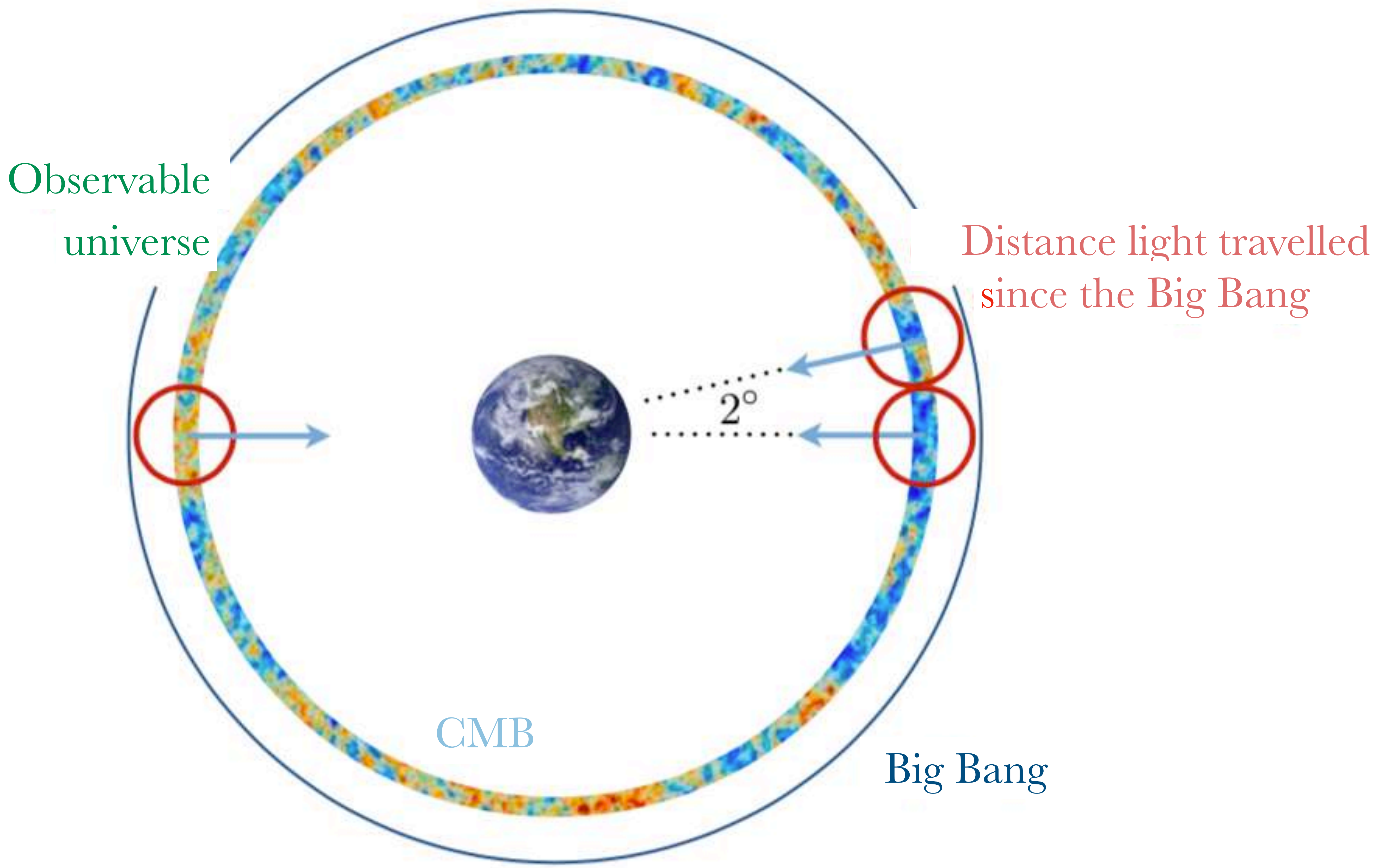


The CMB is made of $10^4 - 10^6$ causally disconnected regions, yet it is observed to be almost perfectly uniform!?

= horizon problem!

Crédito: D. Baumann

Horizon problem

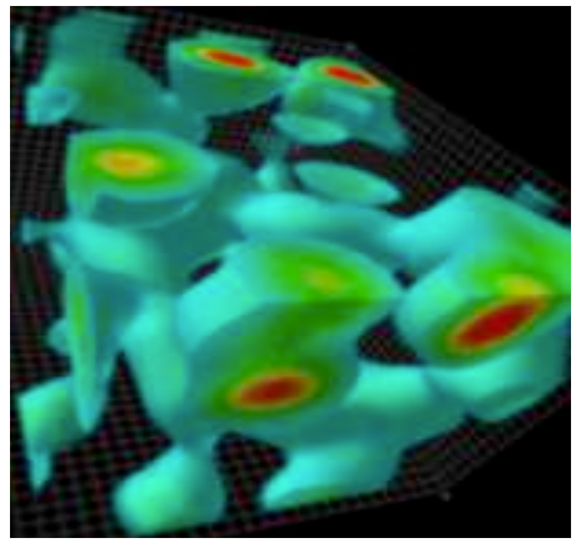


horizon problem

also known as homogeneity and isotropy problem

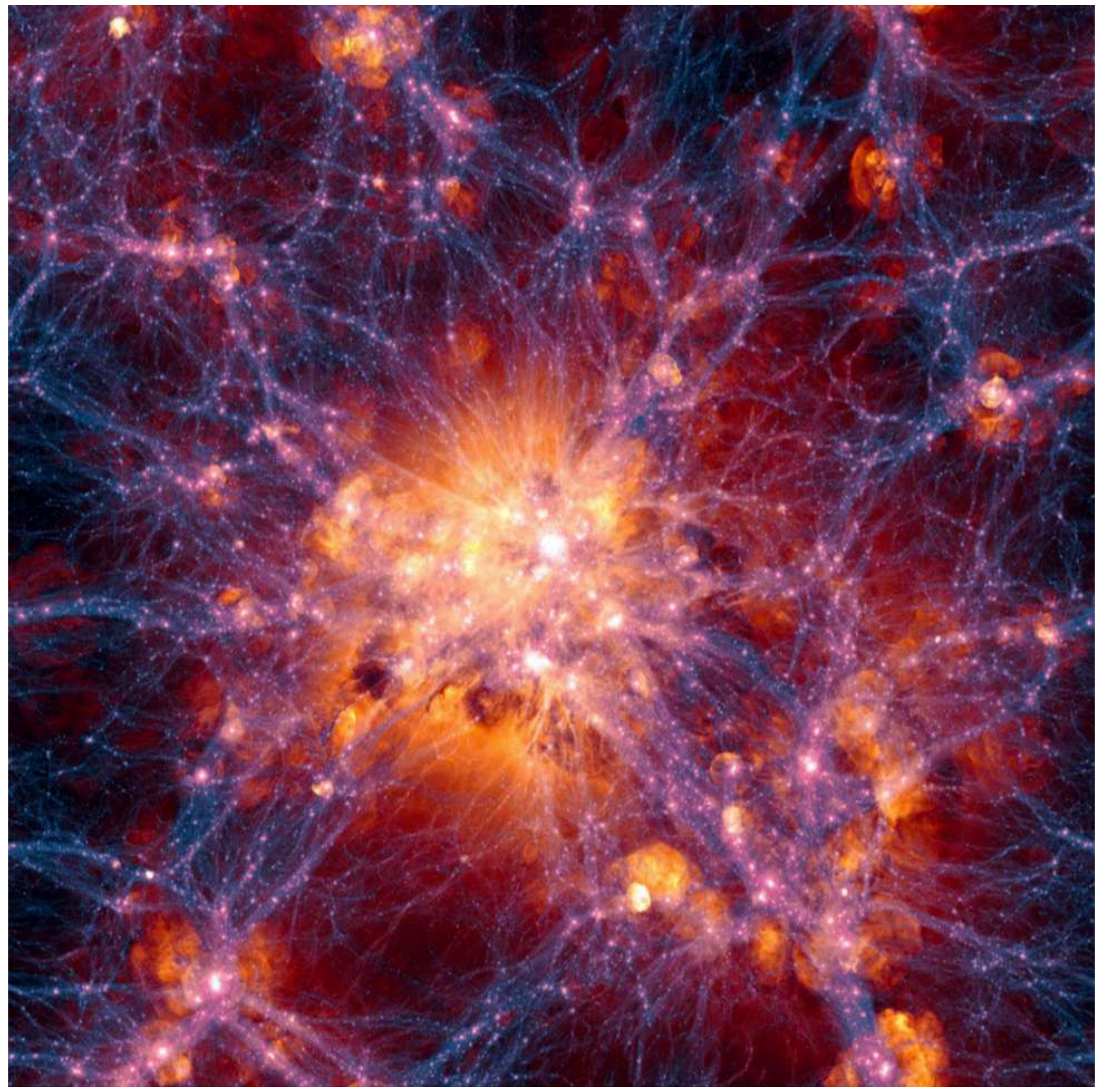
Crédito: D. Baumann

Problem of the origin of structures



Initial conditions
Initial perturbations

10^{-30} m



Structures of the universe

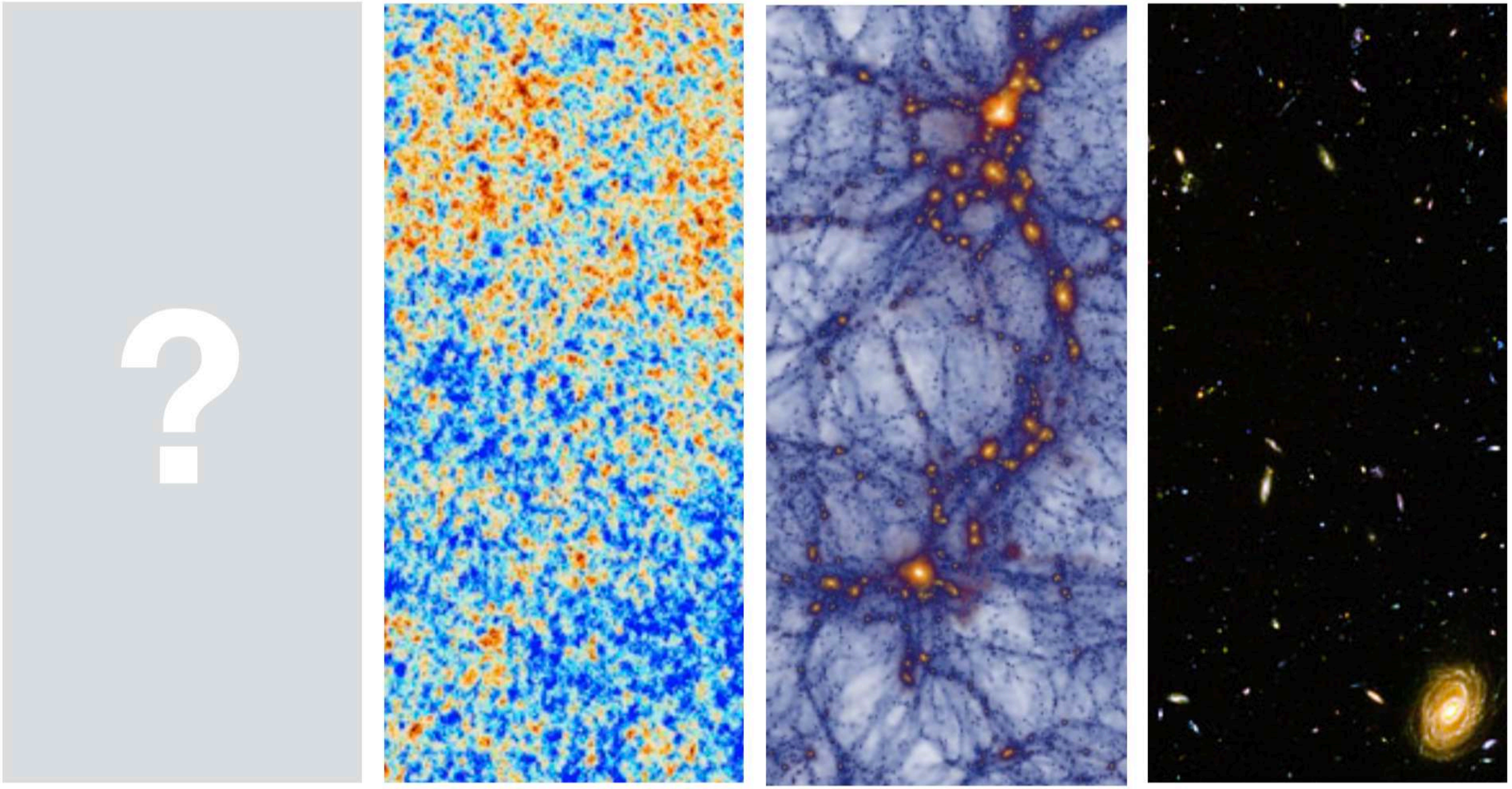
10^{25} m

Crédito: D. Baumann

Problem of the origin of structures

Origin of the small perturbations → Small perturbations → Macroscopic structures

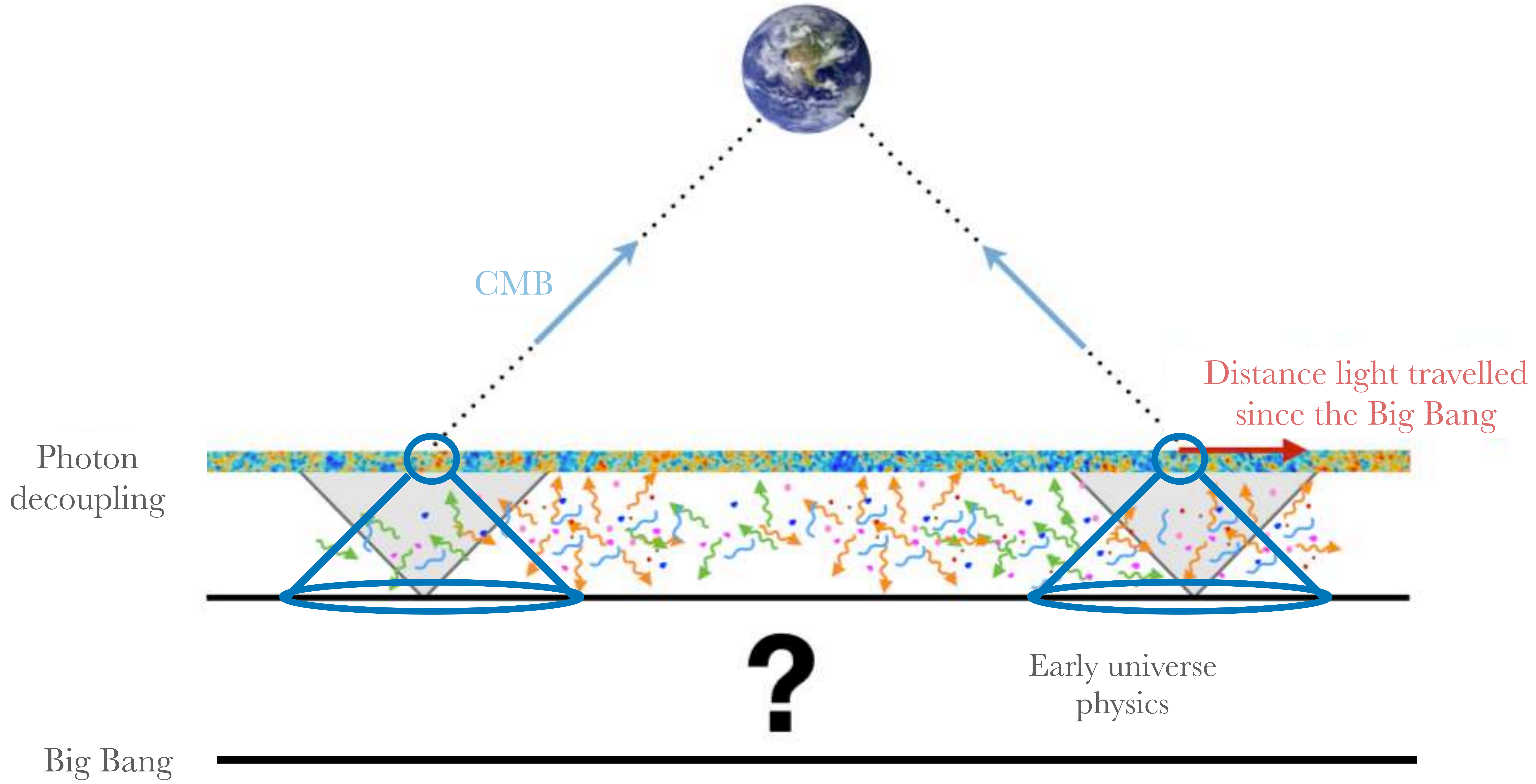
$\delta\rho \sim 10^{-5}\,\bar{\rho}$



10^{-32} s — 380.000 years — 13.8 billion of years

Crédito: D. Baumann

We need to understand the primordial universe, explain the origin of the initial fluctuations and make predictions to test these theories of the early universe evolution



Crédito: D. Baumann

Flatness problem

Observational data tells us our universe is flat or $\rho \simeq \rho_{\rm crit}$

Dynamics - Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}(\rho + 3P)$$

Friedmann equations.
(or Friedmann - Lemaître)

ρ and P here are actually the sum of all the components in the universe $\Longrightarrow$ ρ_{tot}, p_{tot}

We can also rewrite the 1st Friedmann equation as:

$$1 = \Omega_{tot} - \frac{k}{a^2 H^2}$$

Density parameter

$$\Omega_{tot} = \sum_i \Omega_i \,, \qquad \text{onde } \Omega_i = \frac{\rho_i}{\rho_{crit}}$$

Flatness problem

Observational data tells us our universe is flat or $\rho \simeq \rho_{\text{crit}}$ $\Rightarrow$ $(\Omega_0 - 1) \sim \mathcal{O}(1)$

$$(\Omega - 1)a^2H^2 = k$$

$\propto a^2$ rad

$\propto a$ matter

Extrapolating to earlier times:

$$\frac{|\Omega - 1|_{t=t_{pl}}}{|\Omega_0 - 1|} \approx \frac{a_{pl}^2}{a_0^2} \approx \frac{T_0^2}{T_{pl}^2} \sim \mathcal{O}(10^{-64})$$

for BBN, $\mathcal{O}(10^{-16})$

Given the evolution of the universe, so $(\Omega_0 - 1) \sim \mathcal{O}(1)$, then $(\Omega - 1)$ had to be VERY VERY small!

Fine tuning!

Flatness problem

Observational data tells us our universe is flat or $\rho \simeq \rho_{\text{crit}} \quad \Rightarrow \quad (\Omega_0 - 1) \sim \mathcal{O}(1)$

$$(\Omega - 1)a^2 H^2 = k$$

$\propto a^2$ rad

$\propto a$ matter

Extrapolating to earlier times:

$$\frac{|\Omega - 1|_{t=t_{pl}}}{|\Omega_0 - 1|} \approx \frac{a_{pl}^2}{a_0^2} \approx \frac{T_0^2}{T_{pl}^2} \sim \mathcal{O}(10^{-64})$$

for BBN, $\mathcal{O}(10^{-16})$

Given the evolution of the universe, so $(\Omega_0 - 1) \sim \mathcal{O}(1)$, then $(\Omega - 1)$ had to be VERY VERY small! *Fine tuning!*

Can be recast as an "entropy problem" $\longrightarrow$ adiabatic expansion $\quad \Omega - 1 = (k\, m_{pl})/(S^{2/3}\, T^2)$

Flatness problem

Observational data tells us our universe is flat or $\rho \simeq \rho_{\text{crit}}$ $\Rightarrow$ $(\Omega_0 - 1) \sim \mathcal{O}(1)$

$$(\Omega - 1)a^2 H^2 = k$$

$\propto a^2$ rad

$\propto a$ matter

Extrapolating to earlier times:

$$\frac{|\Omega - 1|_{t=t_{pl}}}{|\Omega_0 - 1|} \approx \frac{a_{pl}^2}{a_0^2} \approx \frac{T_0^2}{T_{pl}^2} \sim \mathcal{O}(10^{-64})$$

for BBN, $\mathcal{O}(10^{-16})$

Given the evolution of the universe, so $(\Omega_0 - 1) \sim \mathcal{O}(1)$, then $(\Omega - 1)$ had to be VERY VERY small!

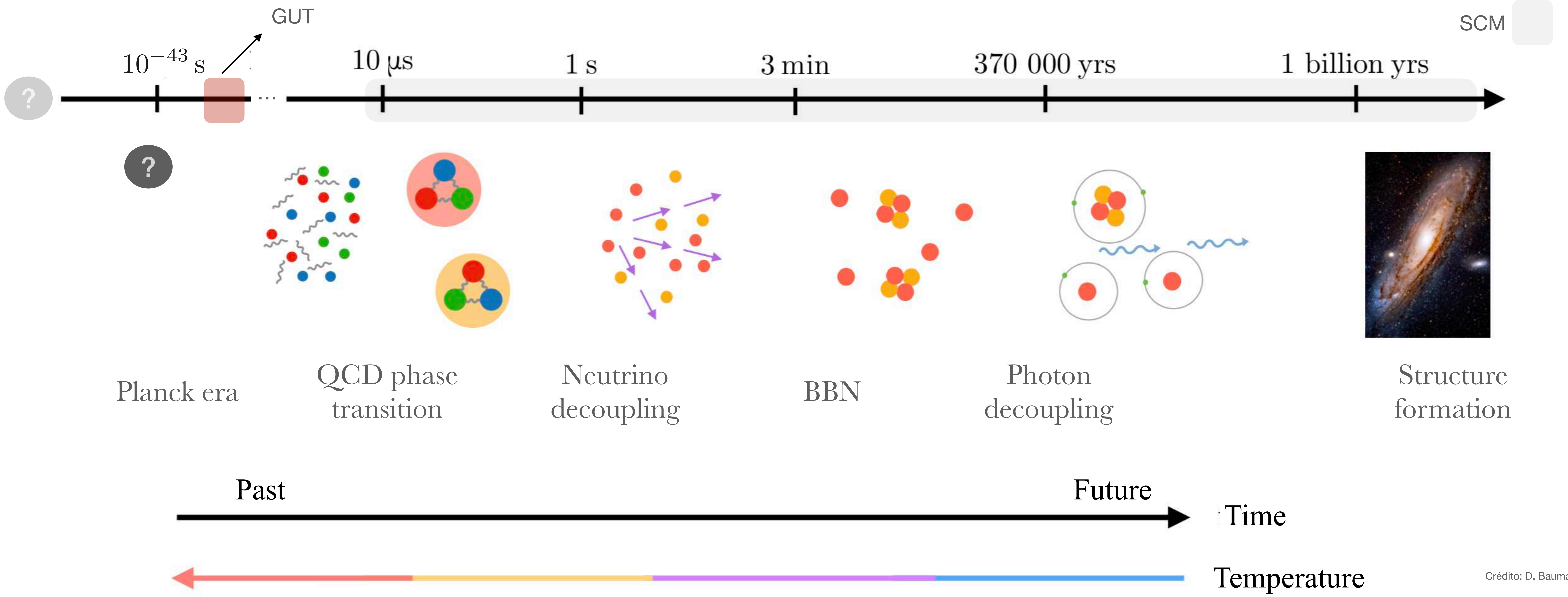
Fine tuning!

Amplification of the curvature radius

$$R = \frac{H^{-1}}{|\Omega - 1|^{1/2}} = \left(\frac{a^2}{k}\right)^{1/2}$$

Magnetic monopoles problem

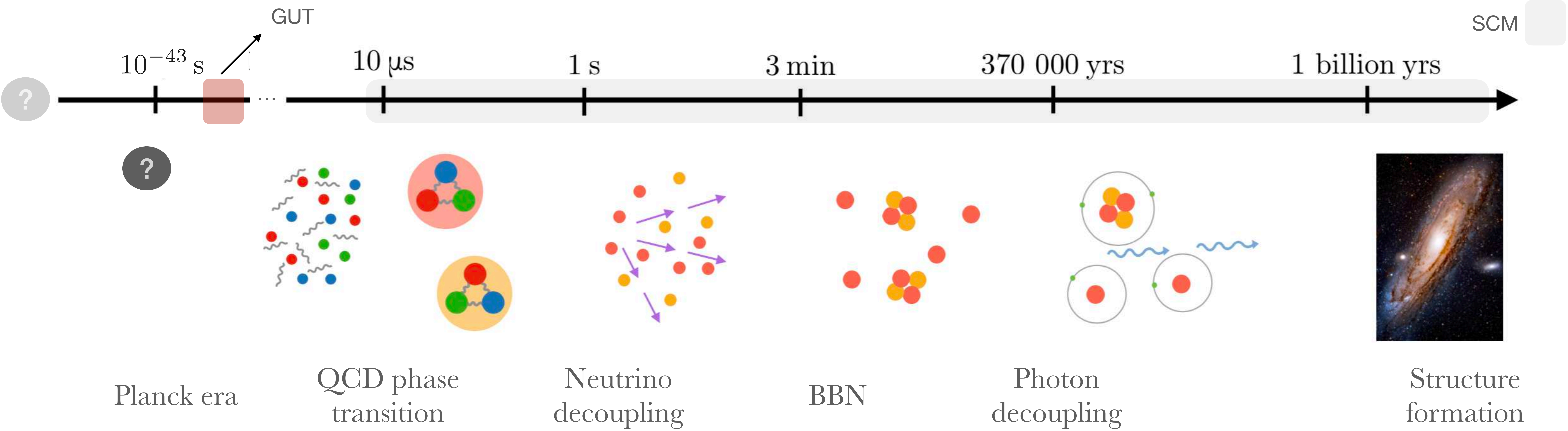
As the universe expands, and the temperature drops, some symmetries that existed at higher temperatures are broken



Crédito: D. Baumann

Magnetic monopoles problem

As the universe expands, and the temperature drops, some symmetries that existed at higher temperatures are broken



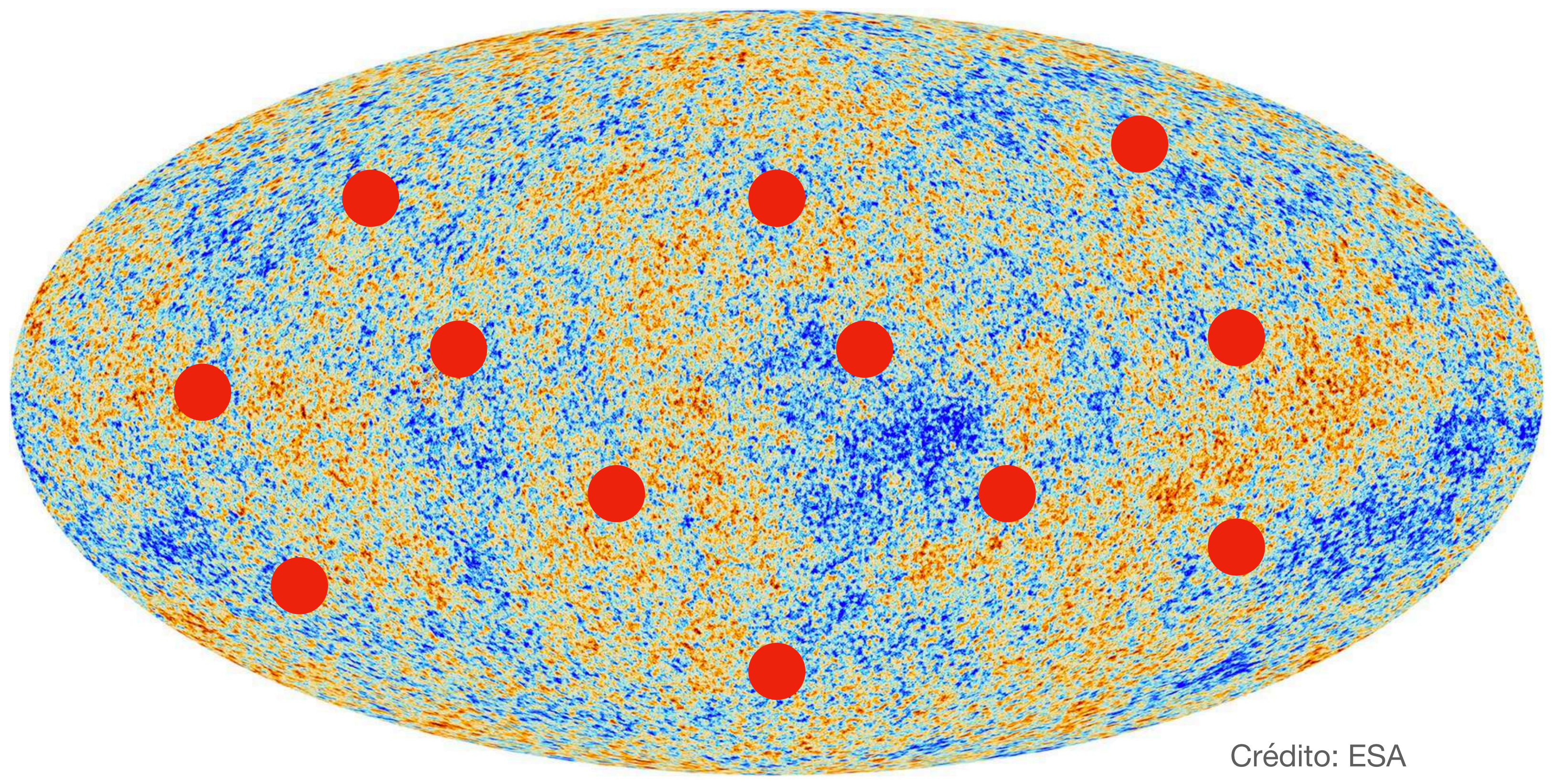
It is predicted by GUT the presence of topological defects like magnetic monopoles

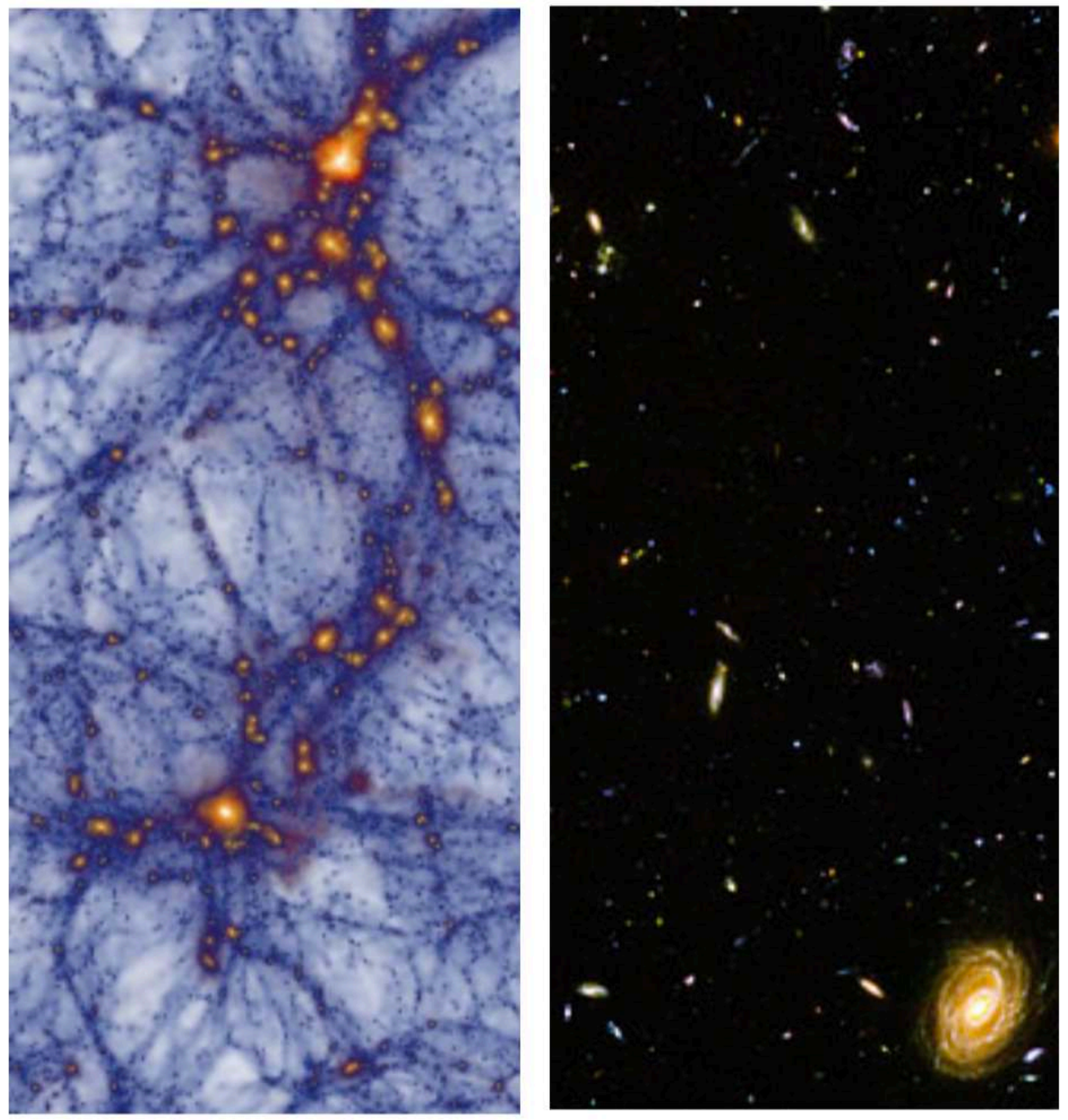
Magnetic monopoles problem

Magnetic monopoles production is very efficient and they are very heavy $10^{16} \times m_p$

Magnetic monopoles problem

Magnetic monopoles production is very efficient and they are very heavy $10^{16} \times m_p$





T<<13.8 billion of years

The prediction of the abundance of magnetic monopoles largely exceeds the observations limits!
Also NO magnetic monopoles has been observed…

Early universe models

solving the SCM problems

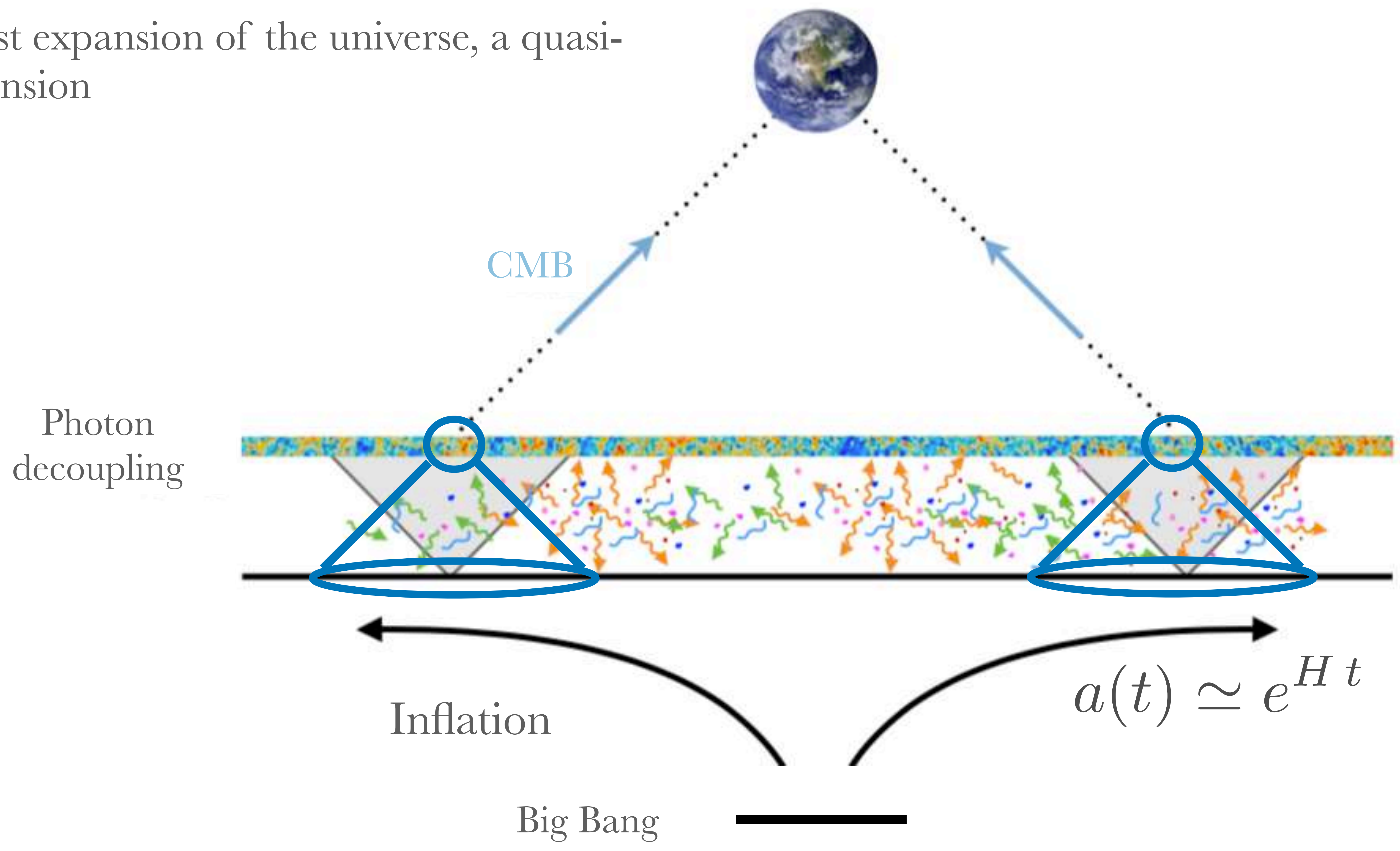
Inflation

Motivation: solve the SCM problems

Inflation

Guth (1980)
Linde(1982)
Albrecht e Steinhardt (1982)

Period of very fast expansion of the universe, a quasi-exponential expansion

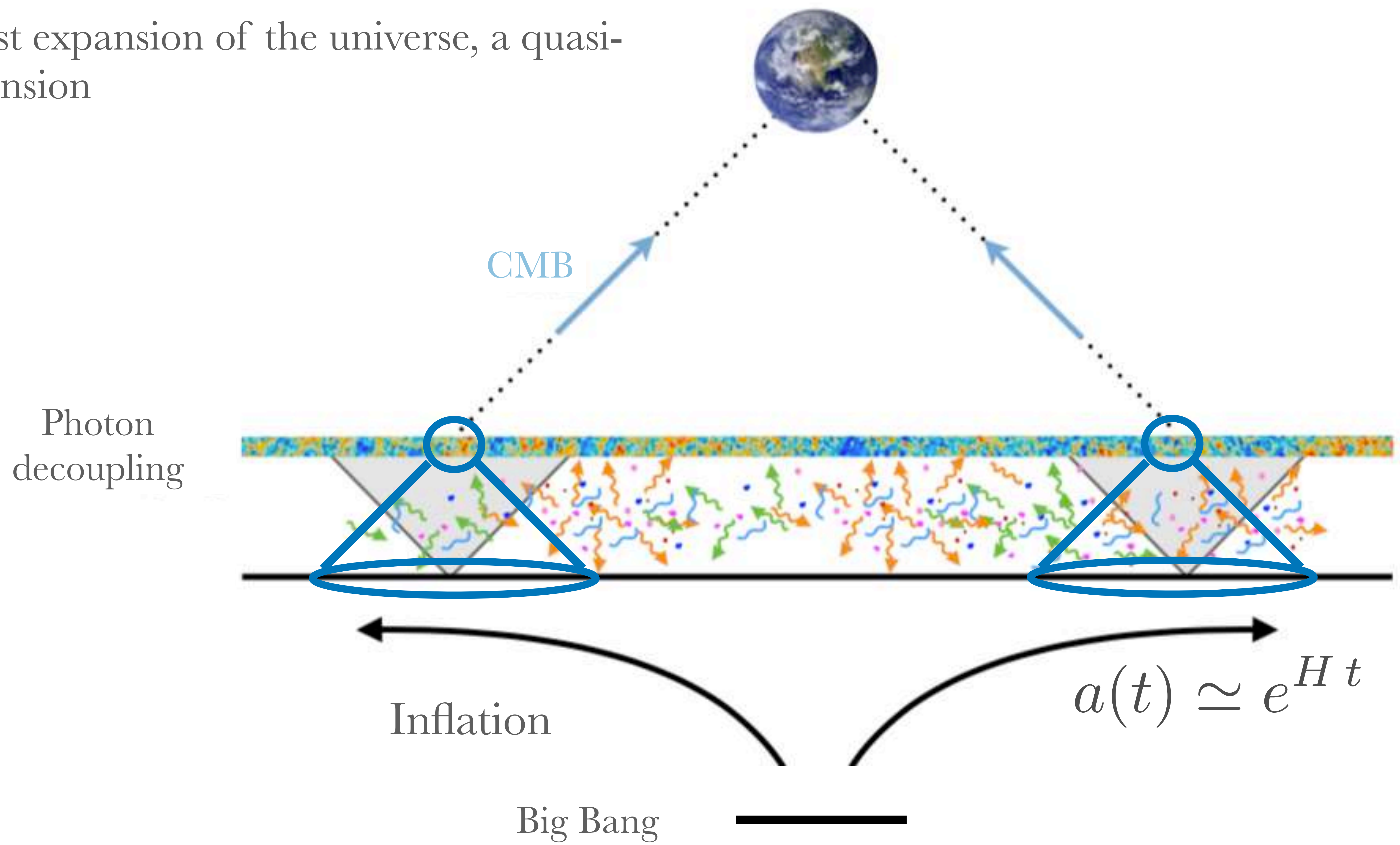


Crédito: D. Baumann

Inflation

Guth (1980)
Linde(1982)
Albrecht e Steinhardt (1982)

Period of very fast expansion of the universe, a quasi-exponential expansion



Originally (Guth 1980) - to solve the magnetic monopoles problem

Crédito: D. Baumann

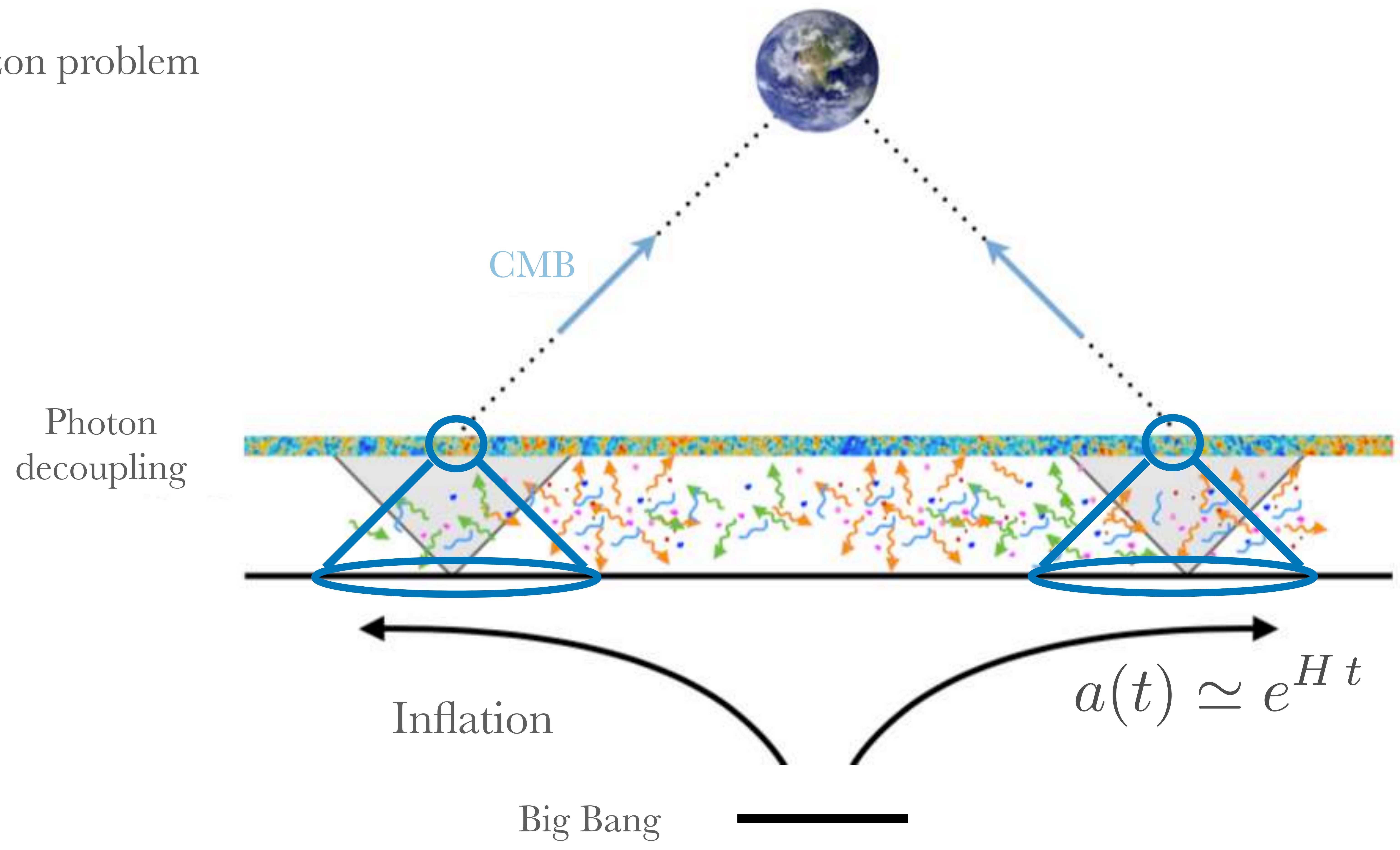
Inflation

Guth (1980)

Linde(1982)

Albrecht e Steinhardt (1982)

Solving the horizon problem



Crédito: D. Baumann

Inflation

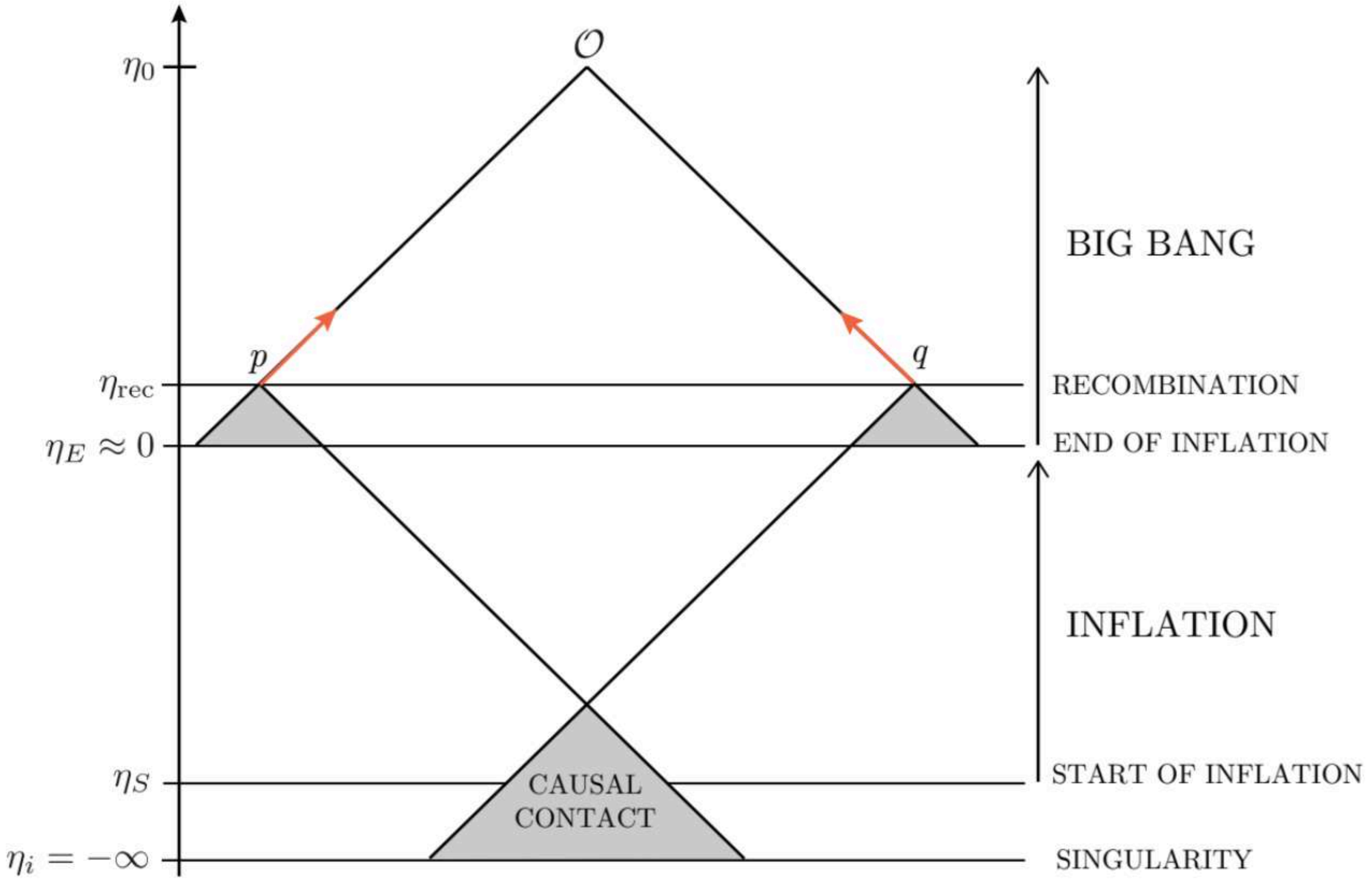
Guth (1980)
Linde(1982)
Albrecht e Steinhardt (1982)

horizon problem ↔ growing Hubble radius

Solving the horizon problem: **decreasing** Hubble radius

$$\frac{d}{dt}(aH)^{-1} < 0$$

If this period lasts long enough, it solves the horizon problem



Crédito: D. Baumann

Inflation

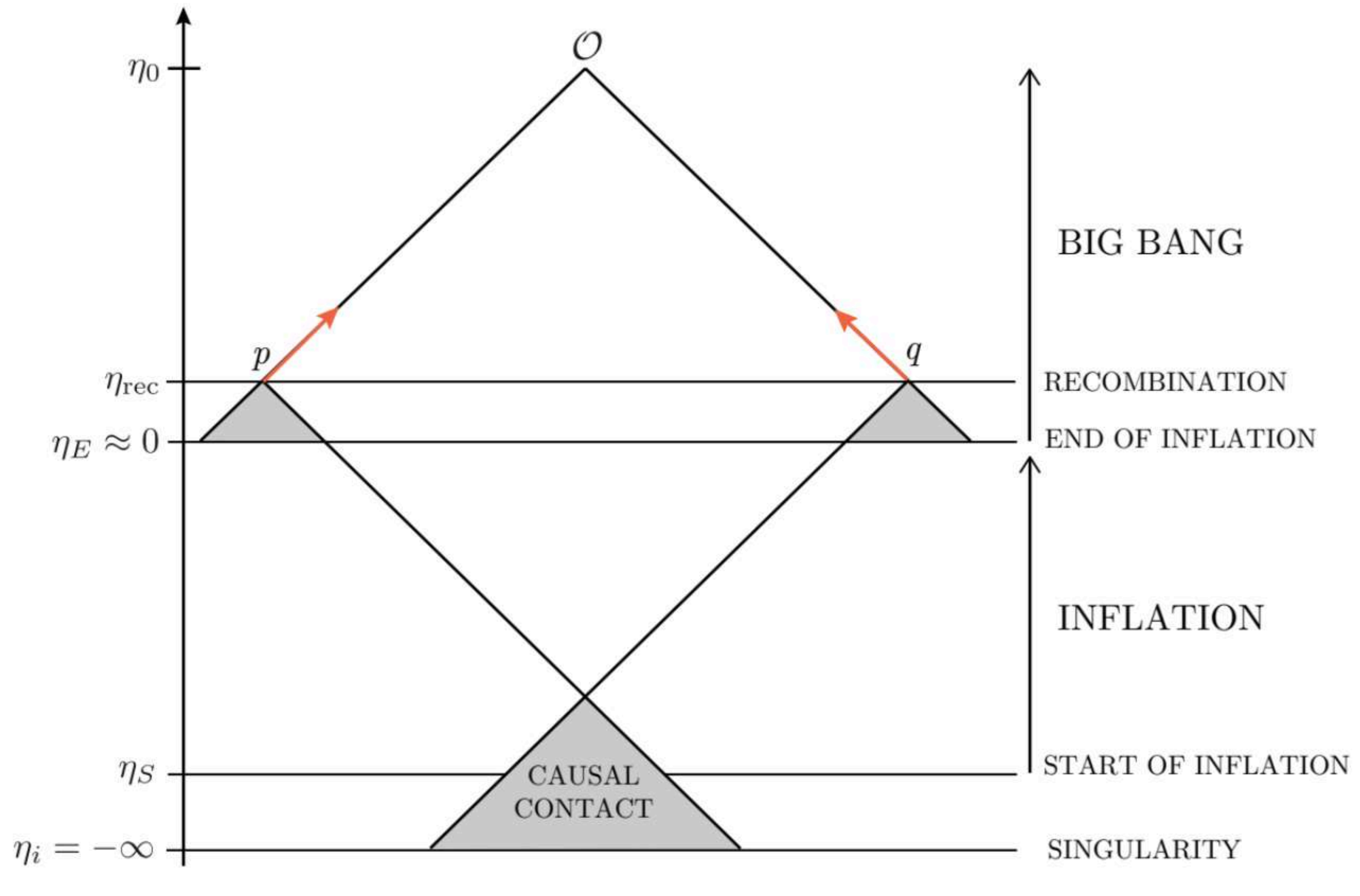
Guth (1980)
Linde(1982)
Albrecht e Steinhardt (1982)

horizon problem $\leftrightarrow$ growing Hubble radius

<u>Solving the horizon problem</u>: **decreasing** Hubble radius

$$\frac{d}{dt}(aH)^{-1} < 0$$

If this period lasts long enough, it solves the horizon problem



Crédito: D. Baumann

HOW?

$$\chi_p(\eta) = \int_{\ln a_i}^{\ln a} (aH)^{-1}\, d\ln a$$

$$(aH)^{-1} = H_0^{-1} a^{(1+3w)/2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) < 0 \quad \Longrightarrow \quad \ddot{a} > 0$$

~~Strong energy condition (SEC) >0~~

Inflation

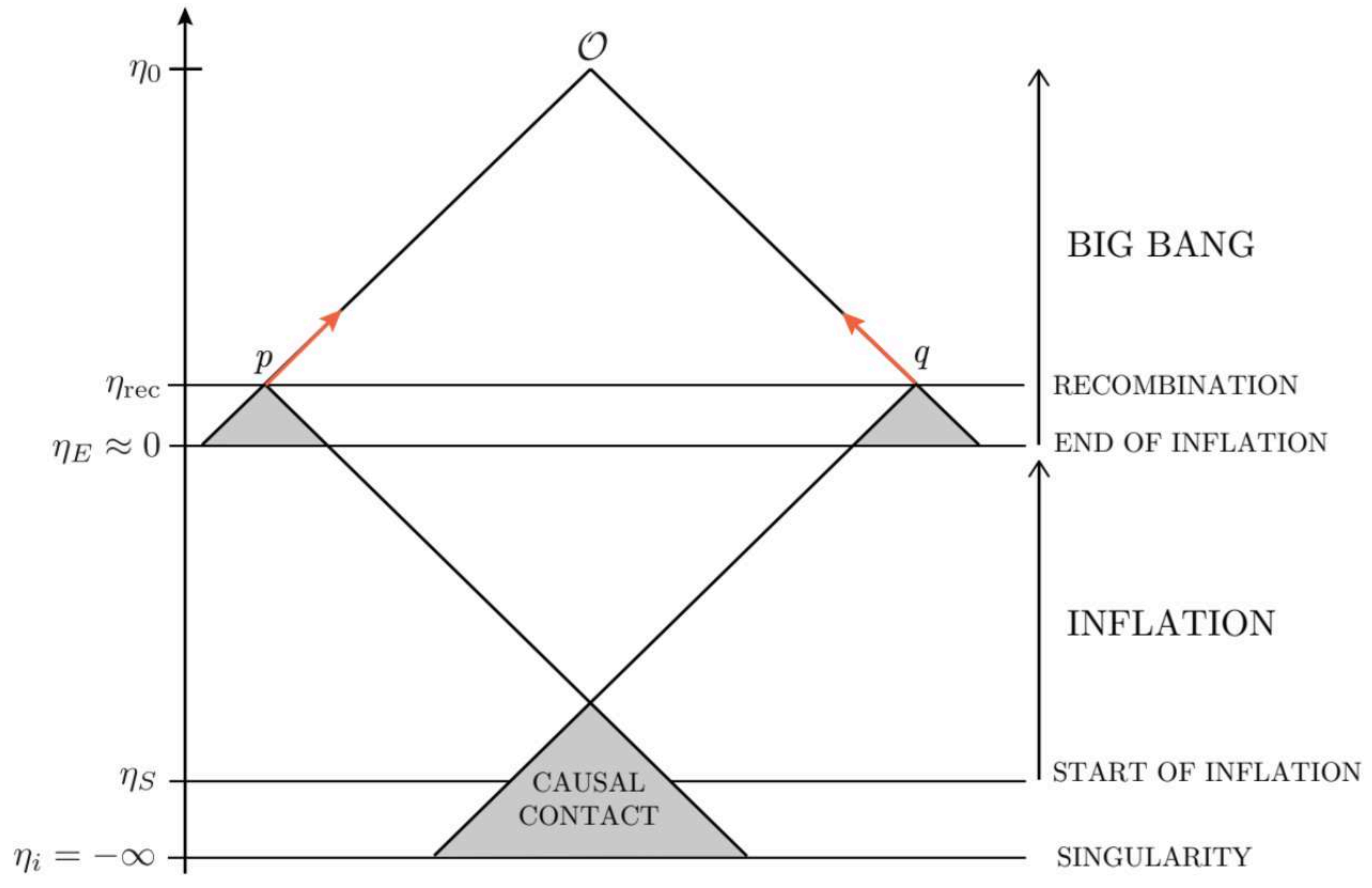
Guth (1980)
Linde(1982)
Albrecht e Steinhardt (1982)

horizon problem $\leftrightarrow$ growing Hubble radius

<u>Solving the horizon problem</u>: **decreasing** Hubble radius

$$\frac{d}{dt}(aH)^{-1} < 0$$

If this period lasts long enough, it solves the horizon problem



Crédito: D. Baumann

HOW?

$$\chi_p(\eta) = \int_{\ln a_i}^{\ln a} (aH)^{-1}\, d\ln a$$

$$(aH)^{-1} = H_0^{-1} a^{(1+3w)/2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) < 0$$

Strong energy condition (SEC)
>0

$$\Rightarrow$$

$$\ddot{a} > 0$$

$$\eta_i \to -\infty$$

Acceleration

How can we obtain such an expansion of the universe?

Remember:

- Grau de liberdade extra: energia escura

acceleration — Dark energy

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[(\rho(t) + p(t))_{R,M} + \rho_{EE}(t)\right] \qquad + \qquad w < -\frac{1}{3}$$

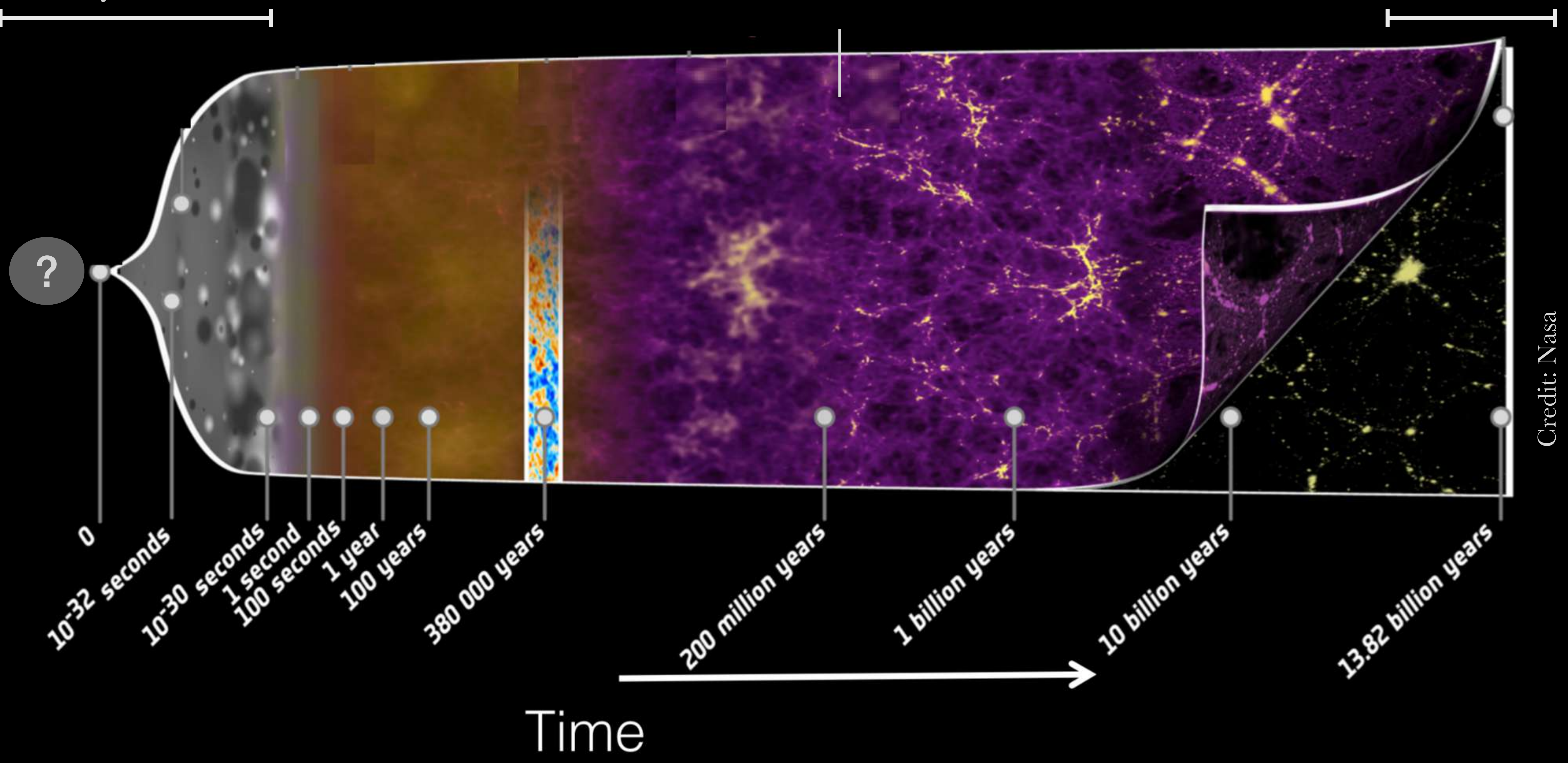
decelerates the expansion

MANY fundamental open questions

Early universe
Big Bang? What is the physics of the early universe?

Dark matter p
What is the dark matter?

Dark energy -
What is the dark energy?



Inflation

- Accelerated expansion

$$\frac{d}{dt}(aH)^{-1} = \frac{d}{dt}(\dot{a})^{-1} = -\frac{\ddot{a}}{(\dot{a})^2} < 0 \;\Rightarrow\; \ddot{a} > 0$$

Shrinking Hubble radius

- Slowly varying Hubble parameter

$$\frac{d}{dt}(aH)^{-1} = -\frac{\dot{a}H + a\dot{H}}{(aH)^2} = \frac{1}{a}(1-\epsilon) < 0$$

Slow-roll parameter: $\epsilon \equiv -\frac{\dot{H}}{H^2} < 1$

- Negative pressure

$$w = \frac{p}{\rho} < -\frac{1}{3}$$

- Constant density

$$\dot{\rho} + 3H\,(\rho + P) = 0$$

$$\Rightarrow \;\; |d\ln\rho/d\ln a| = 2\epsilon < 1$$

- Quasi de Sitter expansion

When $\epsilon \to 0 \;\;\Rightarrow\;\; \text{dS} \;\;\Rightarrow\;\; H = \text{const.}$

Small ϵ, quasi-dS

$$\ddot{a} > 0 \;\leftrightarrow\; p \sim -\rho \;\leftrightarrow\; H = \text{const.} \;\leftrightarrow\; \rho \sim \text{const.} \;\leftrightarrow\; \epsilon < 1 \;\leftrightarrow\; a(t) \simeq \exp(Ht)$$

Constructing inflation

1. Decreasing radius / slowly-varying Hubble parameter

1st slow-roll parameter $$\epsilon \equiv -\frac{\dot{H}}{H^2} < 1$$ Small

2. Inflation persists for long enough (ϵ small for enough time)

2nd slow-roll parameter $$\eta \equiv \frac{d \ln \epsilon}{dN} = \frac{\dot{\epsilon}}{H\epsilon} \quad , \quad |\eta| < 1$$

$N = \ln a$ e-fold

Implementing the inflationary mechanism

How can we implement a microphysical model of the accelerated (exponential

- Adding one (or many) new components that dominate the universe at its begining with $w < -\frac{1}{3} \quad \Longrightarrow \quad \ddot{a} > 0$

$$\rho_{\rm infl}\,, \;\; p_{\rm infl}$$

We call this new component the *inflaton*

Acceleration

How can we obtain such an expansion of the universe?

Remember:

- Grau de liberdade extra: energia escura

acceleration

Dark energy

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[(\rho(t) + p(t))_{R,M} + \rho_{EE}(t)\right] \quad + \quad w < -\frac{1}{3}$$

decelerates the expansion

Implementing the inflationary mechanism

How can we implement a microphysical model of the accelerated (exponential

- Adding one (or many) new components that dominate the universe at its begining with $w < -\frac{1}{3} \quad \Longrightarrow \quad \ddot{a} > 0$

$$\rho_{\rm infl}\,,\;\; p_{\rm infl}$$

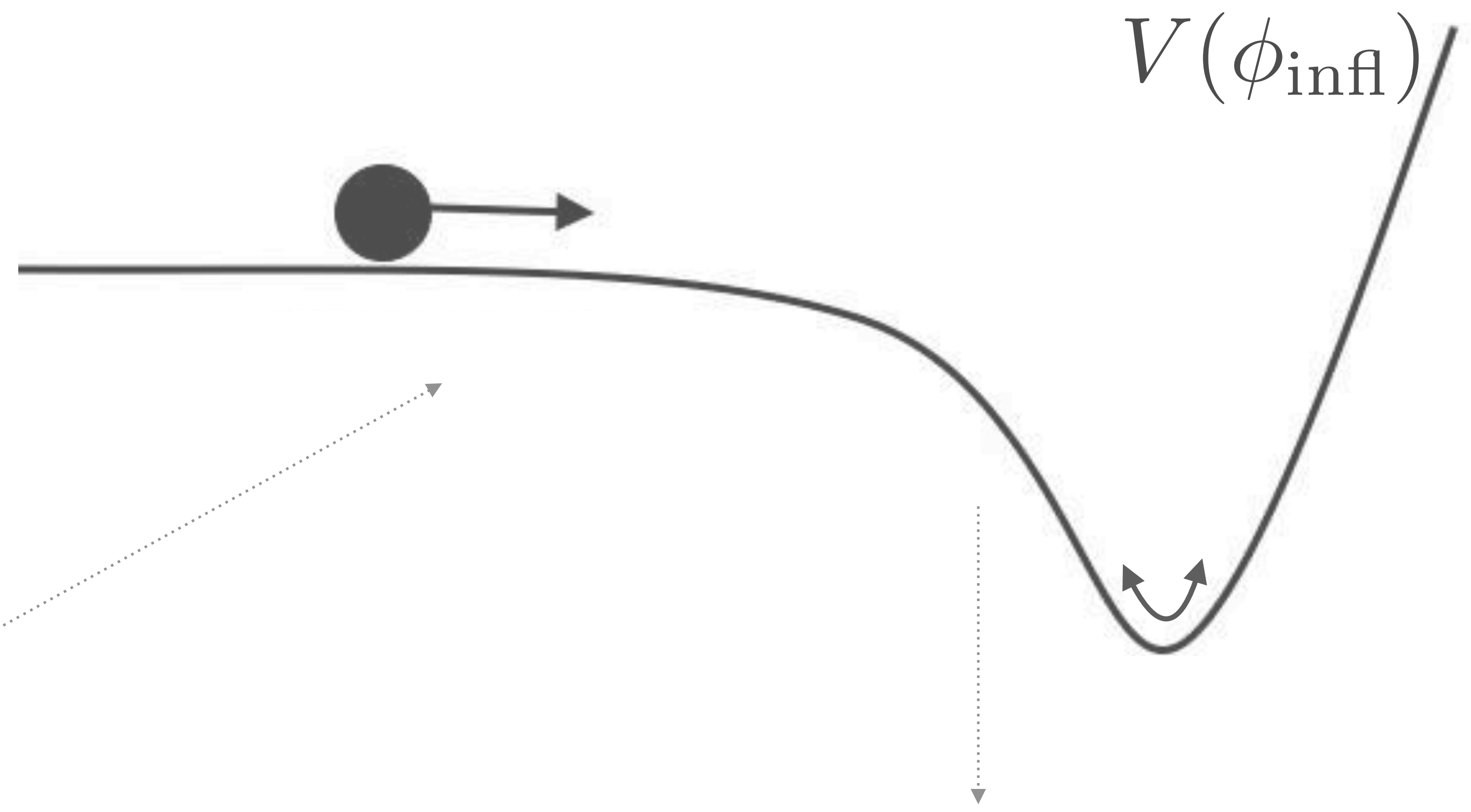
We call this new component the *inflaton*

Inflation

TOY MODEL:

Scalar field (*inflaton*) $\phi(t)$ in a FRW background

To cause the acceleration, the potential has to have the form:



Causes the accelerated expansion - quasi-exponential

However, inflation has to end, so the era of radiation begins - ***graceful exit***

$$\epsilon \sim 1$$

Inflation

Single scalar field inflation

$$S = -\int d^4x\sqrt{-g}\,\mathcal{L} = -\int d^4x\sqrt{-g}\,\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)\right]$$

$-g:$ metric determinant of $g_{\mu\nu}$

Inflation

Single scalar field inflation

$$S = -\int d^4x\sqrt{-g}\,\mathcal{L} = -\int d^4x\sqrt{-g}\,\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)\right]$$

$-g$: metric determinant of $g_{\mu\nu}$

Energy momentum tensor: $$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi)\right)$$

$$\Rightarrow \quad \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Potential energy

dominates

kinetic energy =

$p_\phi < -\rho_\phi/3$

Inflation

Single scalar field inflation

$$S = -\int d^4x\sqrt{-g}\,\mathcal{L} = -\int d^4x\sqrt{-g}\,\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)\right]$$

$-g$: metric determinant of $g_{\mu\nu}$

Equation of motion:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Inflation

Single scalar field inflation

$$S = -\int d^4x\sqrt{-g}\,\mathcal{L} = -\int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)\right]$$

Equation of motion:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Slow-roll parameters:

1st slow-roll parameter

$$\epsilon = \frac{\dot{\phi}^2/2}{M_{pl}^2 H^2}$$

2nd slow-roll parameter

$$\eta = 2\left(\epsilon + \frac{\ddot{\phi}}{H\dot{\phi}}\right)$$

Inflation - slow roll approximation

The conditions for inflation to happen and persists for long enough $(\epsilon, \eta) \ll 1$, can be used to simplify the EoM:

$$\epsilon \ll 1 \quad \Rightarrow \quad \frac{1}{2}\dot{\phi}^2 \ll V \quad \rightarrow \quad H^2 \approx \frac{V}{3M_{pl}^2}$$

$$\left[\delta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}}\right], \quad |\delta| \ll 1 \quad \rightarrow \quad 3H\dot{\phi} \approx -dV/d\phi$$

Potential slow-roll parameters

$$\epsilon_v \equiv \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2, \quad |\eta_v| \equiv M_{pl}^2 \frac{|V''|}{V}$$

e-fold number

$$N \equiv \int_{t_i}^{t_f} H\, dt \simeq -\frac{1}{m_{pl}^2}\int_{\phi_i}^{\phi_f} \frac{V}{dV\ d\phi}\, d\phi$$

Inflation models

Many models!!!

Name	Parameters	Sub-models	$V(\phi)$
HI	0	1	$M^4\left(1-e^{-\sqrt{2/3}\phi/M_{\mathrm{Pl}}}\right)$
RCHI	1	1	$M^4\left(1-2e^{-\sqrt{2/3}\phi/M_{\mathrm{Pl}}}+\frac{A_{\mathrm{I}}}{16\pi^2}\frac{\phi}{\sqrt{6}M_{\mathrm{Pl}}}\right)$
LFI	1	1	$M^4\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^p$
MLFI	1	1	$M^4\frac{\phi^2}{M_{\mathrm{Pl}}^2}\left[1+\alpha\frac{\phi^2}{M_{\mathrm{Pl}}^2}\right]$
RCMI	1	1	$M^4\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^2\left[1-2\alpha\frac{\phi^2}{M_{\mathrm{Pl}}^2}\ln\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]$
RCQI	1	1	$M^4\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^4\left[1-\alpha\ln\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]$
NI	1	1	$M^4\left[1+\cos\left(\frac{\phi}{f}\right)\right]$
ESI	1	1	$M^4\left(1-e^{-q\phi/M_{\mathrm{Pl}}}\right)$
PLI	1	1	$M^4e^{-\alpha\phi/M_{\mathrm{Pl}}}$
KMII	1	2	$M^4\left(1-\alpha\frac{\phi}{M_{\mathrm{Pl}}}e^{-\phi/M_{\mathrm{Pl}}}\right)$
HF1I	1	1	$M^4\left(1+A_1\frac{\phi}{M_{\mathrm{Pl}}}\right)^2\left[1-\frac{2}{3}\left(\frac{A_1}{1+A_1\phi/M_{\mathrm{Pl}}}\right)^2\right]$
CWI	1	1	$M^4\left[1+\alpha\left(\frac{\phi}{Q}\right)^4\ln\left(\frac{\phi}{Q}\right)\right]$
LI	1	2	$M^4\left[1+\alpha\ln\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]$
RpI	1	3	$M^4e^{-2\sqrt{2/3}\phi/M_{\mathrm{Pl}}}\left\lvert e^{\sqrt{2/3}\phi/M_{\mathrm{Pl}}}-1\right\rvert^{2p/(2p-1)}$
DWI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-1\right]^2$
MHI	1	1	$M^4\left[1-\mathrm{sech}\left(\frac{\phi}{\mu}\right)\right]$
RGI	1	1	$M^4\frac{(\phi/M_{\mathrm{Pl}})^2}{\alpha+(\phi/M_{\mathrm{Pl}})^2}$
MSSMI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{2}{3}\left(\frac{\phi}{\phi_0}\right)^6+\frac{1}{5}\left(\frac{\phi}{\phi_0}\right)^{10}\right]$
RIPI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{4}{3}\left(\frac{\phi}{\phi_0}\right)^3+\frac{1}{2}\left(\frac{\phi}{\phi_0}\right)^4\right]$
AI	1	1	$M^4\left[1-\frac{2}{\pi}\arctan\left(\frac{\phi}{\mu}\right)\right]$
CNAI	1	1	$M^4\left[3-(3+\alpha^2)\tanh^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]$
CNBI	1	1	$M^4\left[(3-\alpha^2)\tan^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\mathrm{Pl}}}\right)-3\right]$
OSTI	1	1	$-M^4\left(\frac{\phi}{\phi_0}\right)^2\ln\left[\left(\frac{\phi}{\phi_0}\right)^2\right]$
WRI	1	1	$M^4\ln\left(\frac{\phi}{\phi_0}\right)^2$
SFI	2	1	$M^4\left[1-\left(\frac{\phi}{\mu}\right)^p\right]$
II	2	1	$M^4\left(\frac{\phi-\phi_0}{M_{\mathrm{Pl}}}\right)^{-\beta}-M^4\frac{\beta^2}{6}\left(\frac{\phi-\phi_0}{M_{\mathrm{Pl}}}\right)^{-\beta-2}$
KMIII	2	1	$M^4\left[1-\alpha\frac{\phi}{M_{\mathrm{Pl}}}\exp\left(-\beta\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]$
LMI	2	2	$M^4\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^{\alpha}\exp\left[-\beta(\phi/M_{\mathrm{Pl}})^{\gamma}\right]$
TWI	2	1	$M^4\left[1-A\left(\frac{\phi}{\phi_0}\right)^2e^{-\phi/\phi_0}\right]$
GMSSMI	2	2	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{2}{3}\alpha\left(\frac{\phi}{\phi_0}\right)^6+\frac{\alpha}{5}\left(\frac{\phi}{\phi_0}\right)^{10}\right]$
GRIPI	2	2	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{4}{3}\alpha\left(\frac{\phi}{\phi_0}\right)^3+\frac{\alpha}{2}\left(\frac{\phi}{\phi_0}\right)^4\right]$
BSUSYBI	2	1	$M^4\left(e^{\sqrt{6}\frac{\phi}{M_{\mathrm{Pl}}}}+e^{\sqrt{6}\gamma\frac{\phi}{M_{\mathrm{Pl}}}}\right)$
TI	2	3	$M^4\left(1+\cos\frac{\phi}{\mu}+\alpha\sin^2\frac{\phi}{\mu}\right)$
BEI	2	1	$M^4\exp_{1-\beta}\left(-\lambda\frac{\phi}{M_{\mathrm{Pl}}}\right)$
PSNI	2	1	$M^4\left[1+\alpha\ln\left(\cos\frac{\phi}{f}\right)\right]$
NCKI	2	2	$M^4\left[1+\alpha\ln\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)+\beta\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^2\right]$
CSI	2	1	$\frac{M^4}{\left(1-\alpha\frac{\phi}{M_{\mathrm{Pl}}}\right)^2}$
OI	2	1	$M^4\left(\frac{\phi}{\phi_0}\right)^4\left[\left(\ln\frac{\phi}{\phi_0}\right)^2-\alpha\right]$
CNCI	2	1	$M^4\left[(3+\alpha^2)\coth^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\mathrm{Pl}}}\right)-3\right]$
SBI	2	2	$M^4\left\{1+\left[-\alpha+\beta\ln\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^4\right\}$
SSBI	2	6	$M^4\left[1+\alpha\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^2+\beta\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^4\right]$
IMI	2	1	$M^4\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^{-p}$
BI	2	2	$M^4\left[1-\left(\frac{\phi}{\mu}\right)^{-p}\right]$
RMI	3	4	$M^4\left[1-\frac{c}{2}\left(-\frac{1}{2}+\ln\frac{\phi}{\phi_0}\right)\frac{\phi^2}{M_{\mathrm{Pl}}^2}\right]$
VHI	3	1	$M^4\left[1+\left(\frac{\phi}{\mu}\right)^p\right]$
DSI	3	1	$M^4\left[1+\left(\frac{\phi}{\mu}\right)^{-p}\right]$
GMLFI	3	1	$M^4\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^p\left[1+\alpha\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^q\right]$
LPI	3	3	$M^4\left(\frac{\phi}{\phi_0}\right)^p\left(\ln\frac{\phi}{\phi_0}\right)^q$
CNDI	3	3	$\frac{M^4}{\left\{1+\beta\cos\left[\alpha\left(\frac{\phi-\phi_0}{M_{\mathrm{Pl}}}\right)\right]\right\}^2}$

Encyclopædia Inflationaris J. Martin, et al (2013)

We will continue studying inflation in the next lesson…