Introduction to cosmology:

overview of the standard cosmological model

Elisa G. M. Ferreira

Universidade de Sao Paulo & Kavli IPMU

Early universe cosmology, USP
17/Nov/2022

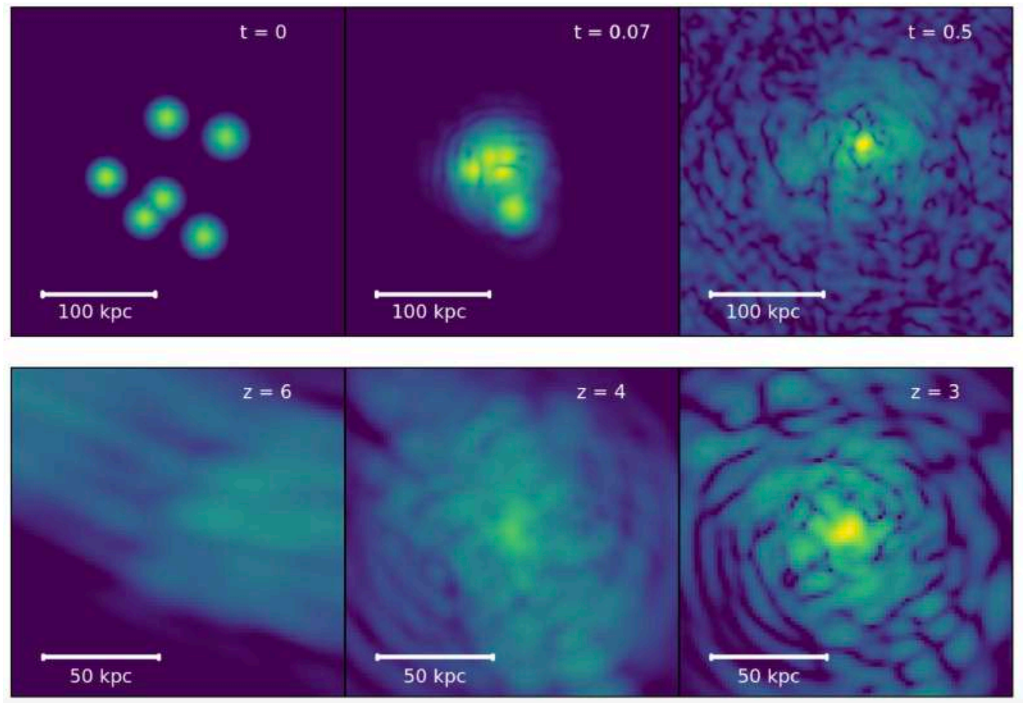
A little bit about me…

Underegrad and masters ⟶ IFUSP

PhD ⟶ Universidade McGill

Postdoc ⟶ Max Planck Institute for Astrophysics

Currently: Professor of IFUSP and Kavli Institute for the Physics and Mathematics of the Universe.



<u>My research:</u>

Theoretical cosmology

- Early universe
- Dark energy
- Dark matter
    - Ultra-light DM, axions

I also use observational data to test cosmological models and simulations.

Early universe cosmology

Important: it is IMPOSSIBLE to teach early universe cosmology in 10 hours! Therefore, I will give an introduction to the topic, giving an overview and pointing to the current problems and open questions

References:

- Daniel Baumann, Cosmology, Cambridge University Press, 2022.
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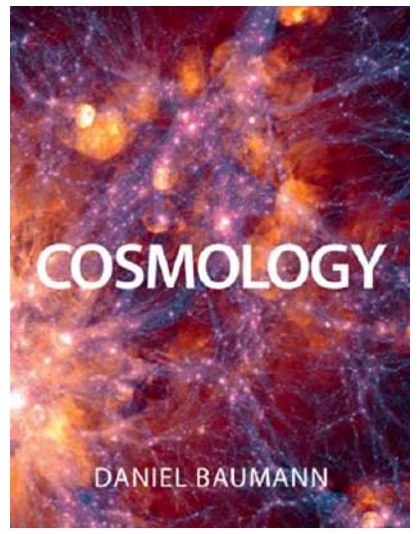
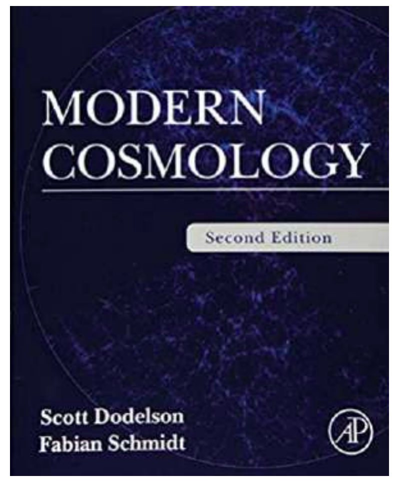
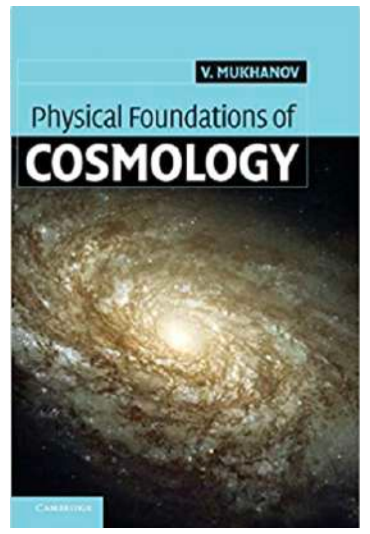
- (Recurso em português) Tese de mestrado Elisa G. M. Ferreira (capítulos 2 e 3)







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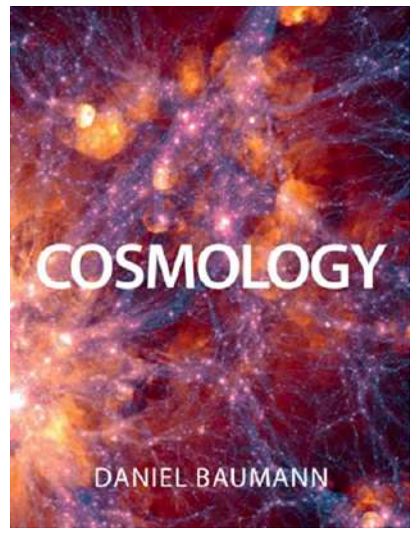

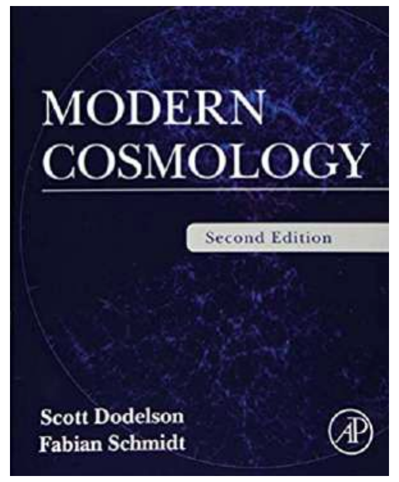

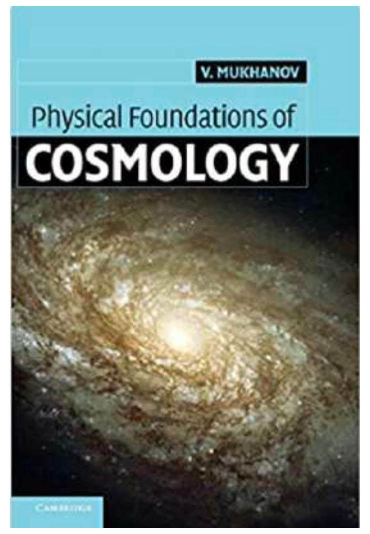

Questions: elisa@if.usp.br

Tentative schedule

Class 1 – Overview of the standard cosmological model. Problems of the MCP

Class 2 - Inflation. Problems of inflation. Preheating and reheating.

Class 3 – Theory of cosmological perturbations. Observables in cosmology.

Class 4 – Alternative models: bouncing models and emergent universes

Class 5 – Gravitational waves. Topological defects in cosmology

Class 6 - Quantum field theory methods in cosmology. Current topics in early universe cosmology

Class 1

The standard cosmological model

Cosmology

– Cosmology studies the evolution and composition of the universe

Cosmology

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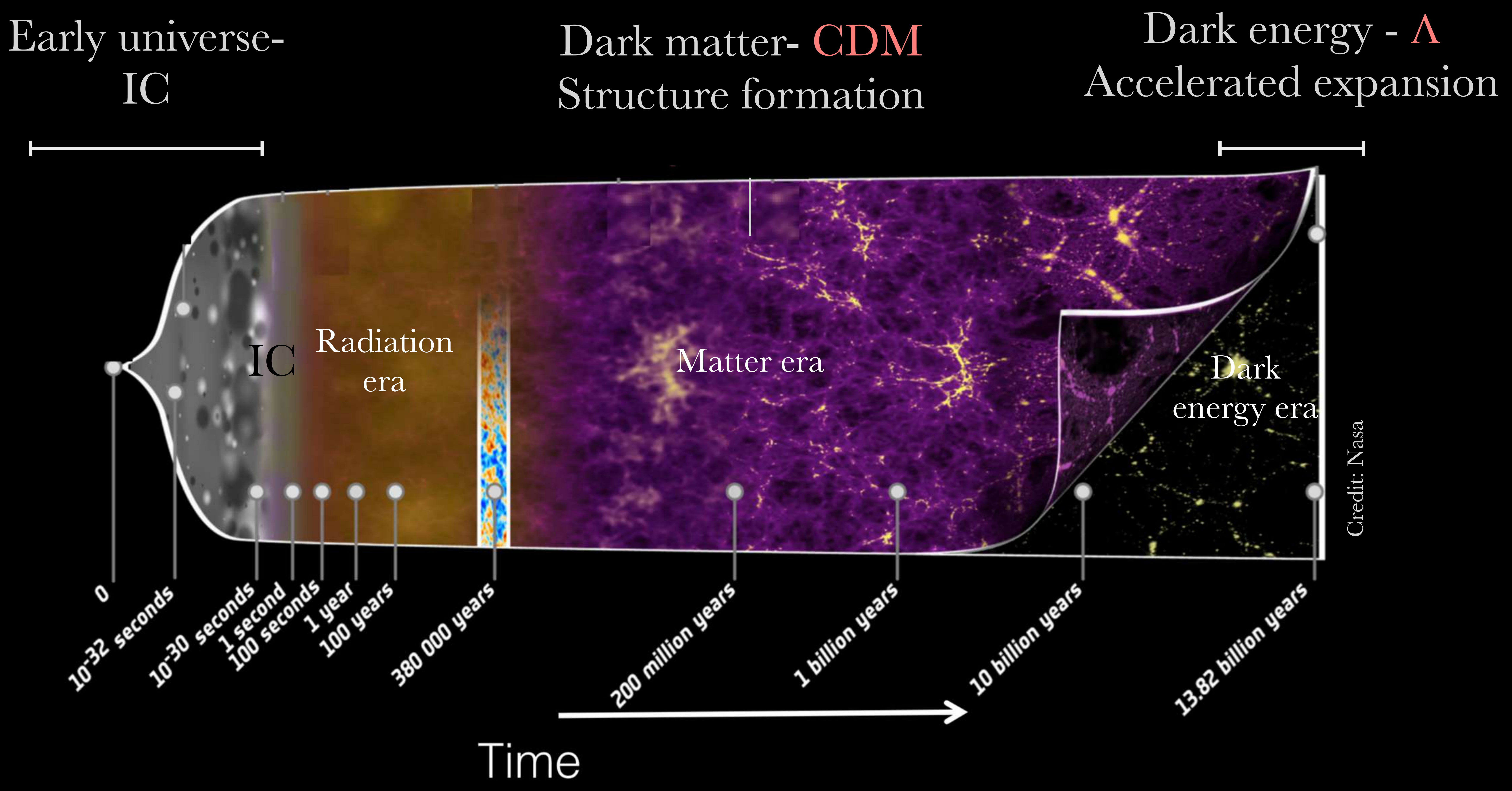
- Huge success! Cosmology became a precision science. (~30 years)
- ΛCDM: standard model, 6 parameters measured with precision ~1%



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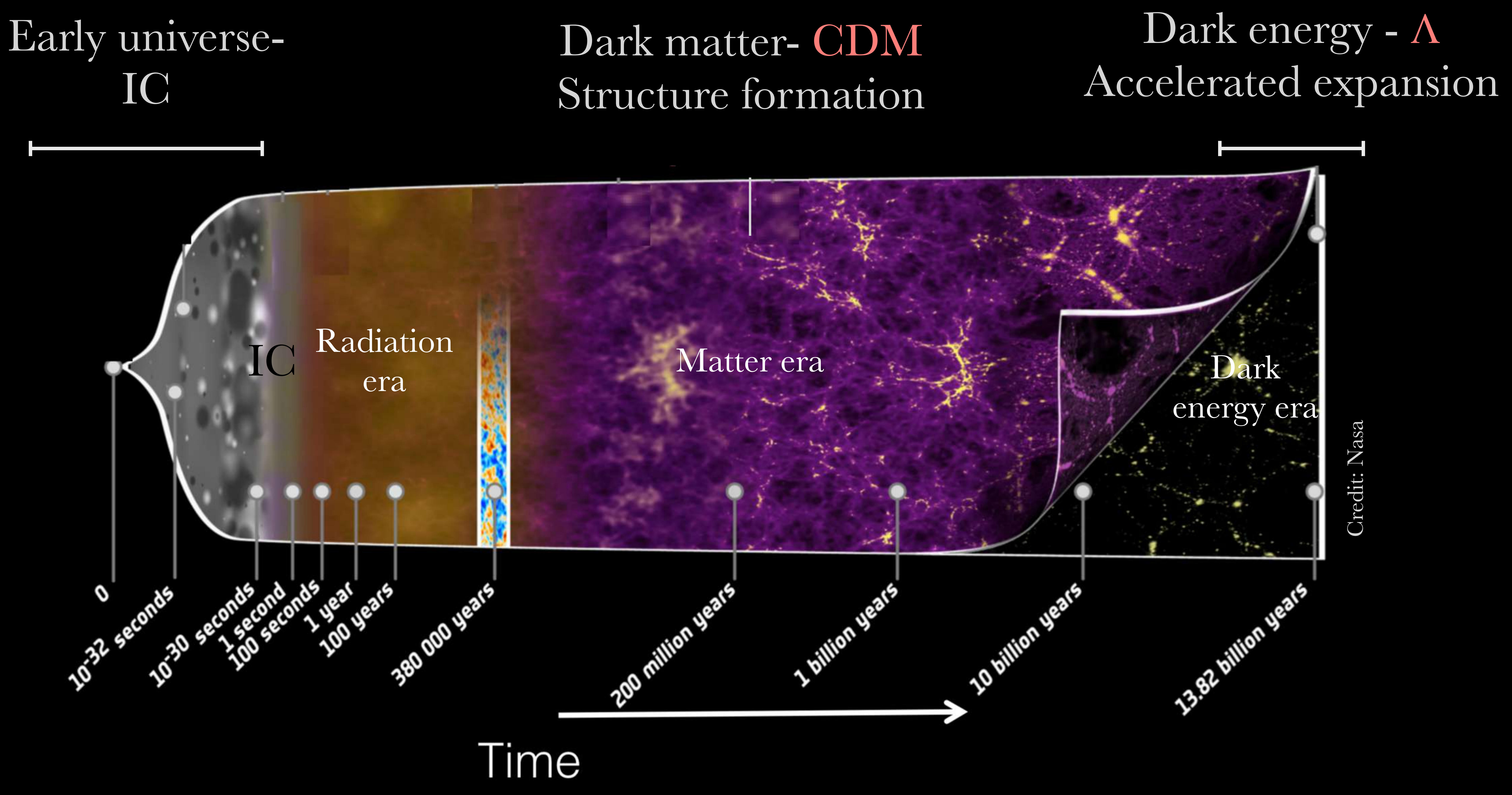
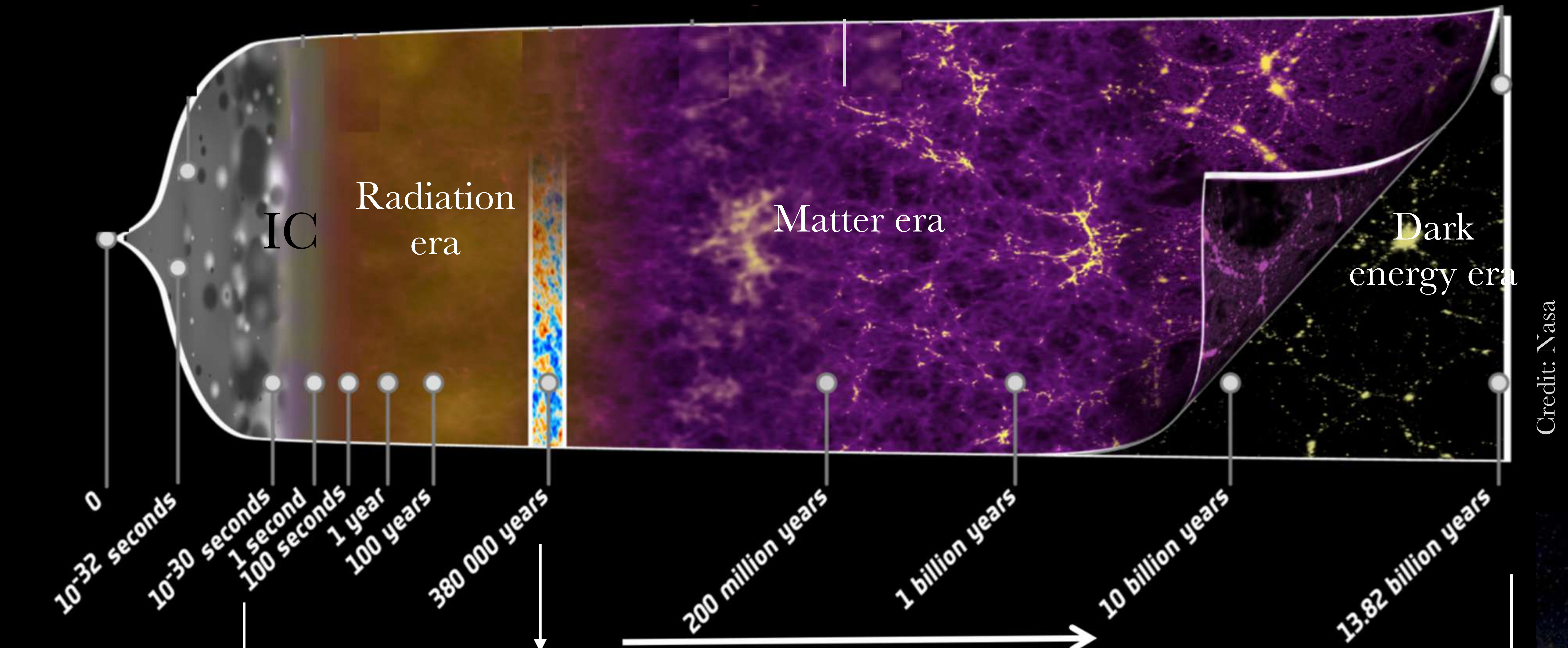


Cosmology

Early universe-
IC

Dark matter- CDM
Structure formation

Dark energy - Λ
Accelerated expansion



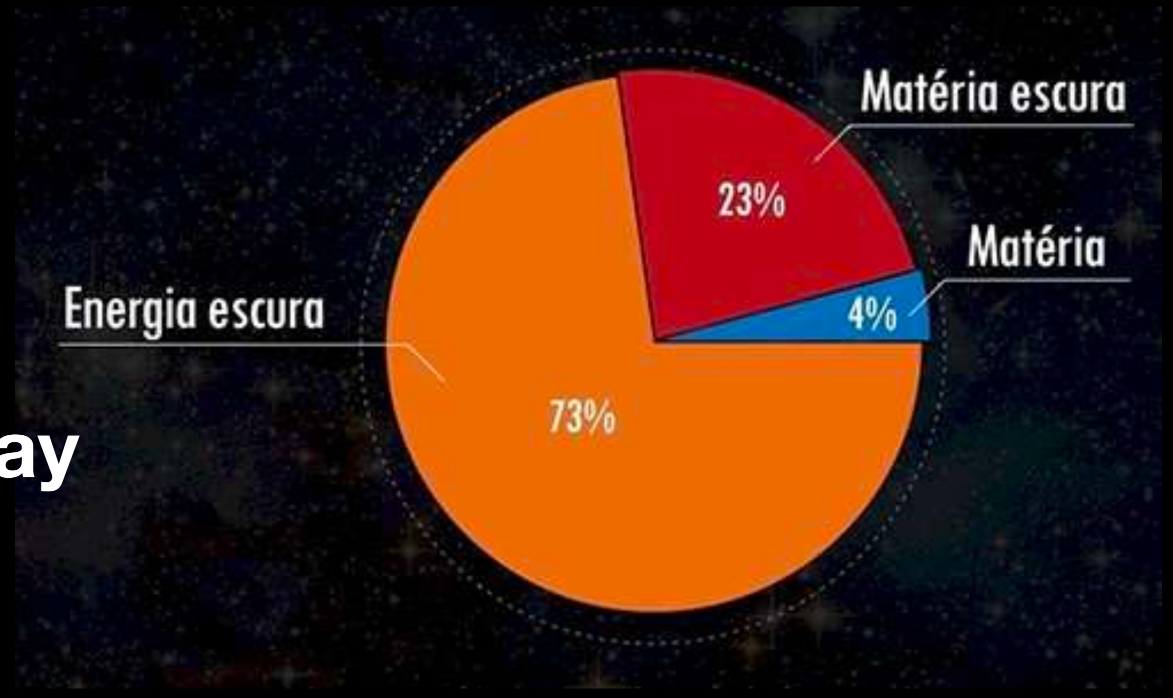
Nucleosynthesis

Time

Recombination/ Decopling

Formation of first stars and galaxies

Today



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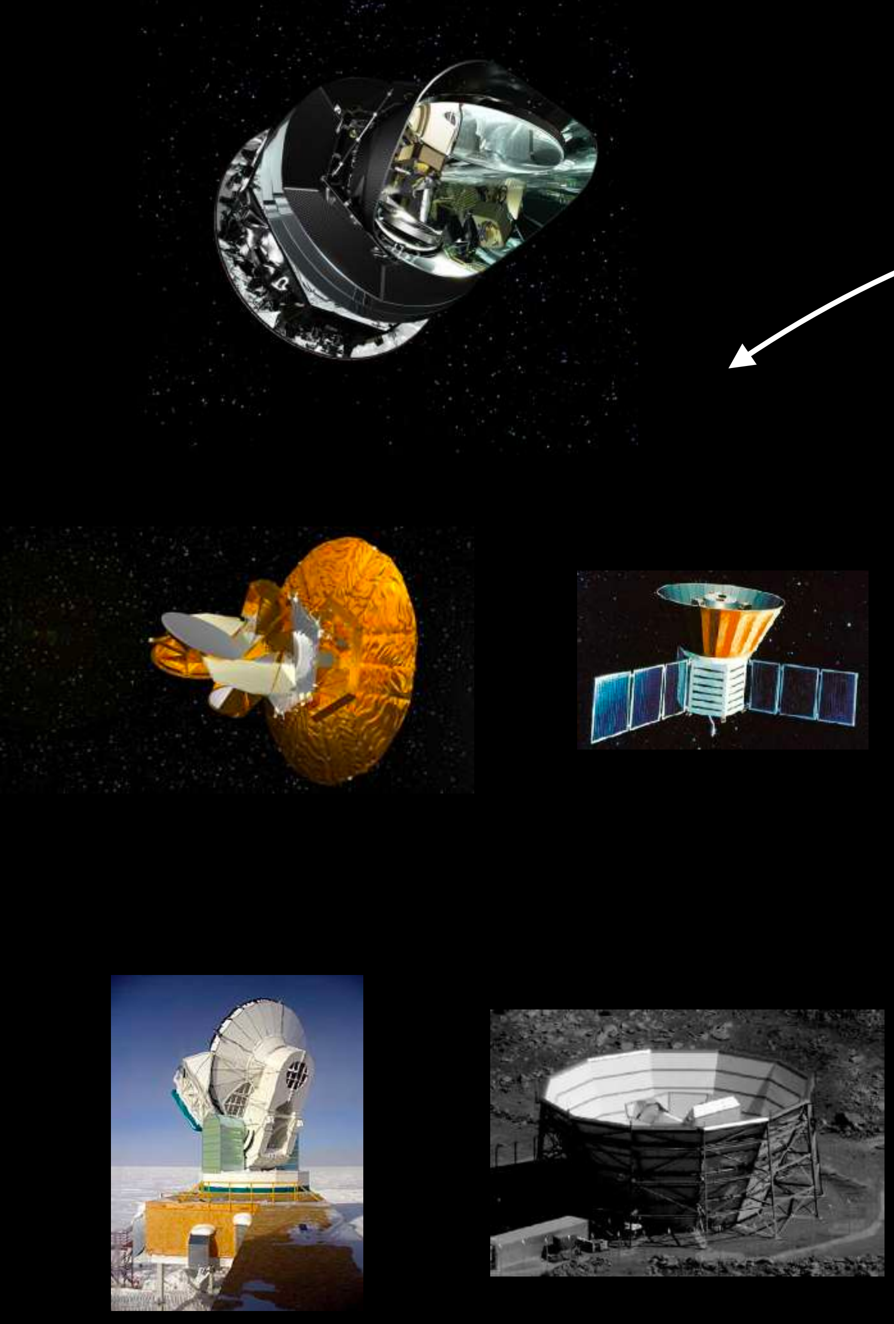
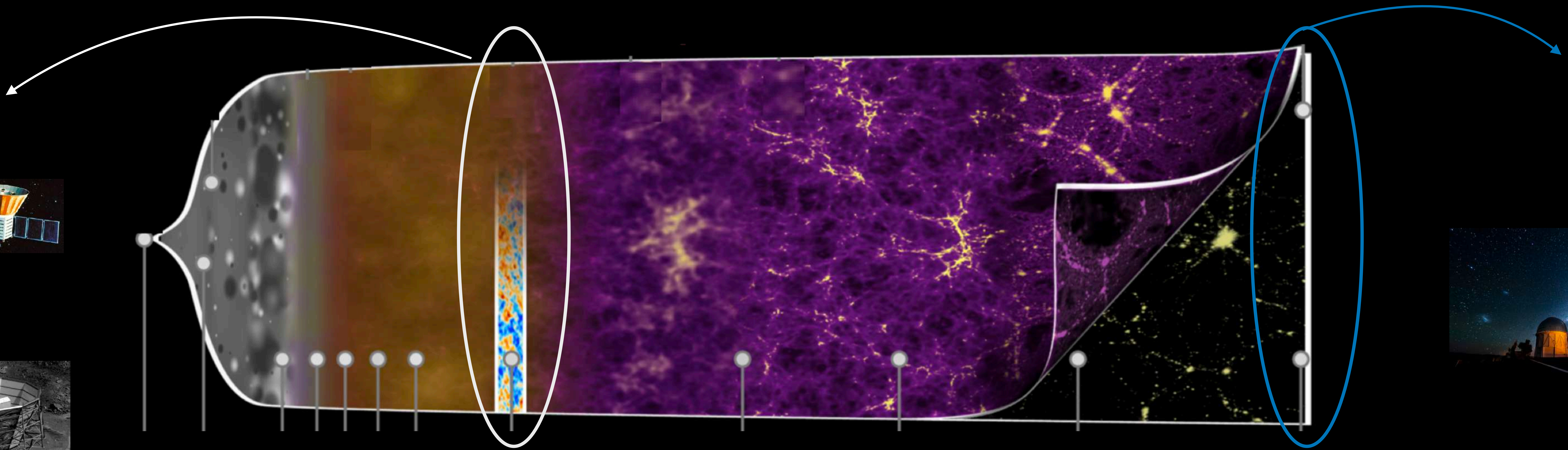
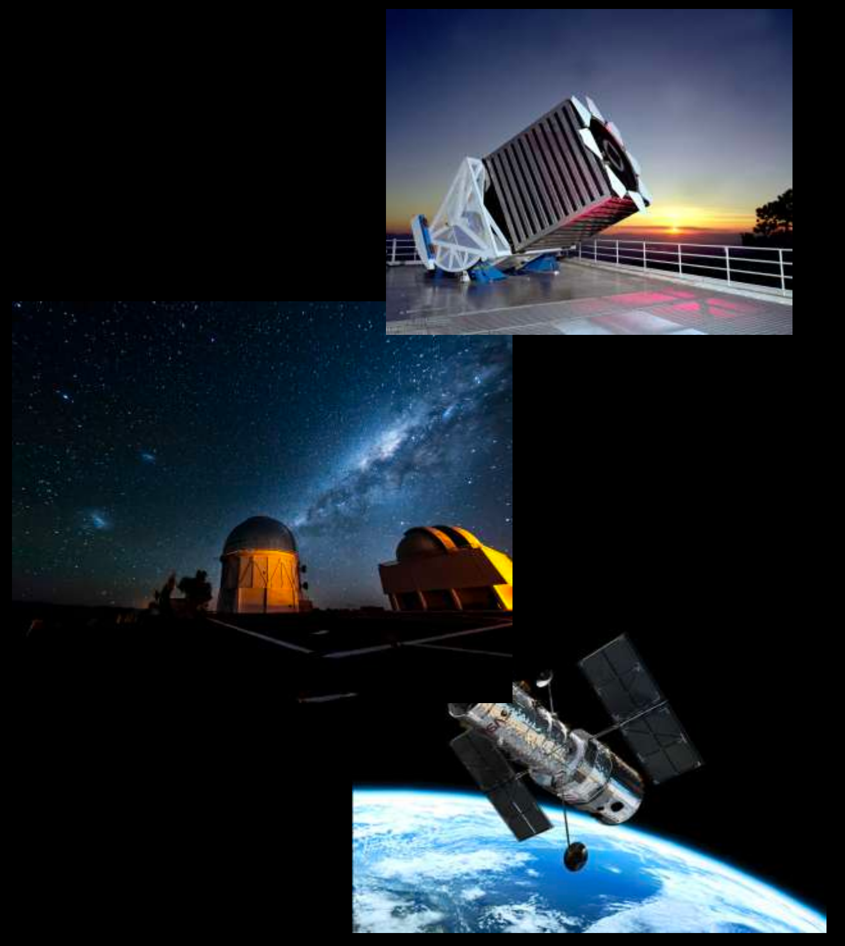






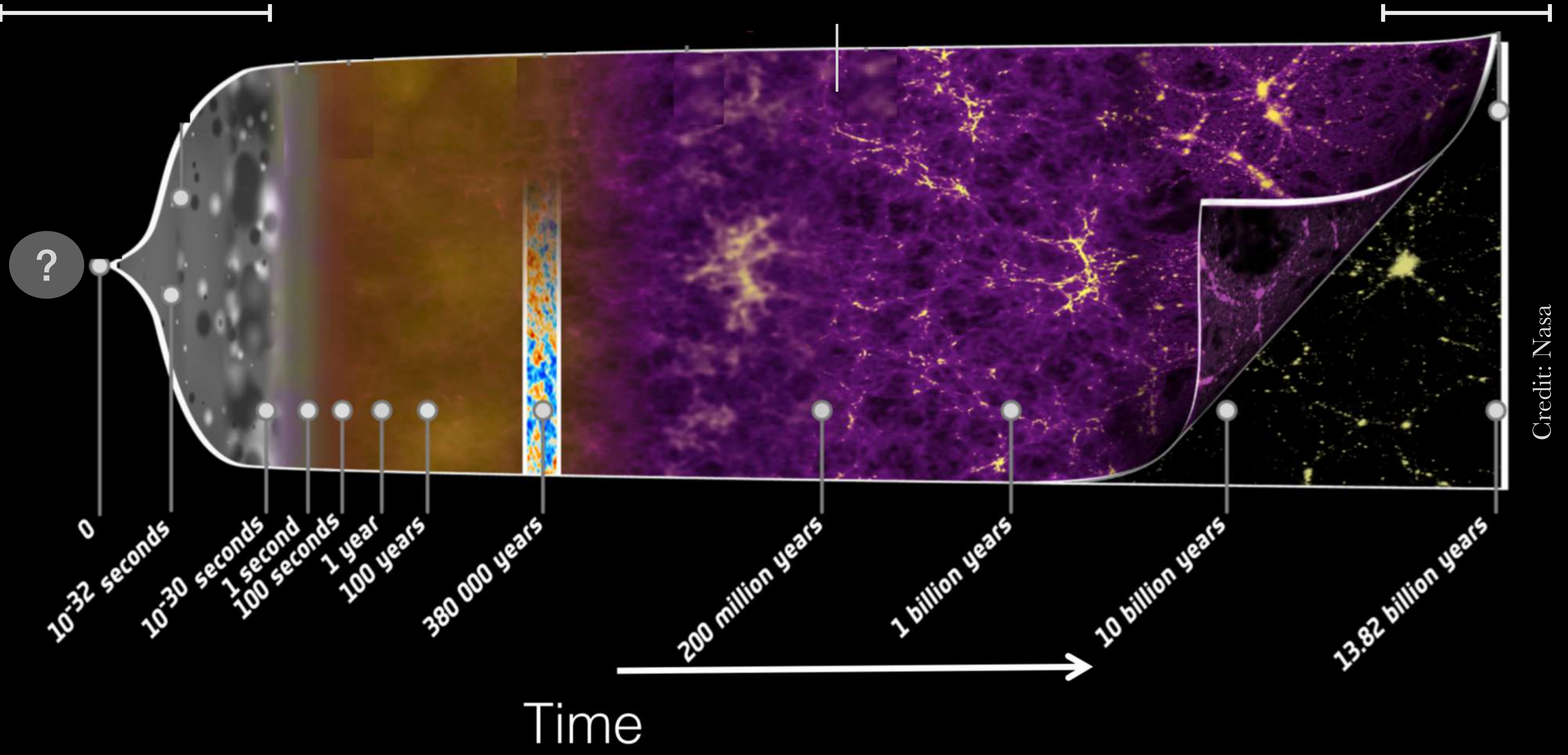
- Theoretical advances
- Observations with growing precision

MANY fundamental open questions

Early universe
Big Bang? What is the physics of the early universe?

Dark matter p
What is the dark matter?

Dark energy -
What is the dark energy?



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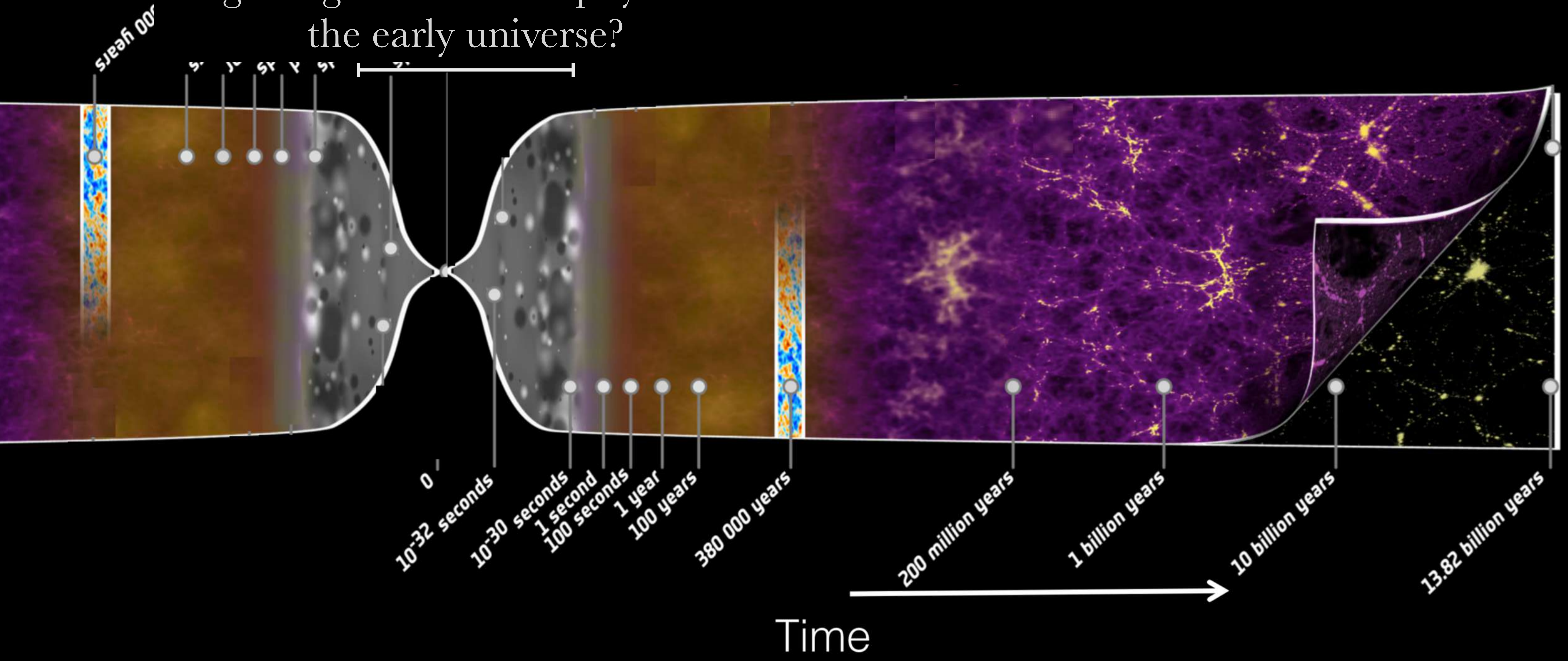

Credit: Nasa

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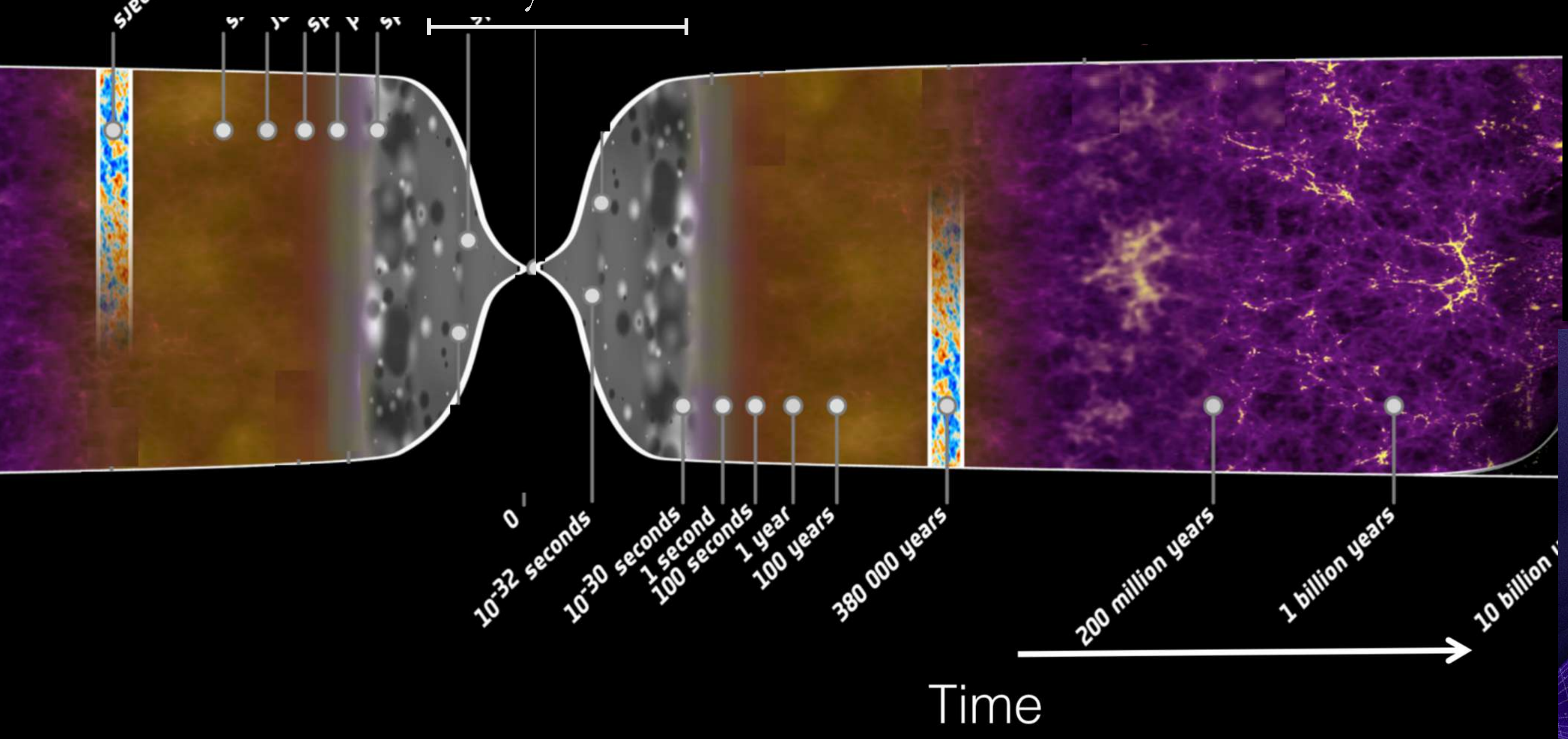
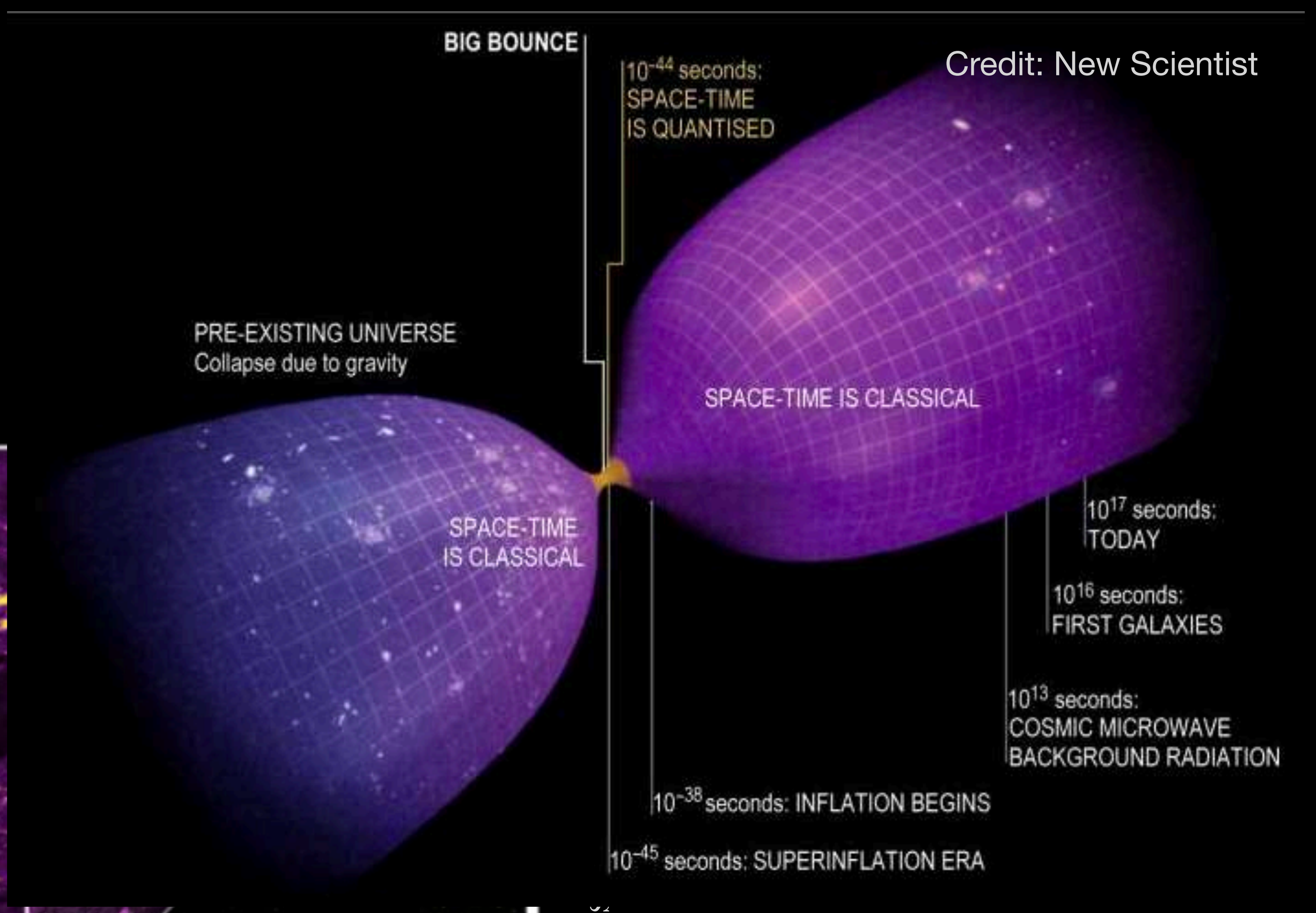
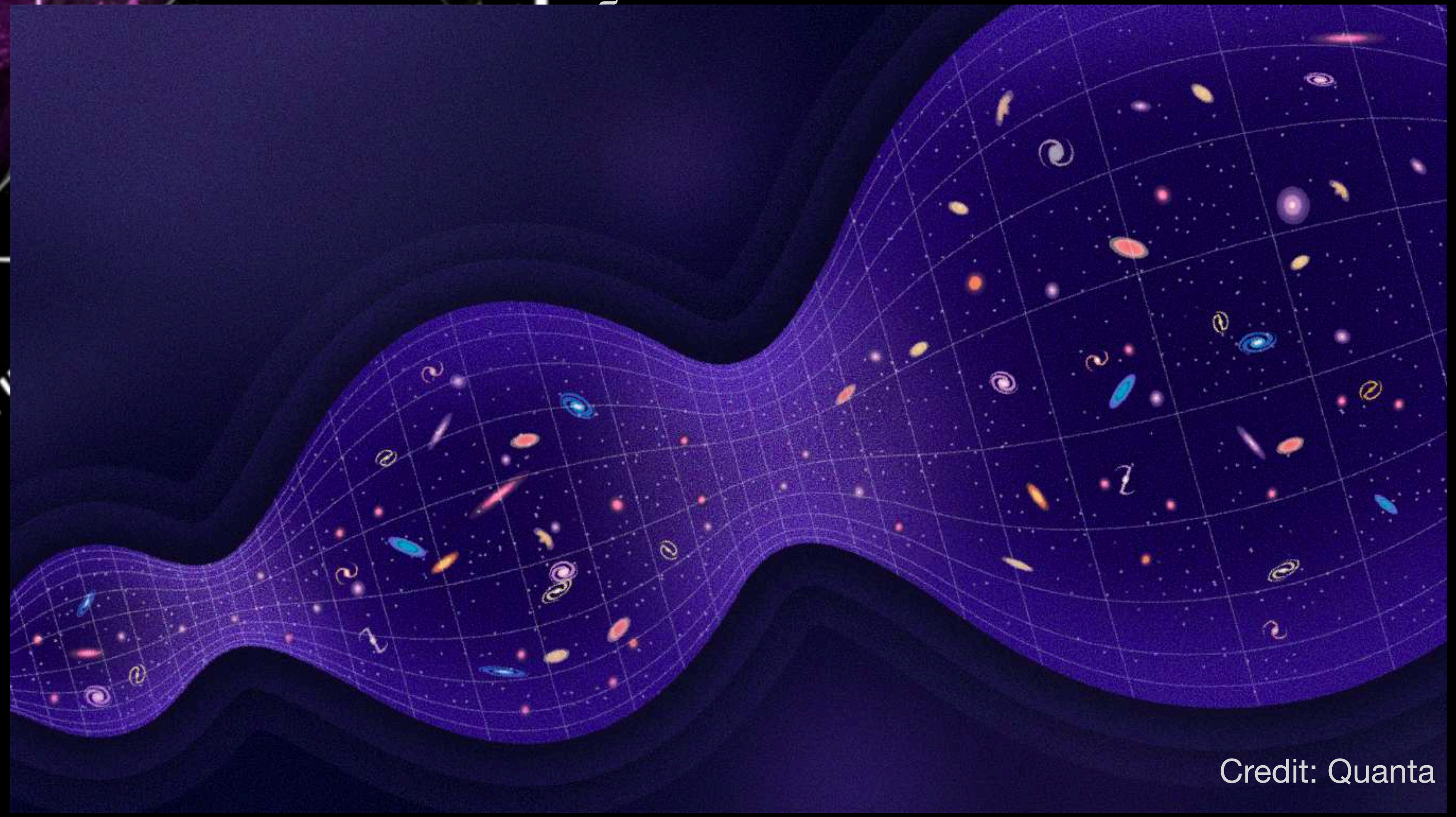




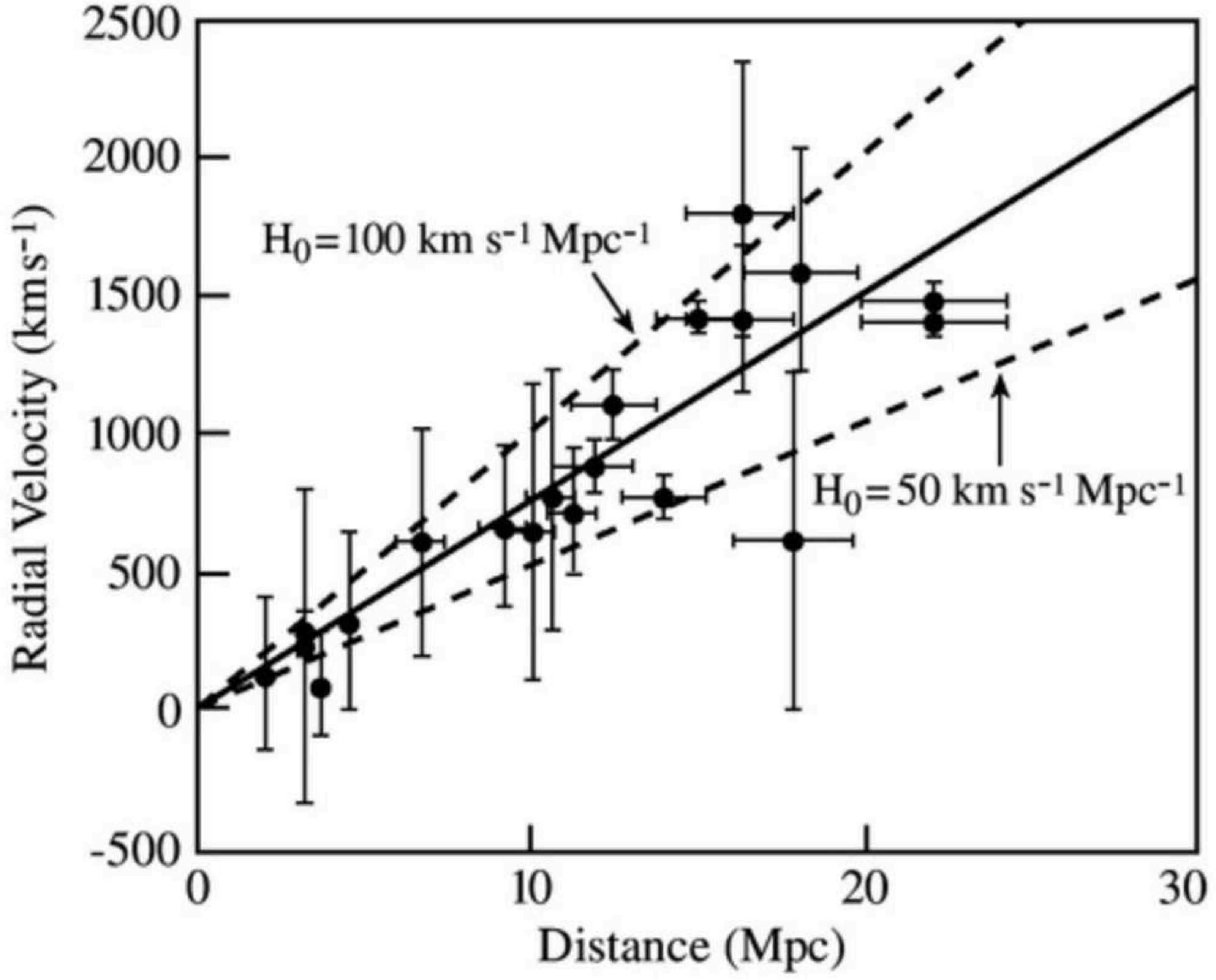


The expansion of the universe

Expanding universe: Hubble-Lemaitre law

Hubble, in 1929, and Lemaître, in 1927, discovered the relation between the recession velocity of galaxies and their distances.

Kenneth R. Lang



$$v = H_0 R$$

Velocity: redshift

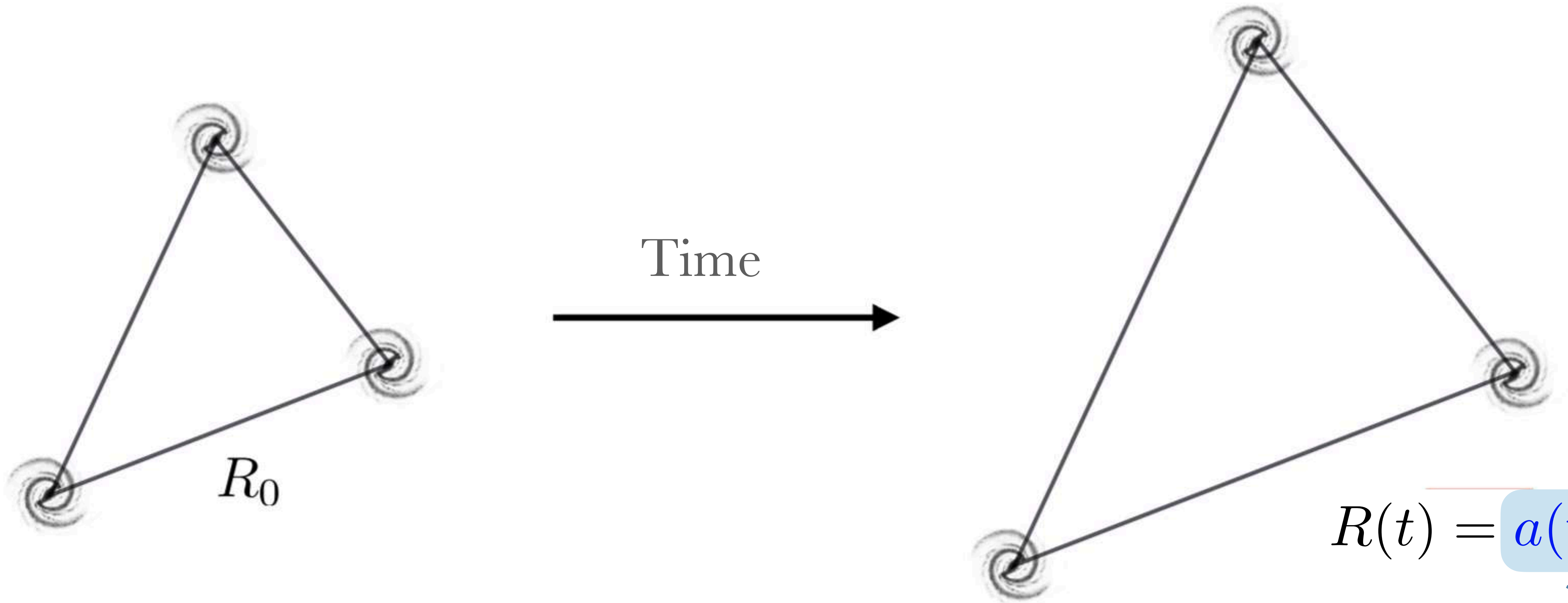
distance: luminosity

H_0 - current rate of expansion

Expanding universe: Hubble-Lemaitre law

In general relativity, we interpret this as the universe expanding. An expansion of the spacebetween galaxies.

Crédito: D. Baumann



$$R(t) = a(t) R_0$$

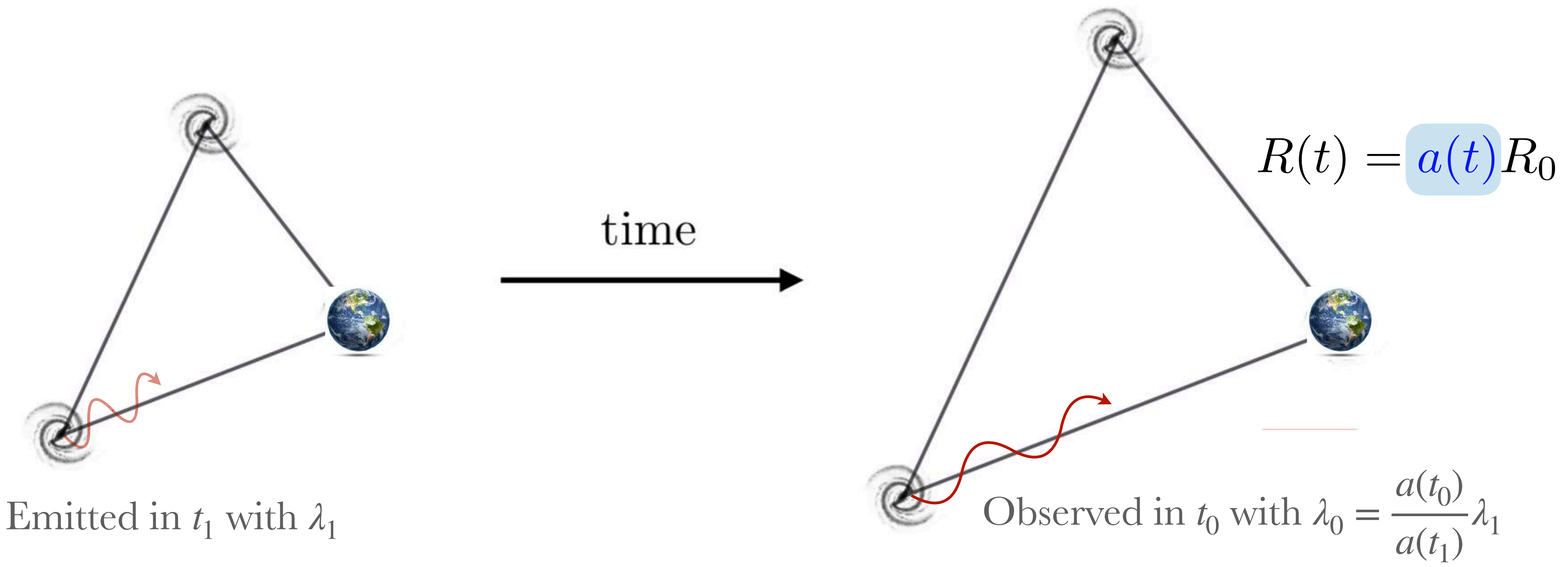
Scale factor

$$v \equiv \dot{R} = \frac{\dot{a}}{a} \equiv H_0 R$$

Hubble parameter (constant): current expansion rate of the universe

Redshift

Hubble's observations of the recession of galaxies was made observing the light emitted by them. This light had a shift in its wavelength towards the red regions in the electromagnetic spectrum, know as *redshift*.



With that, we can define the *redshift* as:

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} = \frac{a(t_0)}{a(t_1)} - 1$$

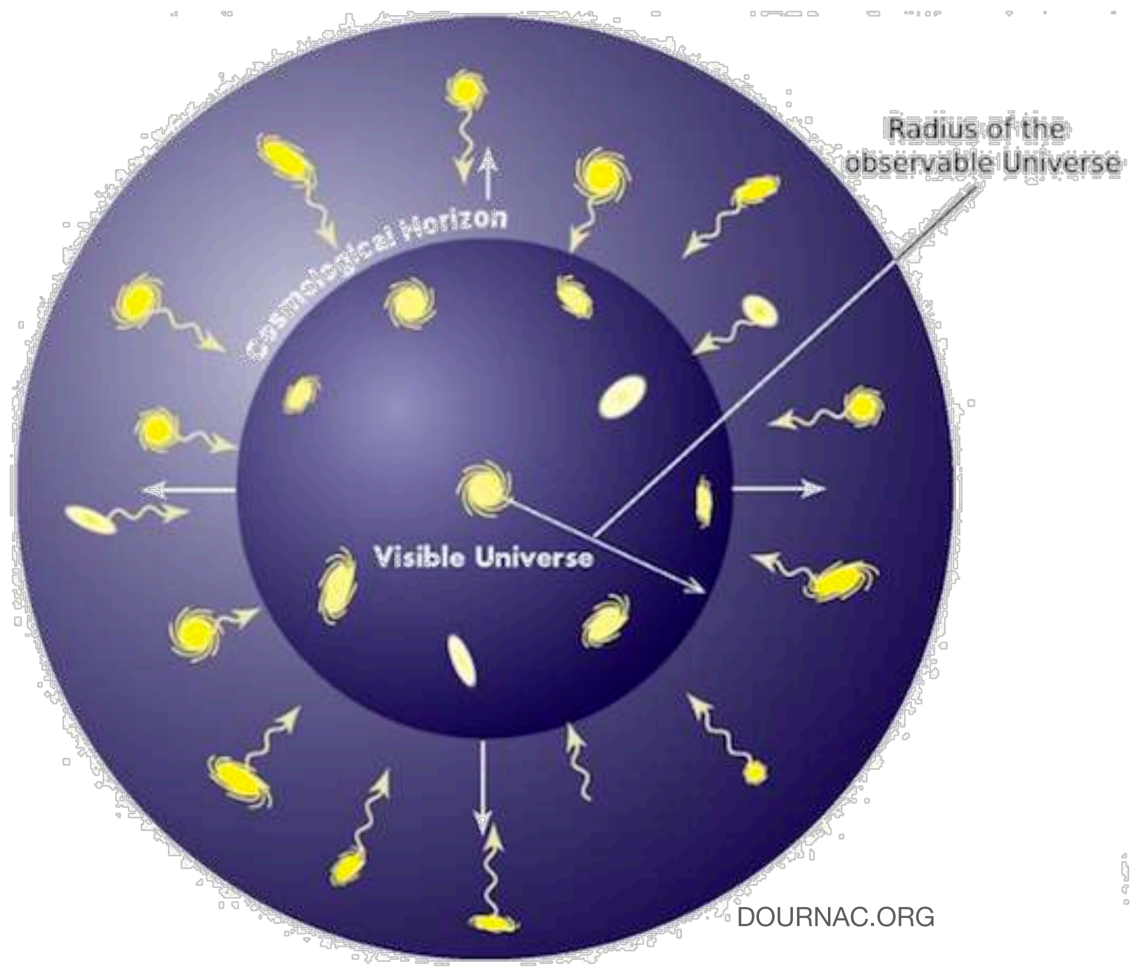
Wavelength ↑
Frequency ↓

Convention: $a(t_0) \equiv 1 \implies z > 0$ *redshift* ; $z < 0$ *blueshift*

Importance of H0

One of the most important numbers in cosmology!

Kinematic quantities in our universe: this number defines the absolute scale of the observable regions of the universe, its characteristic age and size.



If the H_0 value is:

$$H_0 = 68 \pm 2\,\mathrm{km/s/Mpc}$$

$$t_H \equiv H_0^{-1} = 14.38 \pm 0.42\,\mathrm{Gyrs}$$ Hubble age

$$d_H \equiv cH_0^{-1} = 4380 \pm 130\,\mathrm{Mpc} \sim 10^{26}\,\mathrm{m}$$ Hubble radius

Questions?

Dynamics - Friedmann equations

The previous description of the universe is incomplete $\longrightarrow$ it does not provide any prediction about the scale factor, which is the only dynamical quantity present. We need to define the **evolution of the scale factor**.

That is determined by *content of the universe*

Dynamics - Friedmann equations

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That is determined by *content of the universe*

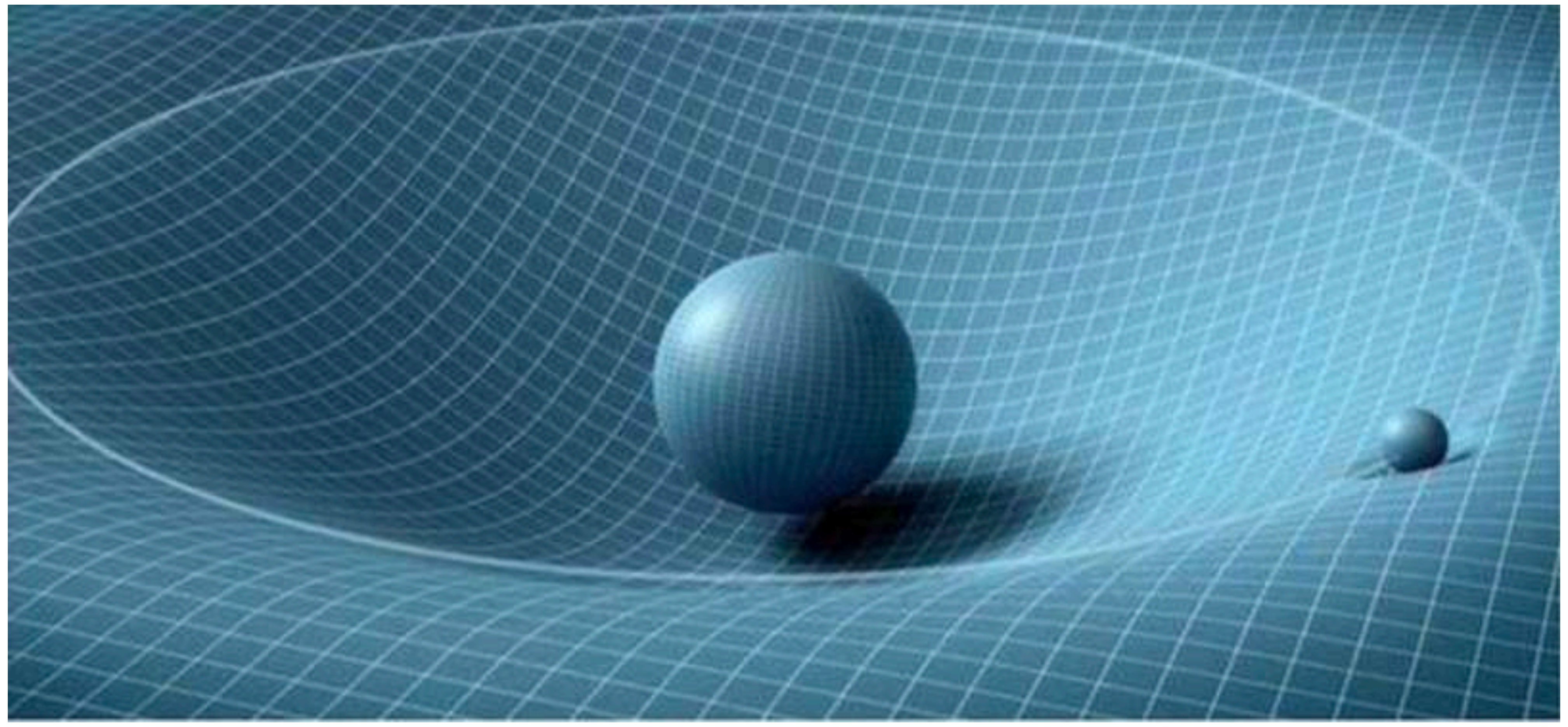
This description is made using **general relativity**

The dynamics and kinematics of our universe are determined by Eisntein's general relativity, where its field equations, valid in all points of the universe, tell us how the content of the universe affects its dynamics.

Useful concepts of general relativity

General relativity: gravity is geometry!

Mass/energy determines the geometry of the space-time
Space geometry determines the movement of mass/energy



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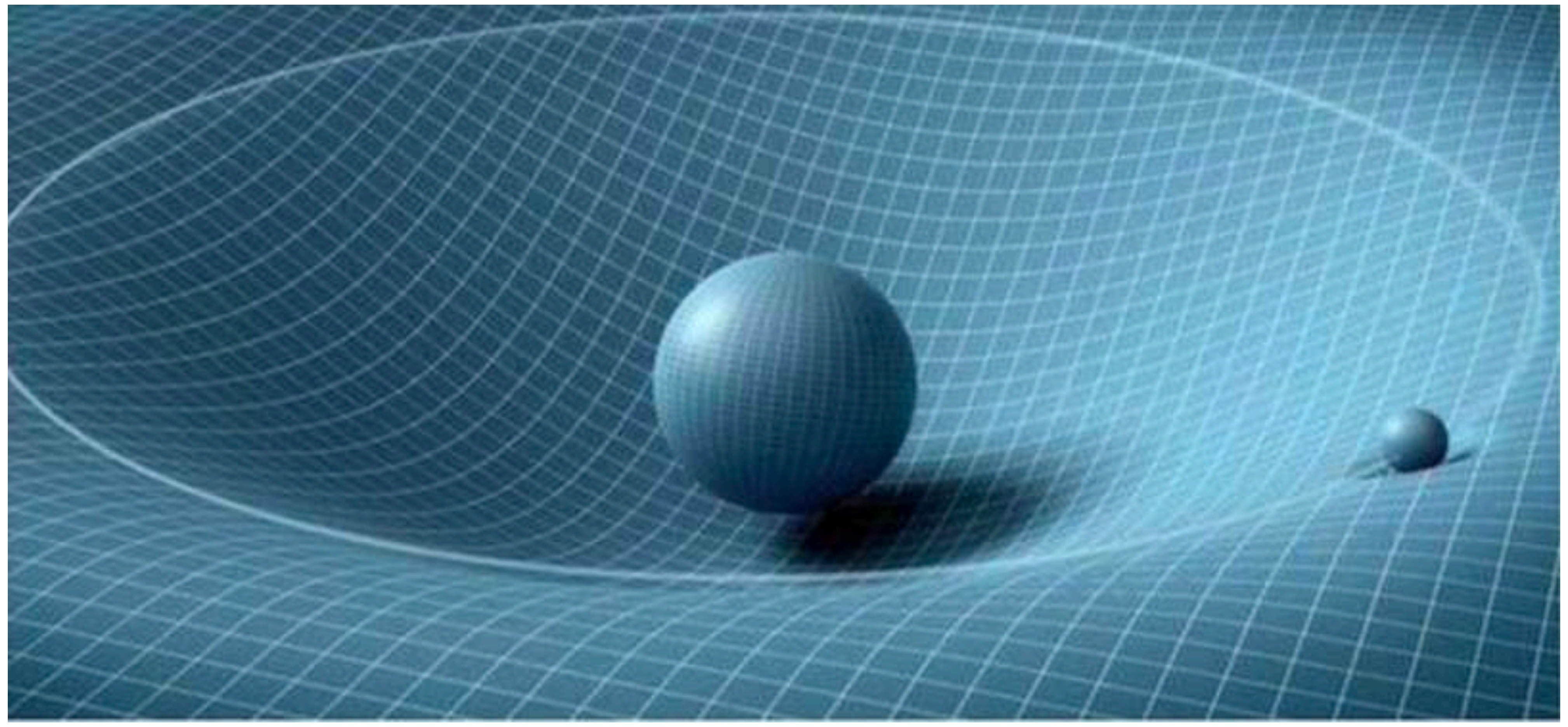
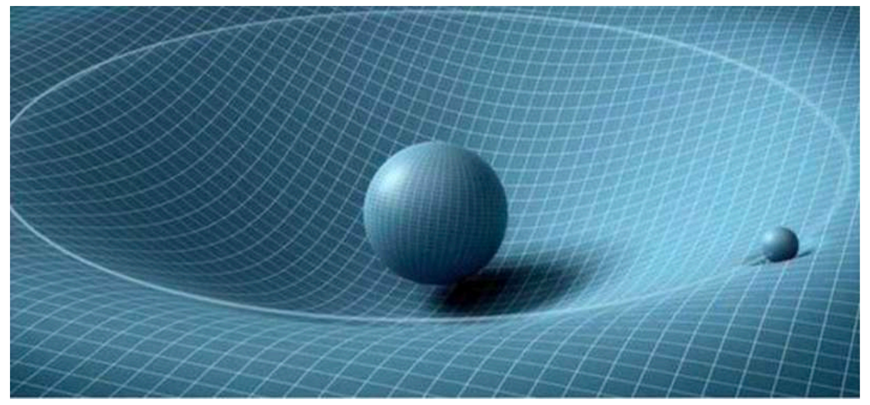


In GR the geometry of the universe is described by the **metric**.

Useful concepts of general relativity



General relativity: gravity is geometry!

In GR the geometry of the universe is described by the **metric.**

Metric: object that transforms distances between coordinates into physical distances

Ex.: In a 3 dimensional Euclidean space, the physical distance between two points separated by infinitesimal distances dx, dy and dz is given by:

$$\mathrm{d}\ell^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 = \sum_{ij=1}^{3} \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j \ ,$$

coordinates: $(x^1, x^2, x^3) = (x, y, z)$

metric: $\delta_{ij} = \mathrm{diag}(1, 1, 1)$

If we use spherical coordinates:

$$\mathrm{d}\ell^2 = \mathrm{d}r^2 + r^2 \mathrm{d}\theta^2 + r^2 \sin^2\theta \, \mathrm{d}\phi^2 \equiv \sum_{ij=1}^{3} g_{ij} \mathrm{d}x^i \mathrm{d}x^j \ ,$$

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Useful concepts of general relativity

Observers using different coordinate systems do not necessarily agree of what is the coordinate distance between two points, but they will ALWAYS agree what is the physical distance $d\ell$!
$d\ell$ is an invariant.

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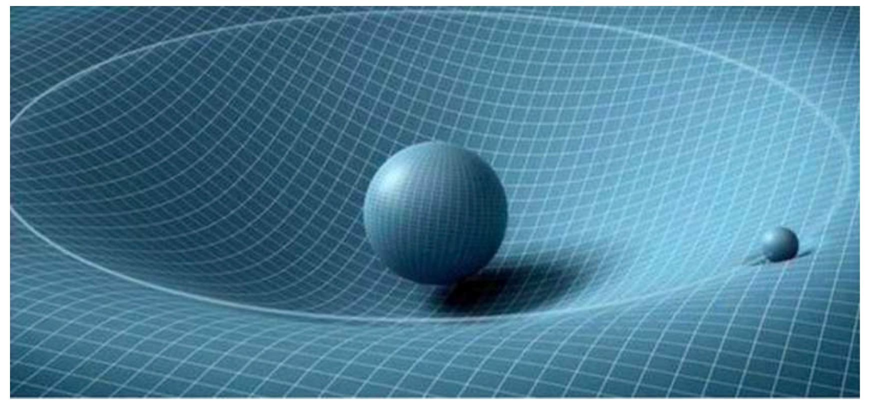

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Metric: object that transforms distances between coordinates into physical distances

Transforms space-time coordinates that depend on the observer $X^\mu = (t, x^i)$ into an invariant line element:

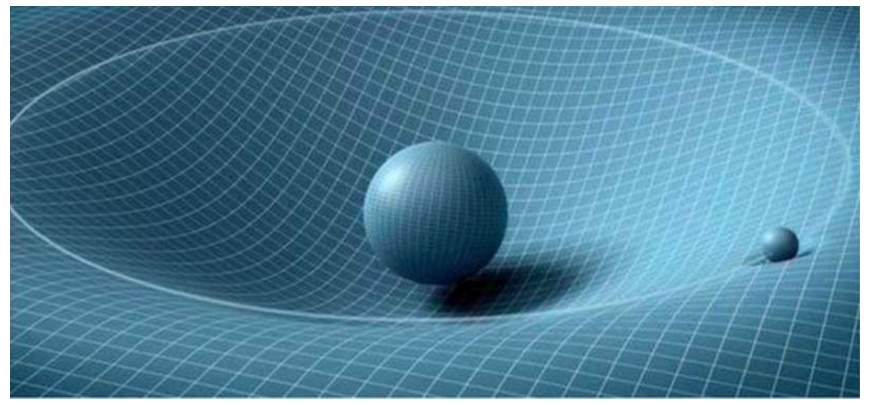
$$ds^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu} dX^\mu dX^\nu \equiv g_{\mu\nu} dX^\mu dX^\nu$$

$g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$: em relatividade especial - métrica de Minkowski — The same in all space-time points

$g_{\mu\nu}(t, \mathbf{x})$: em relatividade geral - métrica depende da local no espaço-tempo → Efeitos da gravidade

As the metric depends on the position, it is determined by the matter/energy content of the universe

Useful concepts of general relativity



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Efeitos da gravidade

Tensors: object which is invariant under change of coordinates and that the coordinates change in a special way given a change of coordinate.

Useful concepts of general relativity

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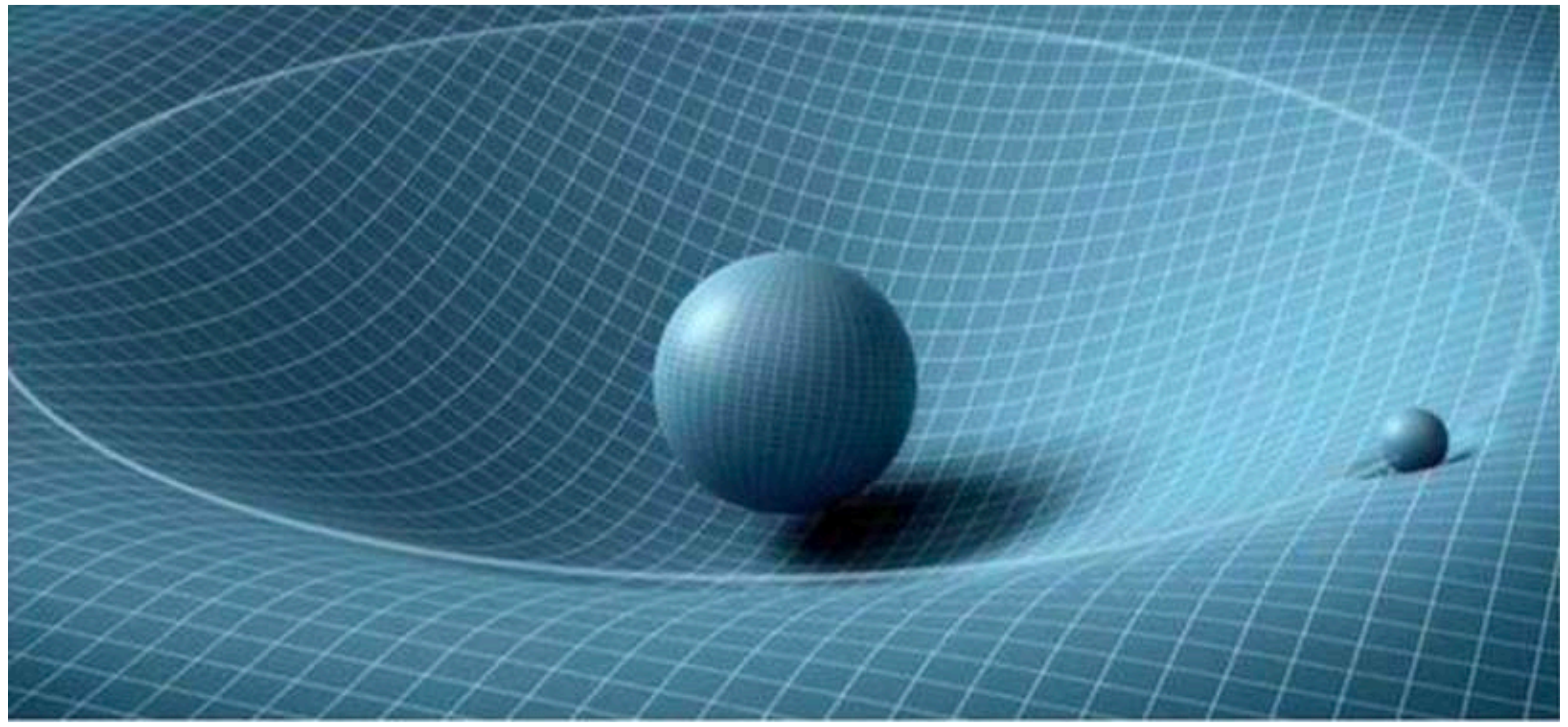
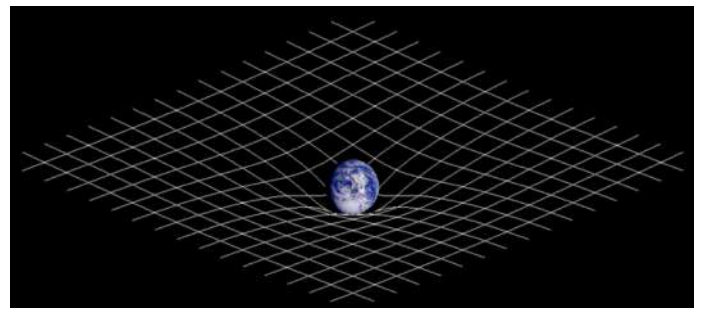
Dynamics



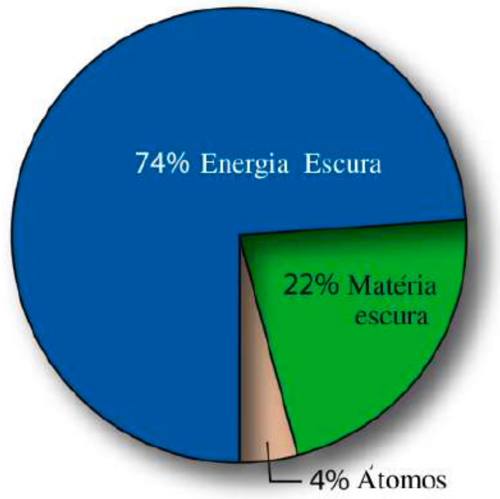
Einstein equations

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$



Geometry-
How universe expands

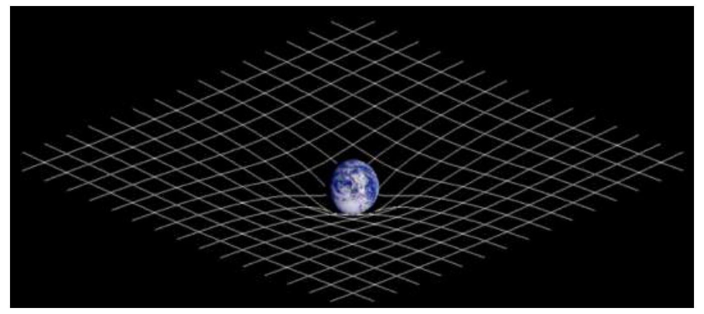
Components



Useful concepts of general relativity

The dynamics and kinematics are determined by the general relativity, given by Einstein's equations:

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$



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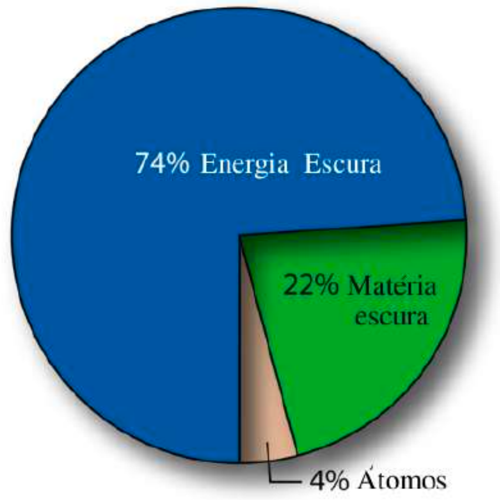

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R : \text{Tensor de Einstein}$$

$$R_{\mu\nu} : \text{Tensor de Ricci}$$

$$g_{\mu\nu} : \text{Métrica}$$

$$R : \text{Escalar de Ricci}$$

$$T_{\mu\nu} : \text{Tensor Energia-Momentum}$$

Questions?

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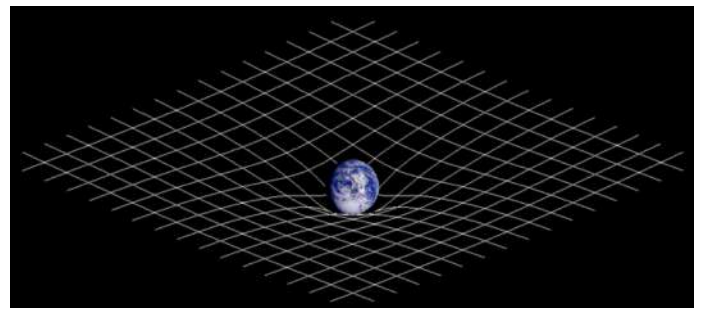

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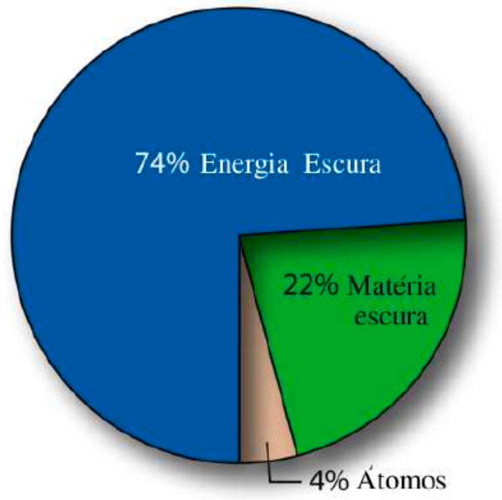

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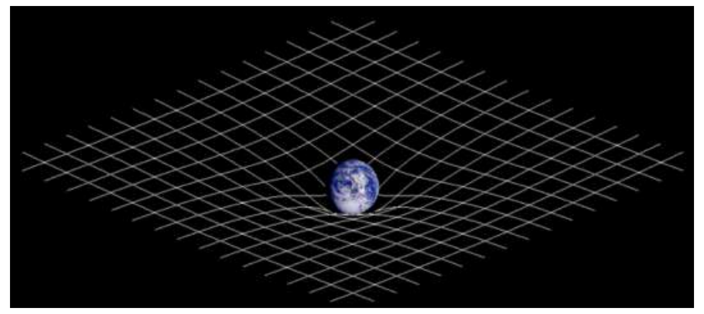

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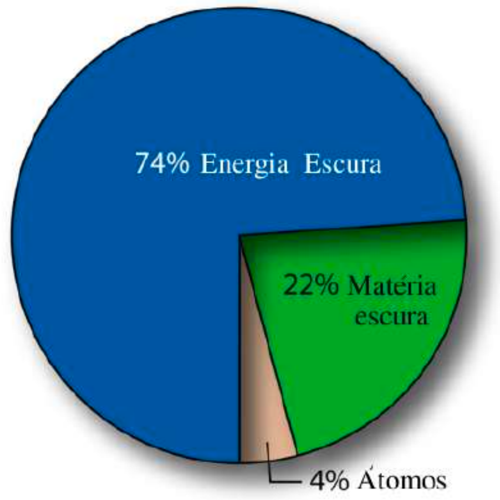


BUT only Einstein's equations are not enough to describe our universe

Structure and evolution of our universe

The dynamics and kinematics are determined by the general relativity, given by Einstein's equations:

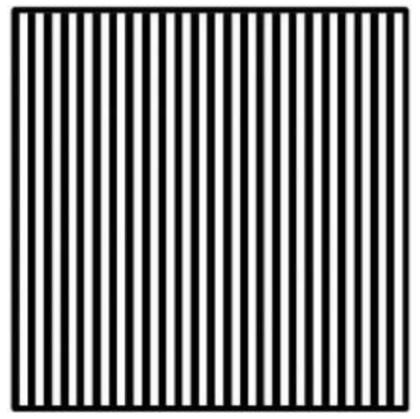
1.

$$\underbrace{G_{\mu\nu}}_{\text{Geometry- How universe expands}} = 8\pi G \underbrace{T_{\mu\nu}}_{\text{Components}}$$

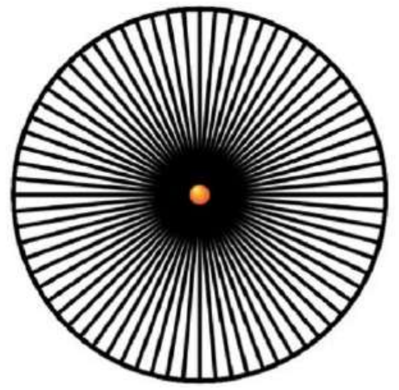
Mas somente as equações de Einstein não são suficientes para chegarmos em equações para a descrição do nosso universo.

2. Cosmological principle: the universe is homogeneous and isotropic on large scales

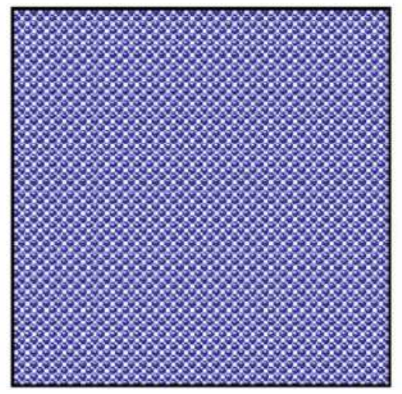
Translation invariance — *Rotations invariance*



Homogeneous (not isotropic)



Isotropic(not homogeneous)



Homogeneous and isotropic

Structure and evolution of our universe

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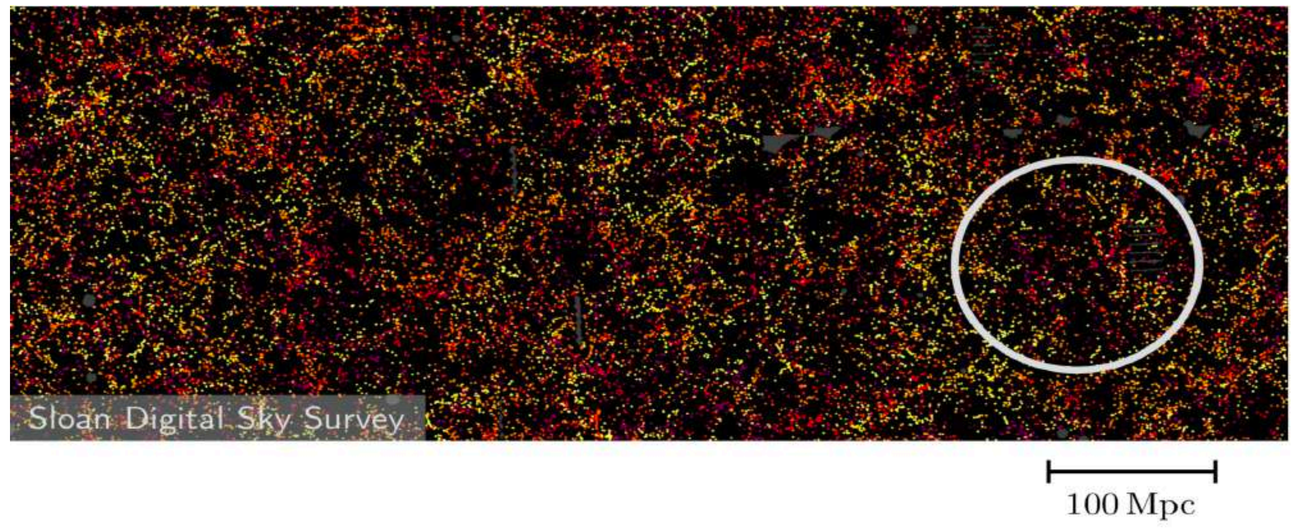
Translation invariance (homogeneous) — *Rotations invariance* (isotropic)



At each time, the universe is the same in each place and direction; the dynamics is the same in all of the universe, except for local irregularities (perturbations - class 3) → spatial properties

Structure and evolution of our universe

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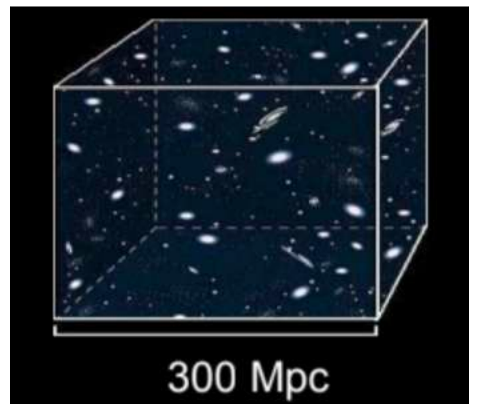
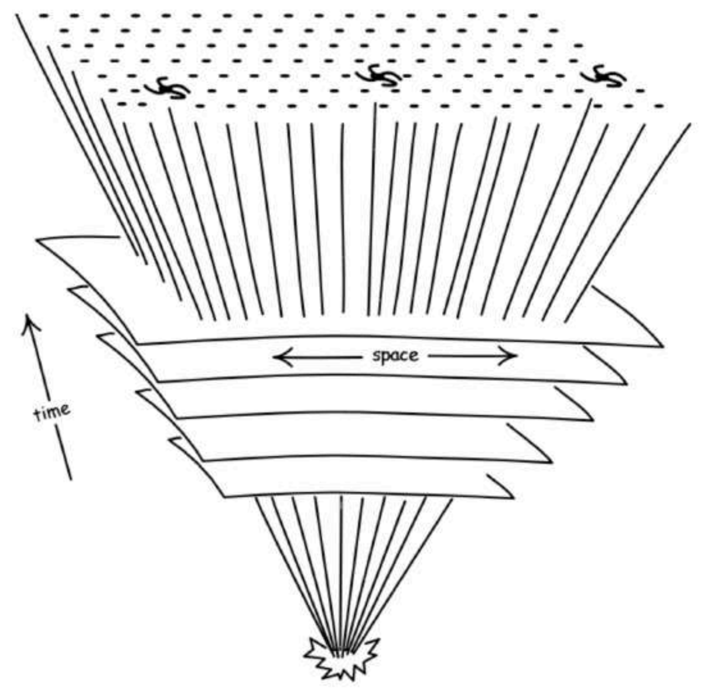
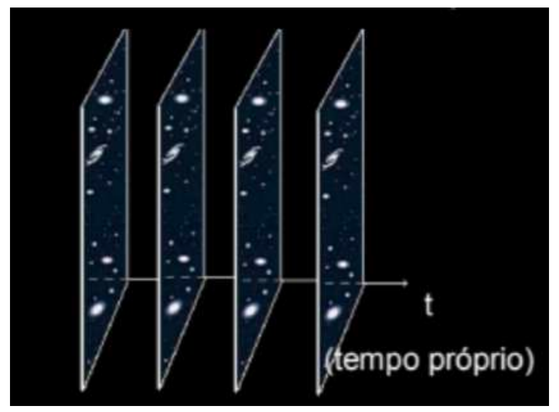
Translation invariance *Rotations invariance*





3. Weyl's postulate:



Comoving observer: it moves together with the expansion of the universe. It is at rest with respect to the subtract that fills the universe.

Structure and evolution of our universe

The dynamics and kinematics are determined by the general relativity, given by Einstein's equations:

1.

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

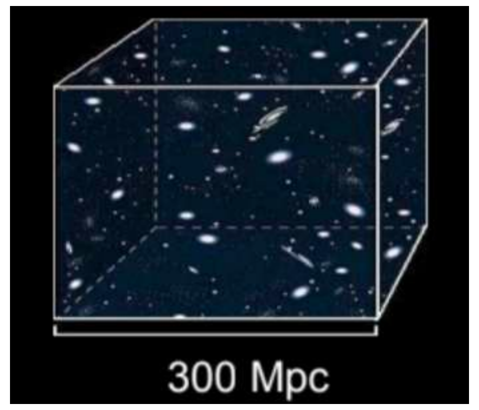
Geometry-
How universe expands

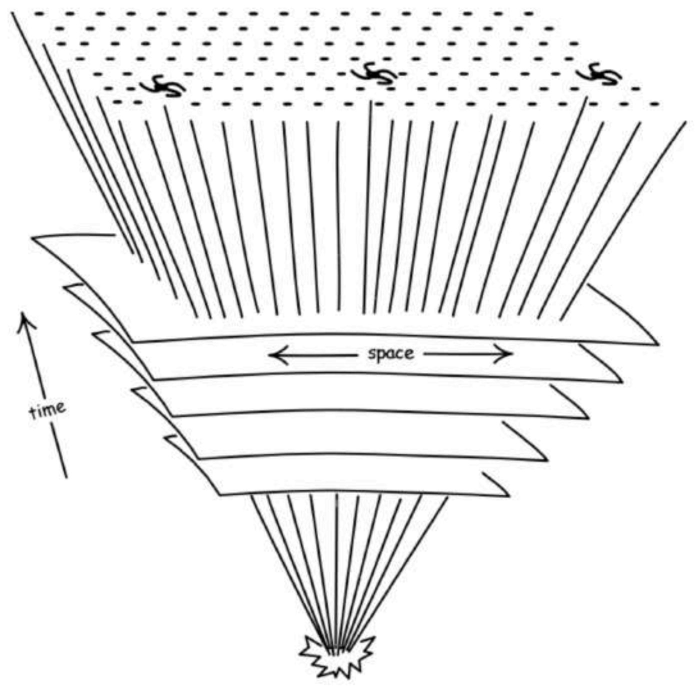
Components

2. Cosmological principle: the universe is homogeneous and isotropic on large scales

Translation invariance

Rotations invariance





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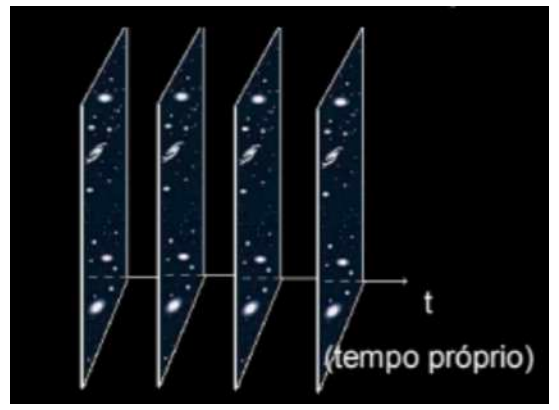
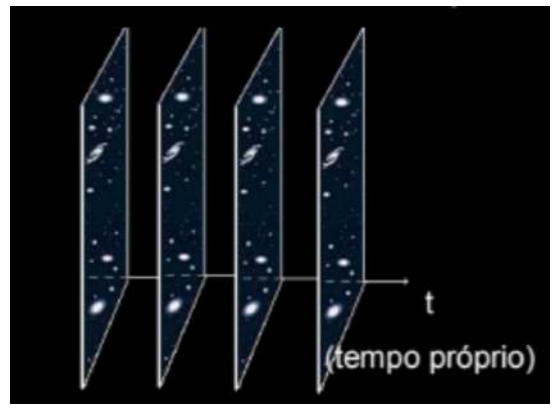


Quantities only depend on **time**!

Friedmann Robertson Walker metric



2.Cosmological principle: homogeneous and isotropic

3. Weyl postulate

These symmetries fix the metric:

$$ds^2 = dt^2 - \boxed{a^2(t)} \times \boxed{d\ell^2}$$

Scale factor

Symmetric 3 dimensional space: we have 3 possible 3d maximally symmetric spaces

$$d\ell^2 = \frac{dr^2}{1 - kr^2} + r^2 \underbrace{d\theta^2 + \sin^2\theta \, d\phi^2}_{d\Omega^2} \;, \qquad k = \begin{cases} 0 & \mathrm{E}^3 \quad \text{(Flat)} \\ +1 & \mathrm{S}^3 \quad \text{(Spherical)} \\ -1 & \mathrm{H}^3 \quad \text{(Hyperbolic)} \end{cases}$$

$$\boxed{ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{a - kr^2} + r^2 d\Omega^2\right]}$$

FRW metric

Friedmann Robertson Walker metric

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FRW metric

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{a^2(t)}{1-kr^2} & 0 & 0 \\ 0 & 0 & a^2(t)\, r^2 & 0 \\ 0 & 0 & 0 & a^2(t)\, r^2 \mathrm{sen}^2\theta \end{pmatrix} .$$

Friedmann Robertson Walker metric

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$(+,-,-,-)$

$(-,+,+,+)$

Friedmann Robertson Walker metric

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$$\boxed{ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{a - kr^2} + r^2 d\Omega^2\right]}$$

FRW metric

$$g_{\mu\nu} = diag\left(-1\,, \frac{a(t)^2}{1 - kr^2}\,, a(t)^2\, r^2\,, a(t)^2 r^2 \sin^2\theta\right)$$

$(+ , - , - , -)$

$(- , + , + , +)$

Friedmann Robertson Walker metric

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Properties:

$$a \to \lambda a\,, \quad r \to r/\lambda\,, \quad k \to \lambda^2 k\,,$$

- Parametrization invariant

Allows us to choose: $a(t_0) = 1$

$$\boxed{ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{a - kr^2} + r^2 d\Omega^2\right]}$$

FRW metric

$(+,-,-,-)$

Friedmann Robertson Walker metric

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$$ds^2 = dt^2 - \underbrace{a^2(t)}_{\text{Scale factor}} \times d\ell^2$$

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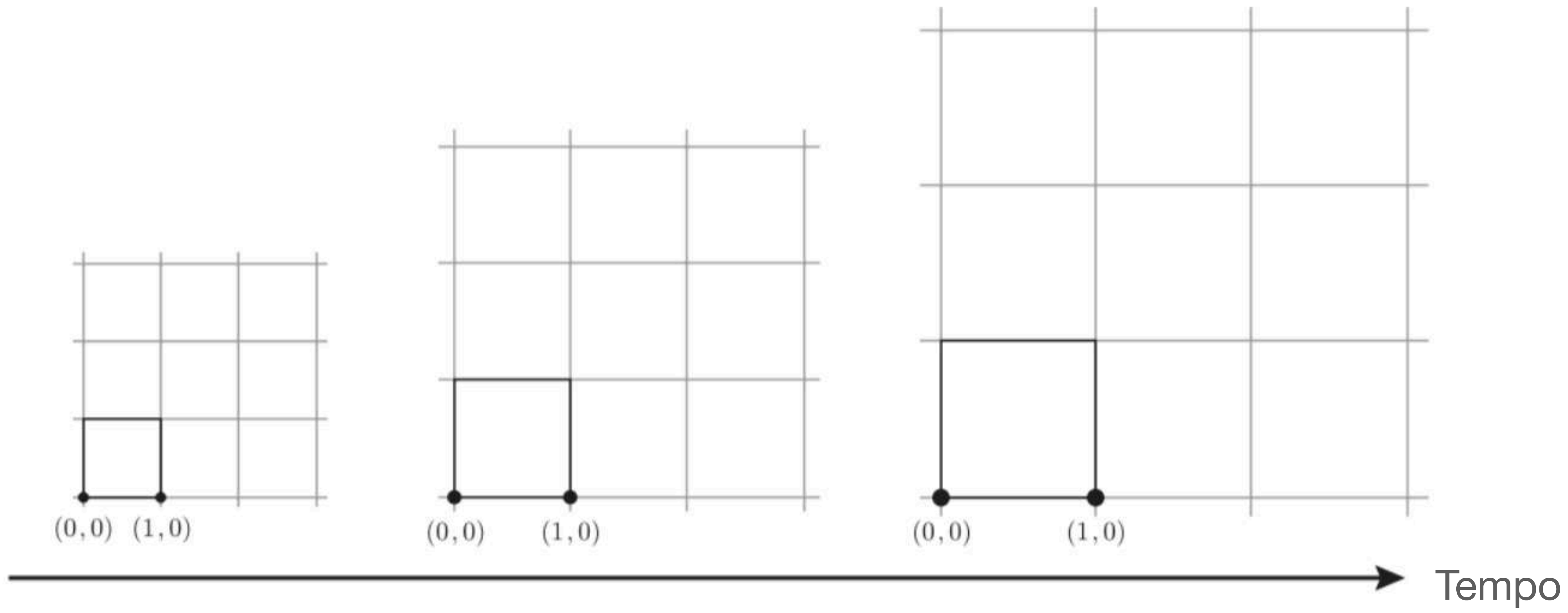

Properties:

- Reparametrization invariant: $a \to \lambda a \,, \quad r \to r/\lambda \,, \quad k \to \lambda^2 k \,,$
- r is a comoving coordinate. The physics depends only on the physical coord.: $r_{fis} = a(t) r$

(Physical curvature) $k_{fis} = k/a^2(t)$

$$v_{fis} = \frac{dr_{fis}}{dt} = \frac{da}{dt} r + a\frac{dr}{dt} = H a r + a\frac{dr}{dt} = H\, r_{fis} + v_{pec}$$ Hubble's law

Crédito: D. Baumann

Friedmann Robertson Walker metric

2.Cosmological principle: homogeneous and isotropic

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- r is a comoving coordinate. The physics depends only on the physical coord.: $r_{fis} = a(t)r$

  (Physical curvature) $k_{fis} = k/a^2(t)$
- Comoving time is called ***conformal time*** $d\eta = dt/a(t)$

$$ds^2 = a^2(\eta)\left[d\eta^2 - \frac{dr^2}{1-kr^2} - r^2 d\Omega^2\right] = a^2(\eta) \times \text{static metric}$$

Questions?

Dynamics - Friedmann equations

The dynamics and kinematics are determined by the general relativity, given by Einstein's equations:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Energy momentum tensor

GR cannot define the characteristics for the form of the energy momentum tensor

$$T_{\mu\nu} = \left(\begin{array}{c|c} T_{00} & T_{0i} \\ \hline T_{i0} & T_{ij} \end{array}\right) = \left(\begin{array}{c|c} \text{energy density} & \text{energy flux} \\ \hline \text{momentum density} & \text{stress tensor} \end{array}\right)$$

Given the previous hypotheses, the EMT has to obey the following properties:

A comoving observer only sees a homogeneous and isotropic universe if

- The scalar part is a function only of time. $T_{00} = \epsilon(t)$
- Vetor part is absent $T_{0i} = 0$
- Tensorial part is proportional to g_{ij} $T_{ij} = -P(t)\, g_{ij}$

Before we had mass density ρ.
Now we have energy density:

$$\epsilon = \rho c^2$$

As most of the time we use $c = 1$ (natural units), they are used interchangeably.
We are going to use ρ!

Energy momentum tensor

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$$T^{\mu}{}_{\nu} \equiv g^{\mu\lambda} T_{\lambda\nu} = \begin{pmatrix} \rho & & & \\ & -P & & \\ & & -P & \\ & & & -P \end{pmatrix}$$

Perfect fluid EMT

$$U_\mu = (1, 0, 0, 0)$$

$\rho(t)$: densidade de energia

$P(t)$: pressão

In the fluid rest frame

U_μ : 4-velocidade relativa

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Perfect fluid EMT

$$U_\mu = (1, 0, 0, 0)$$

For a ***general observer:***

$$T^{\mu}{}_{\nu} = (\rho + P) U^{\mu} U_{\nu} - P \delta^{\mu}_{\nu}$$

$\rho(t)$: densidade de energia

$P(t)$: pressão

In the fluid rest frame

U_μ : 4-velocidade relativa

Dynamics - Friedmann equations

The dynamics and kinematics are determined by the general relativity, given by Einstein's equations:

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Using:

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{a - kr^2} + r^2 d\Omega^2\right]$$

FRW metric

+

$$T^{\mu}{}_{\nu} = (\rho + P)U^{\mu}U_{\nu} - P\delta^{\mu}_{\nu}$$

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$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}(\rho + 3P)$$

Friedmann equations.
(or Friedmann - Lemaître)

Dynamics - Friedmann equations

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Combining these equations (taking the derivative of the first and using the second)

$$\dot{\rho} + 3H(\rho + P) = 0$$

Continuity equation:

conservation of the energy density

Dynamics - Friedmann equations

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Continuity equation:

conservation of the energy density

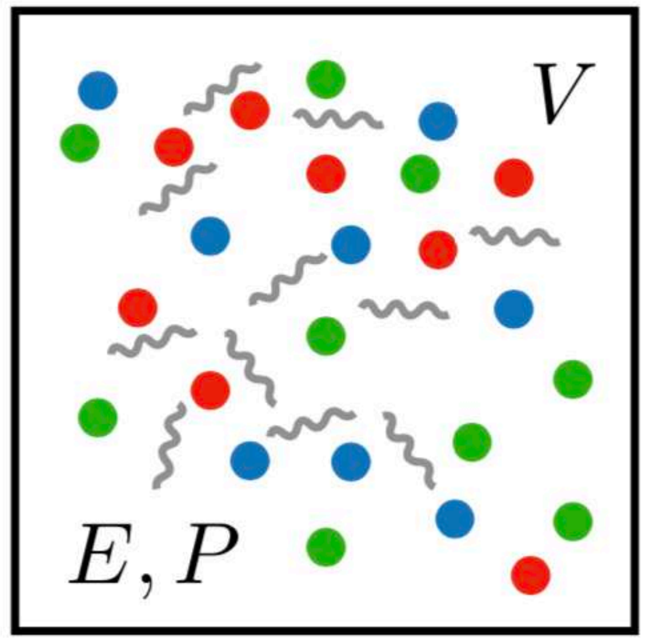
Comes from the conservations of the EMT:

$$T^{\mu}_{\nu;\mu} = 0$$

This equation is also called "adiabaticity condition"

PS: These 3 equations are linearly dependent - we can only use two of them at the same time!

From the first law of thermodynamics we have that:



Crédito: D. Baumann

$$dE = TdS - pdV$$

em que E é a energia total, T é a temperatura, S a entropia e V o volume. Como $\rho = E/V$, ela assume a forma:

$$d\rho = TdS - (\rho + p)\,\frac{dV}{V}\,.$$

Sabendo que o volume do universo é da ordem $V \propto a^3$, assim $dV/V = 3da/a$. Substituindo isso na equação da continuidade (2.22), obtemos que $\dot{S} = 0$, ou seja, a entropia do universo é constante. Podemos ver que a expansão do universo, regida pelas equações de Friedmann, é adiabática.

Dynamics - Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$
$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}(\rho + 3P)$$

Friedmann equations.
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ρ and P here are actually the sum of all the components in the universe $\Longrightarrow$ ρ_{tot}, p_{tot}

We can also rewrite the 1st Friedmann equation as:

$$1 = \Omega_{tot} - \frac{k}{a^2 H^2}$$

Density parameter

$$\Omega_{tot} = \sum_i \Omega_i \, , \qquad \text{onde } \Omega_i = \frac{\rho_i}{\rho_{crit}}$$

Dynamics - Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$
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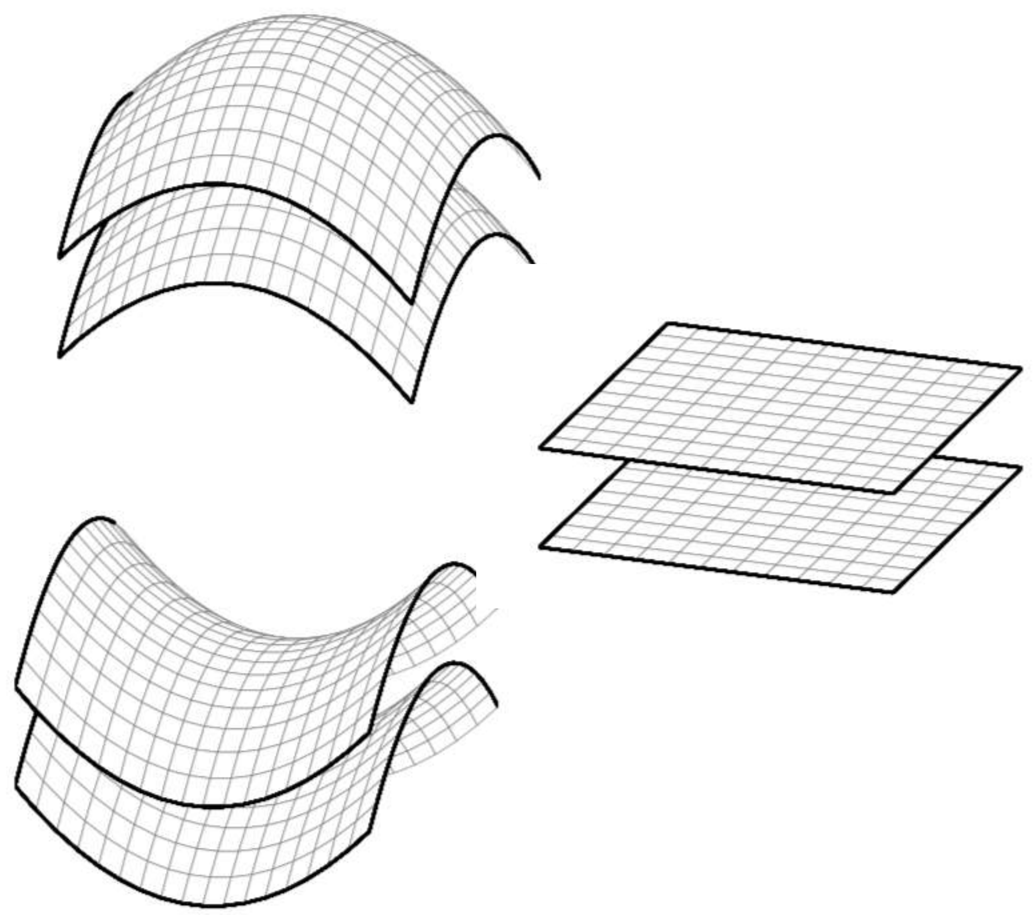
$\Omega_{total} > 1 \Leftrightarrow k = +1\,,$ Universo fechado

$\Omega_{total} = 1 \Leftrightarrow k = 0\,,$ Universo plano

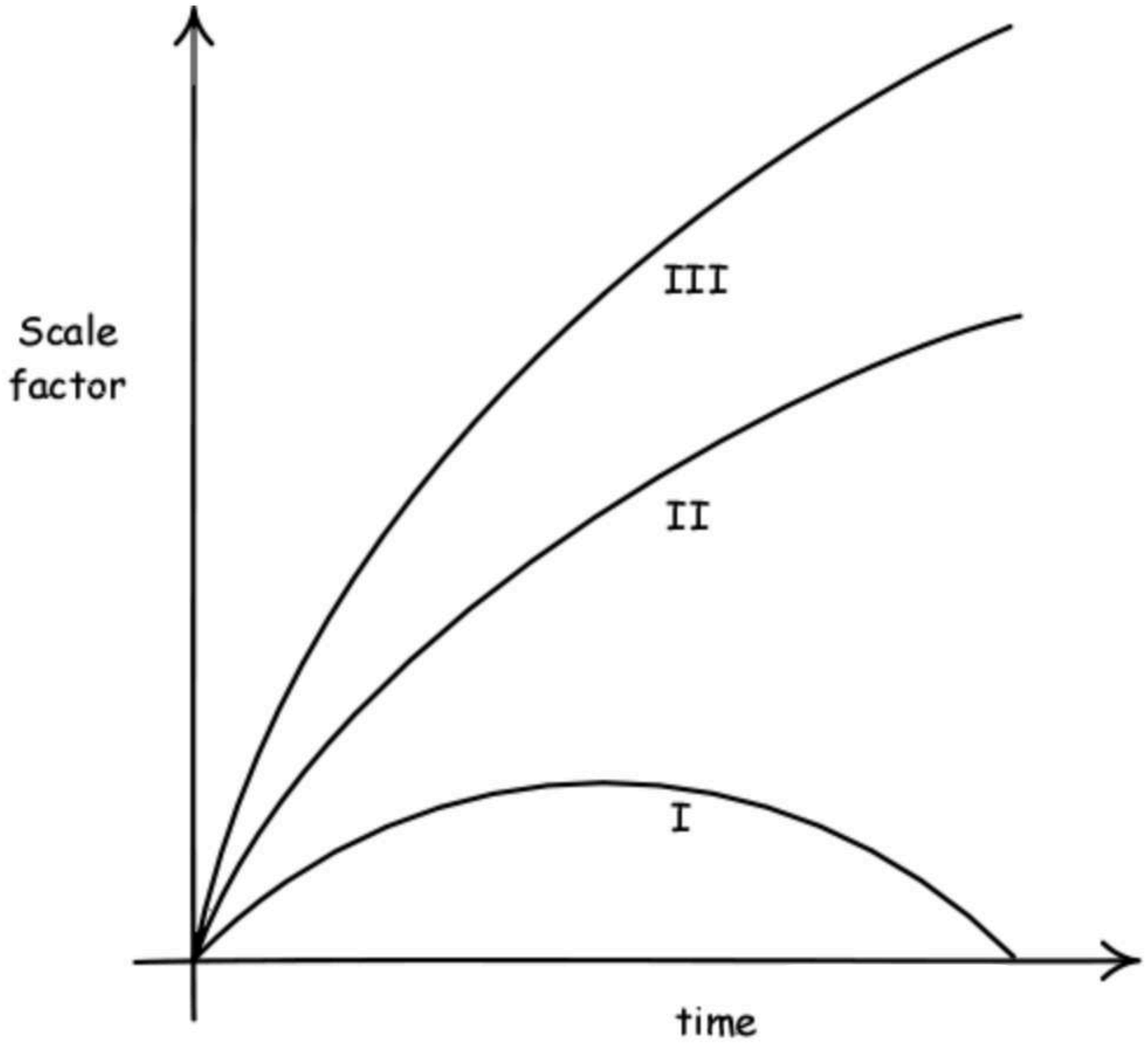
$\Omega_{total} < 1 \Leftrightarrow k = -1\,,$ Universo aberto



Crédito: D. Baumann

Observations favor a flat universe!

Dynamics - Friedmann *equations*



Crédito: J. D. Horton

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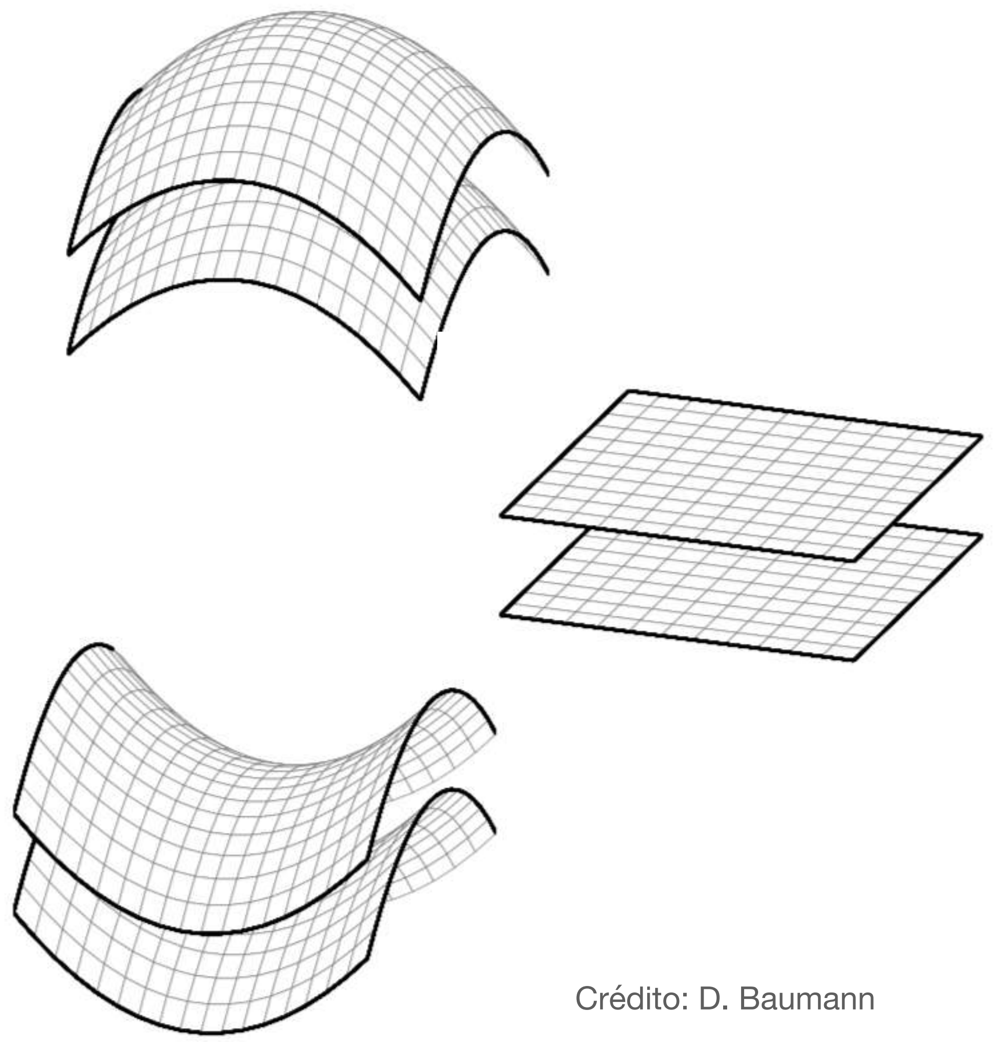

Crédito: D. Baumann

Dynamics - Friedmann equations

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$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}(\rho + 3P)$$

Friedmann equations.
(or Friedmann - Lemaître)

The Friedmann equations (and continutiy) sare a system of equations that provide the dynamics of *a(t), ρ(t) and p(t)*.

However, to fully determine it (3 variable, 2 equations) we need a system of 3 linearly independent equations

What is missing is the information about the **characteristic of the fluids** present in the universe.

Each fluid will induce a different behaviour of the universe.

Questions?

Components of the universe

To describe a homogenous universe, we use perfect fluids, following the equation:

$$\dot{\rho} + 3H\left(\rho + P\right) = 0$$

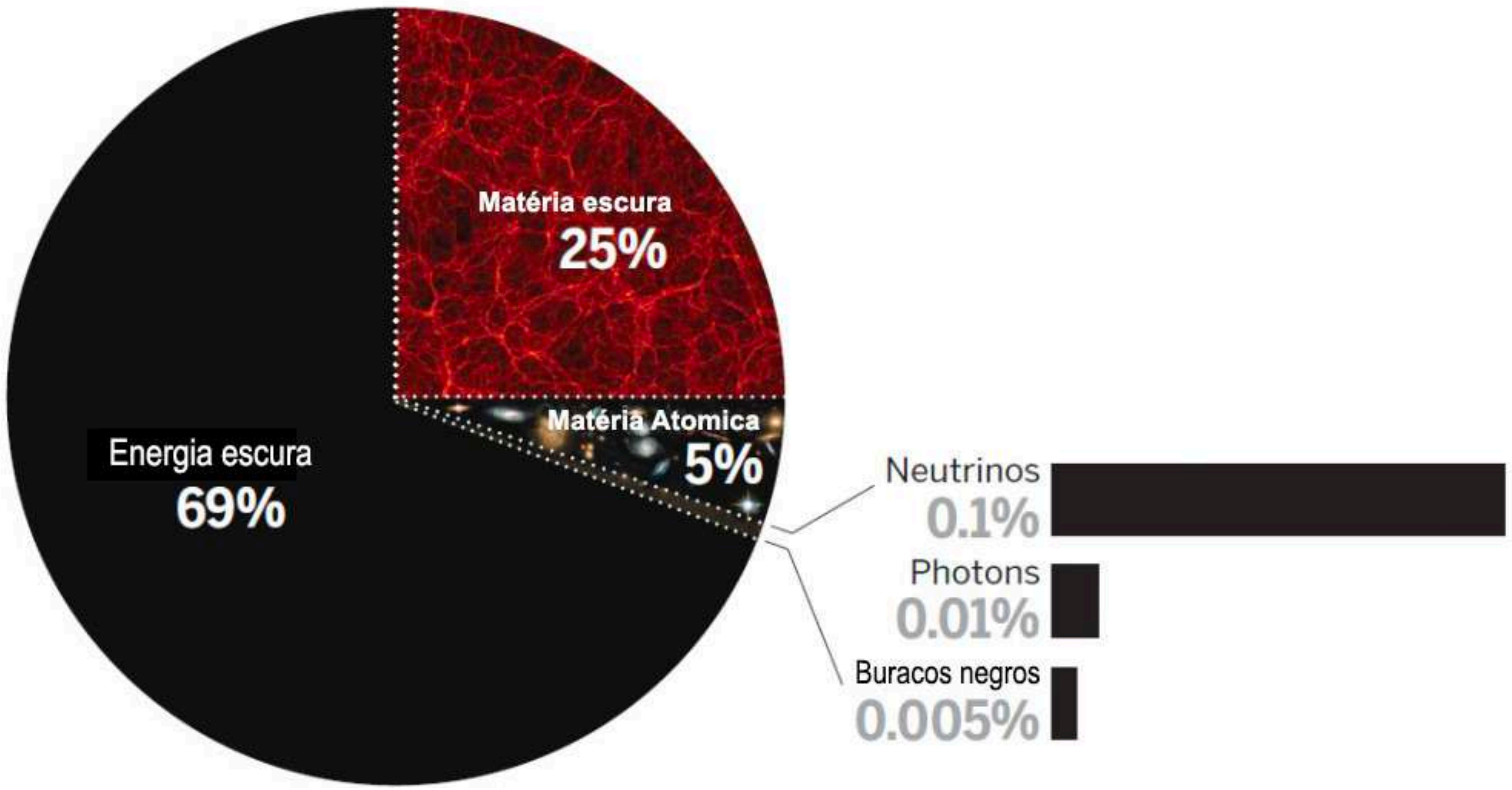
Cosmological fluids are described by a constant ***equation of state (EoS)***

$$\omega = \frac{P}{\rho}$$

Leading to:

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a} \longrightarrow \boxed{\rho \propto a^{-3(1+w)}}$$

Components of the universe



Crédito: Science/AAAS

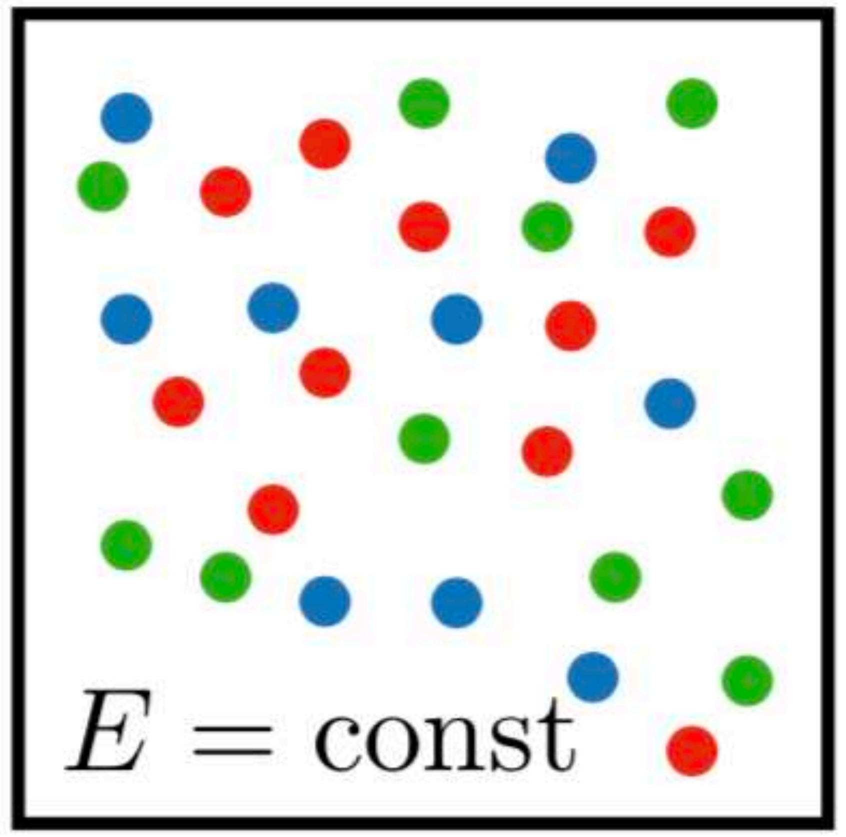
Each component evolves and leads to a different expansion of the universe.

Lets study how each component evolves

Matter

Matter is a fluid with zero pressure ($\omega = 0$):

$$P = 0 \quad \longleftrightarrow \quad \boxed{\rho \equiv \frac{E}{V} \propto a^{-3}}$$



Crédito: D. Baumann

Inserting in Friedmann's eq, matter evolves as:

$$\left(\frac{\dot{a}}{a}\right)^2 \propto \rho \propto a^{-3} \quad \longrightarrow \quad \boxed{a \propto t^{2/3}}$$

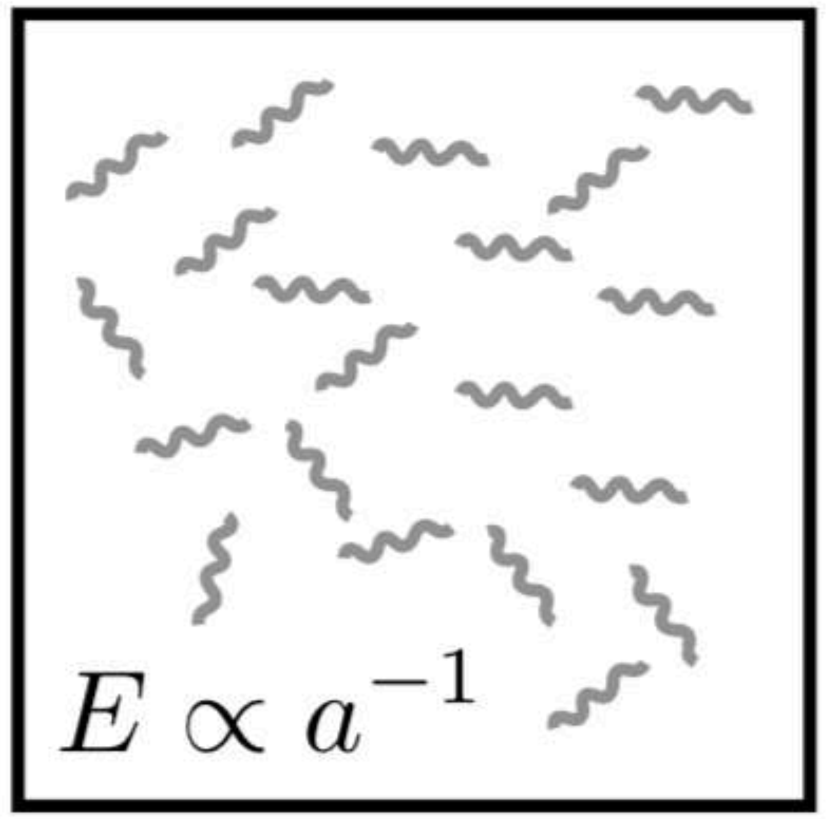
In our universeo, 2 components behave as matter:

dark matter (25%) e a ordinary matter - baryons (5%).

Radiations

Radiation is a relativistic fluid ($\omega = 1/3$):

$$P = \frac{1}{3}\rho \longleftrightarrow \boxed{\rho \equiv \frac{E}{V} \propto a^{-4}}$$



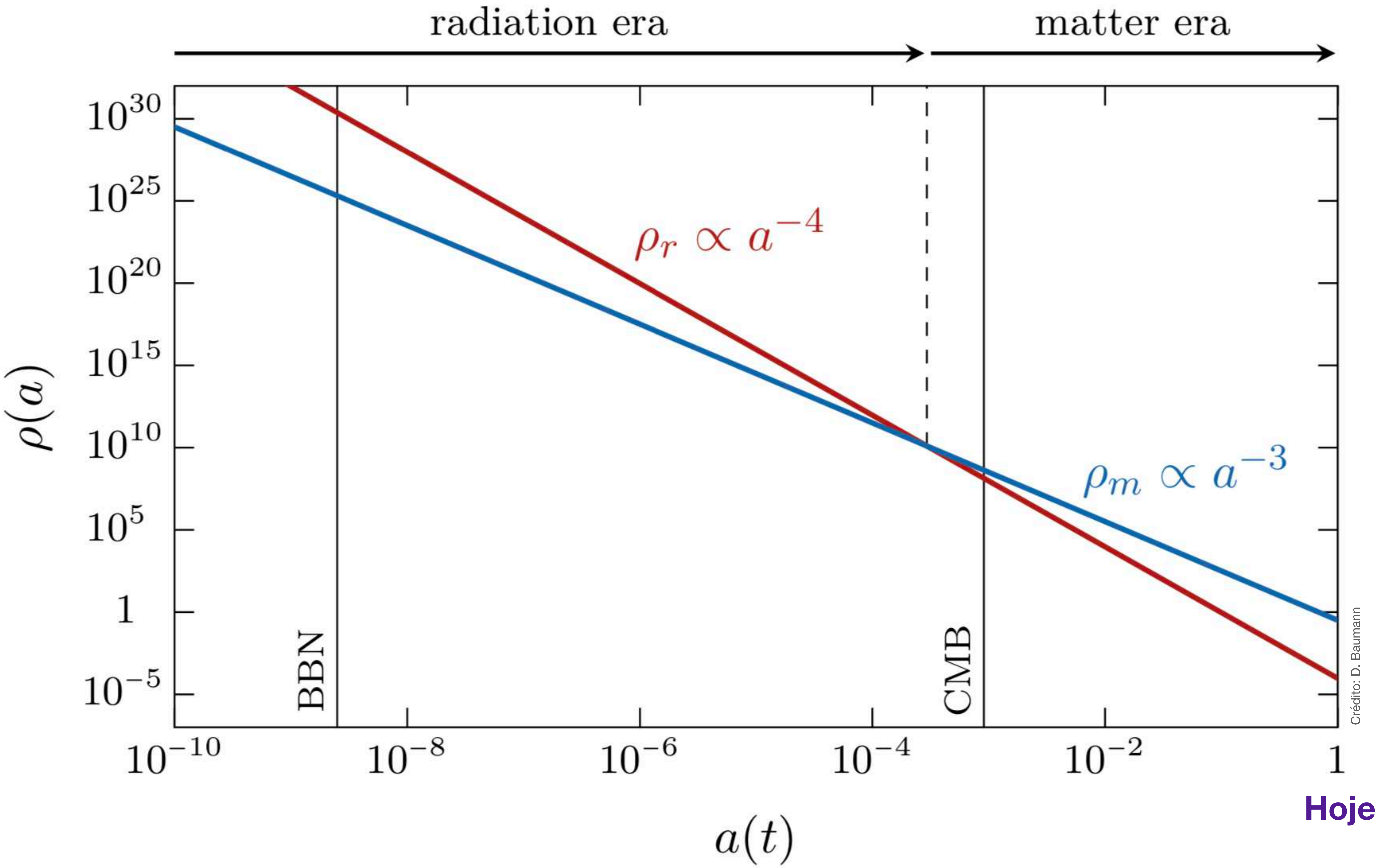
Crédito: D. Baumann

Inserting in Friedmann's eq, raditaion evolves as:

$$\left(\frac{\dot{a}}{a}\right)^2 \propto \rho \propto a^{-4} \longrightarrow \boxed{a \propto t^{1/2}}$$

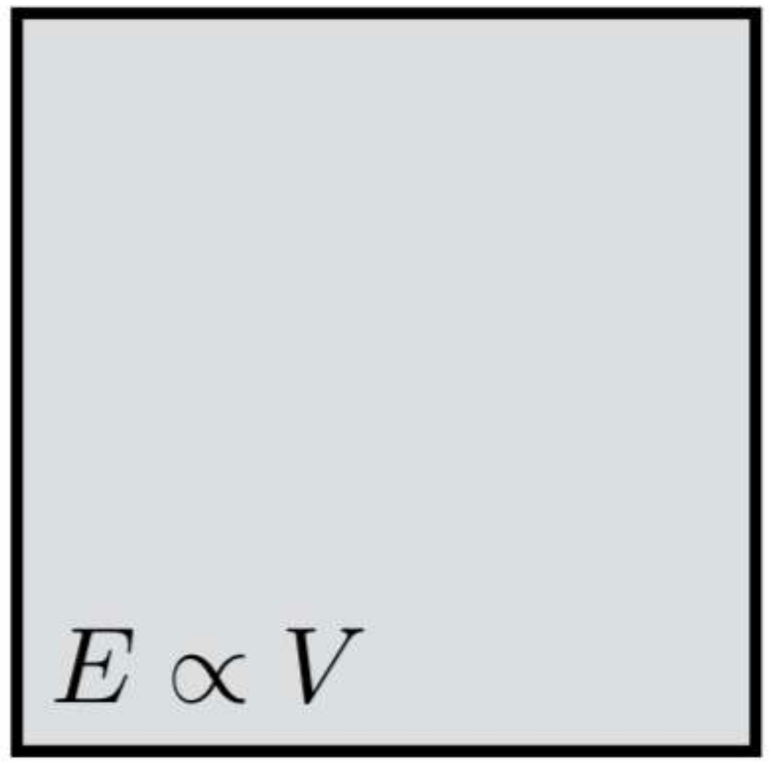
Radiation dominates the evolution of the universe at early stages, before matter.

Matter and radiation



Dark energy

Observational data indicates that the universe is expanding in an **accelerated** way $\ddot{a} > 0$



Crédito: D. Baumann

Acceleration

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

Diceleration

$$w = \frac{p}{\rho} < -\frac{1}{3}$$

The component which is the source of this accelerated expansion we call dark energy

$\ddot{a} < 0$

Decelerated expansion

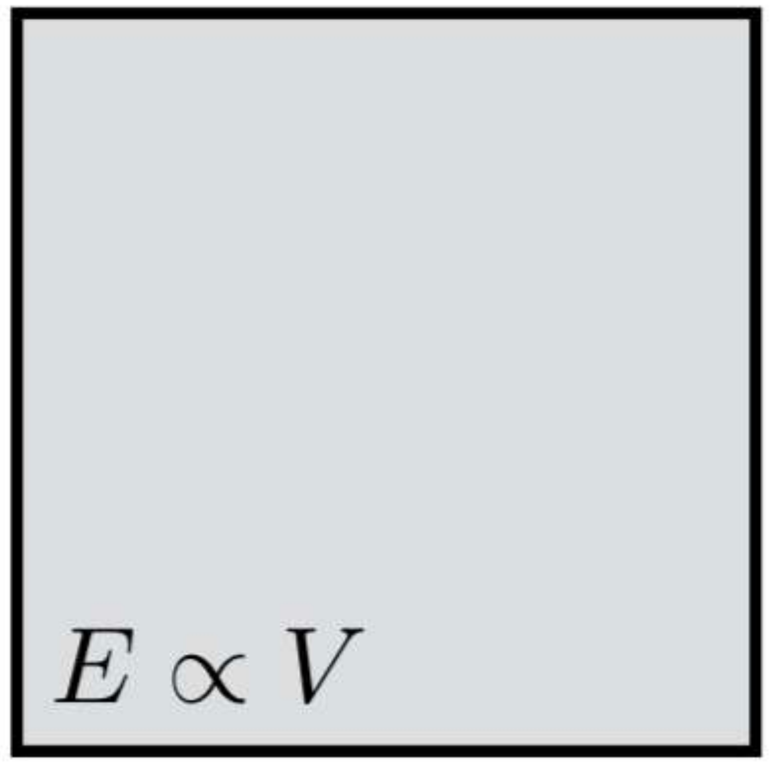
$$P > -\frac{1}{3}\rho$$

$\ddot{a} > 0$

Accelerated expansion

$$P < -\frac{1}{3}\rho$$

Dark energy



Crédito: D. Baumann

Observational data indicates that the universe is expanding in an **accelerated** way $\ddot{a} > 0$

Acceleration

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

Diceleration

$$\Rightarrow \quad w = \frac{p}{\rho} < -\frac{1}{3}$$

The component which is the source of this accelerated expansion we call dark energy

<u>Cosmological constant</u>

Cosmological const.

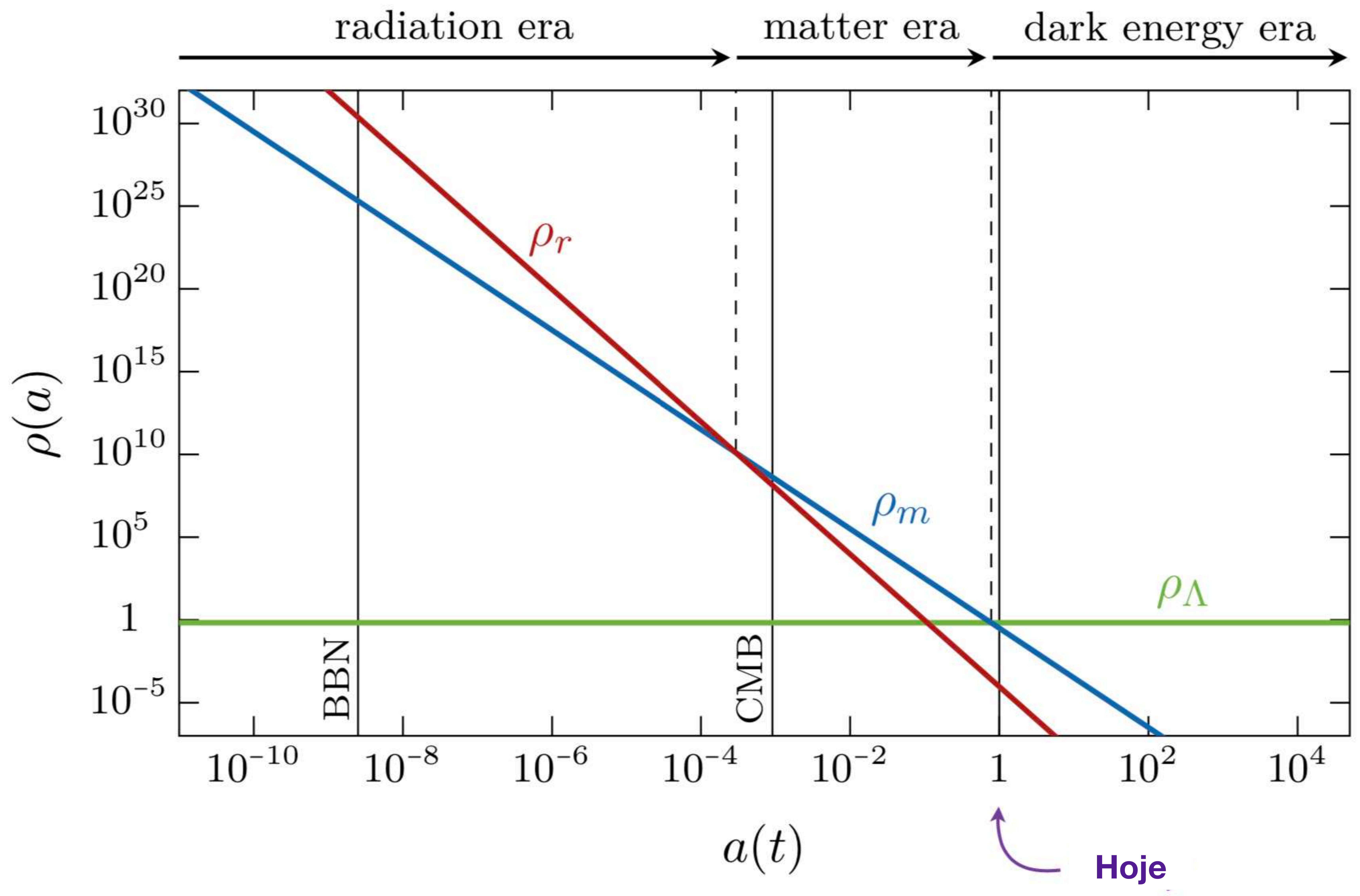
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \quad + \quad w = -1 \quad \Longrightarrow \quad \left(\frac{\dot{a}}{a}\right)^2 \propto \rho = \text{const}$$

accelerates expansion

$$\downarrow$$

$$a \propto e^{H_0 t}$$

Matter, radiation and dark energy



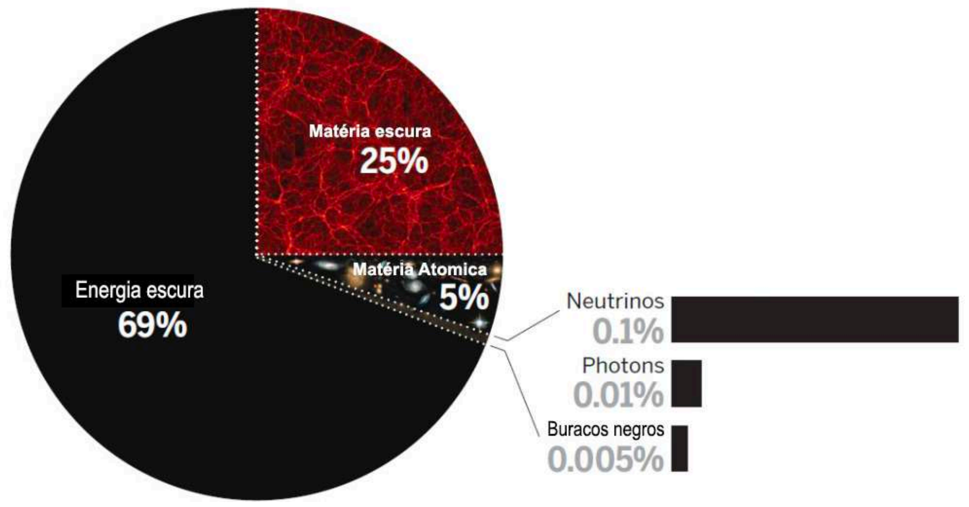
Crédito: D. Baumann

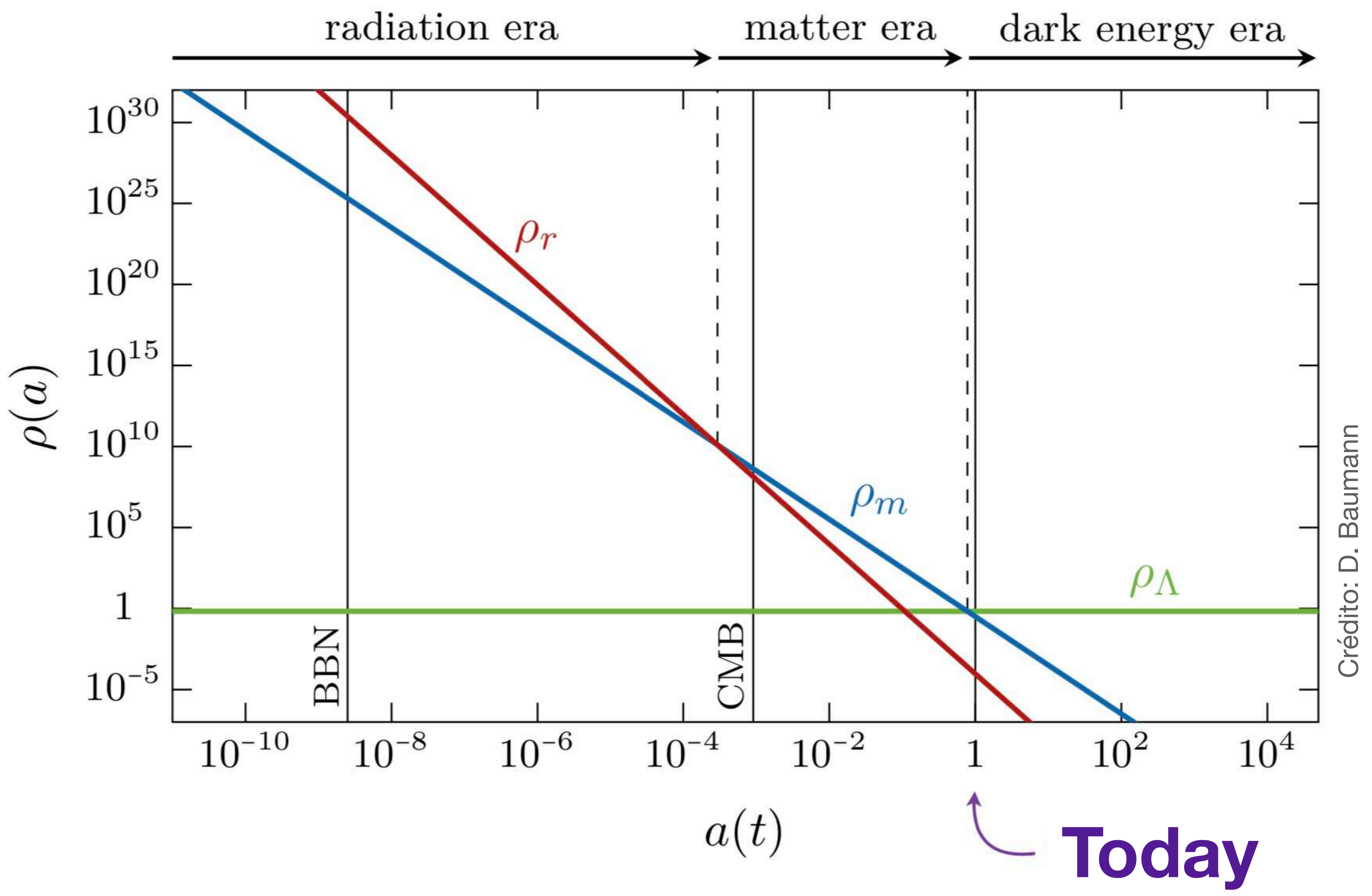
Summary

Homogeneous universe (Cosm. Background) → Homogeneous and isotropic + Perfect fluid → Expansion of the universe

$$\Rightarrow \qquad \frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a} \longrightarrow \boxed{\rho \propto a^{-3(1+w)}}$$

Components of the universe





radiation era → matter era → dark energy era

$$\Omega_{tot} = \Omega_m + \Omega_{rad} + \Omega_{de}$$

Standard cosmological model

Standard cosmological model

The SCM describes the structure, evolution and composition of our universe. It also explains what we see and have in our universe today. It includes, then, the standard model of elementary particles, and explains the evolution and formation of the particles and structures we have today.

2 theoretical Pilars:

- GR
- Cosmological principle

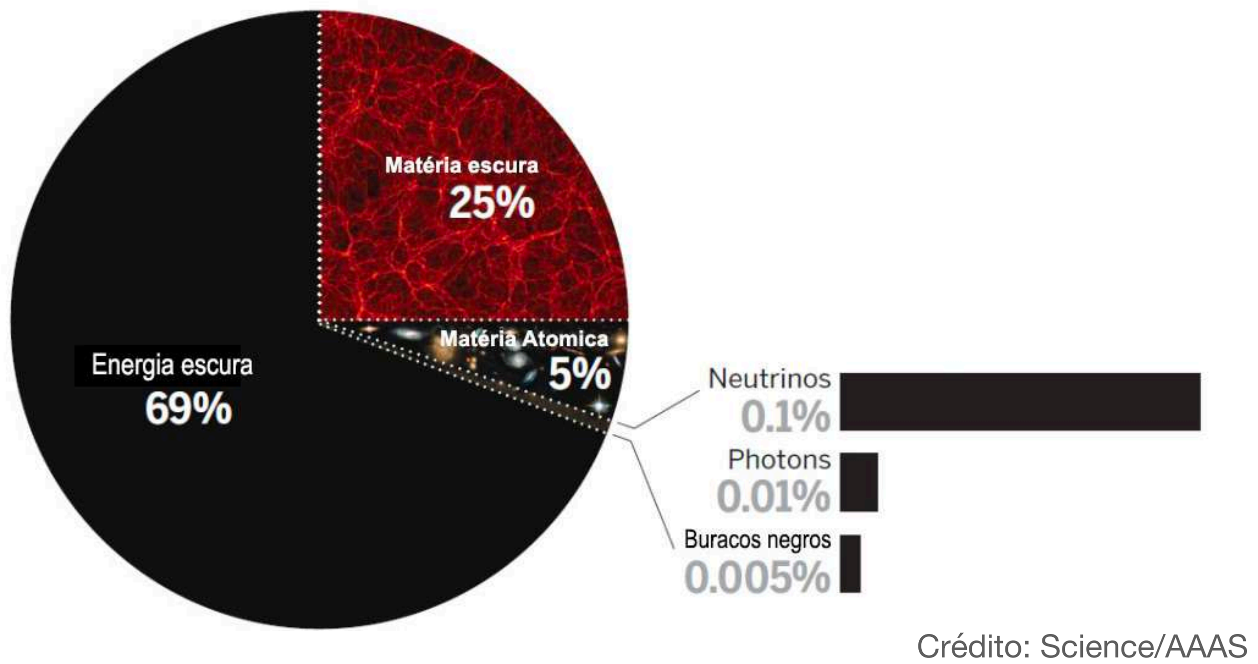
3 observational pilars:

- Hubble - Lemaître Law
- Nucleosynthesis
- Cosmic Microwave Background

\+

a.k.a. ΛCDM model

Parametrization: 6 parameters



Crédito: Science/AAAS

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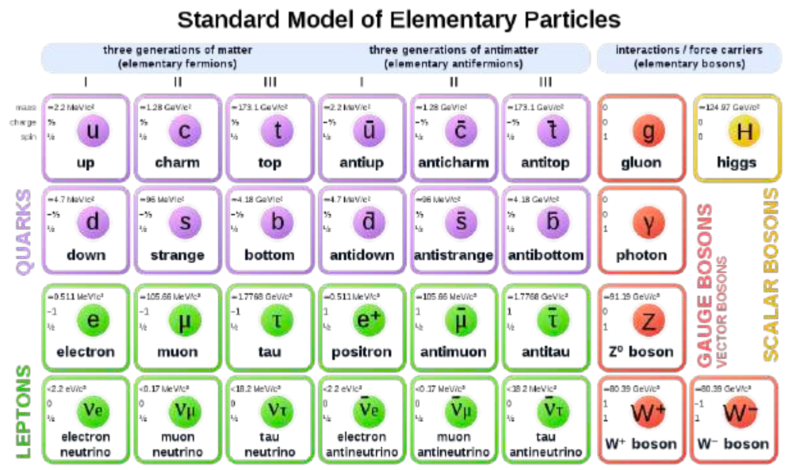
Standard model of elementary particles



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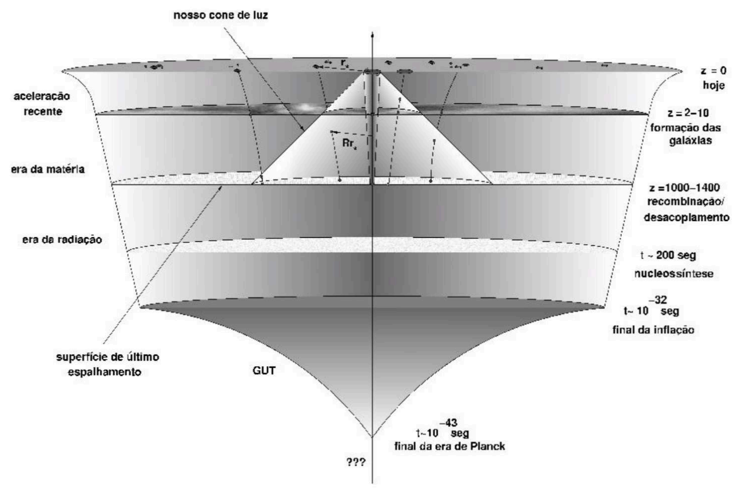
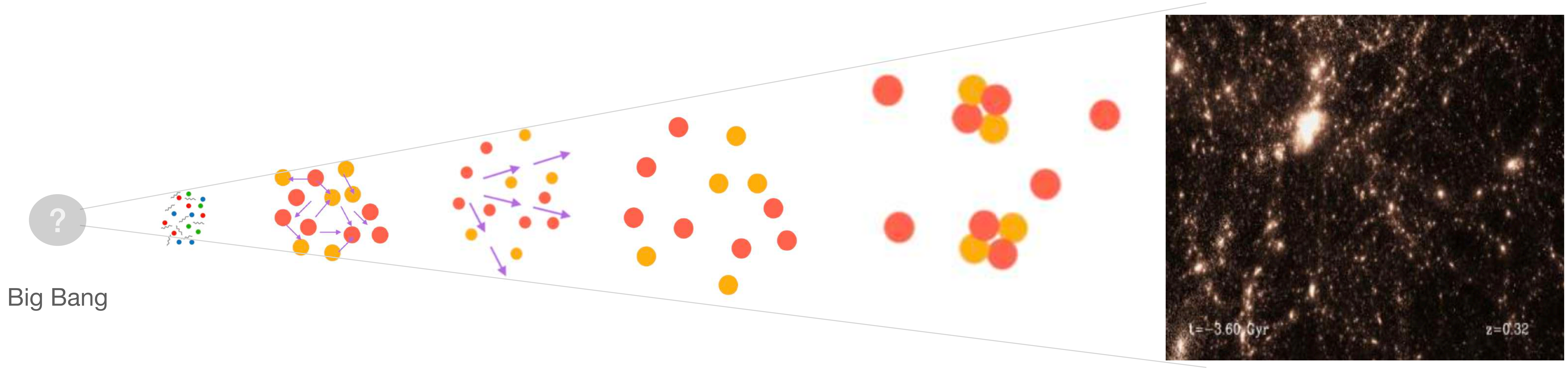
Thermal history



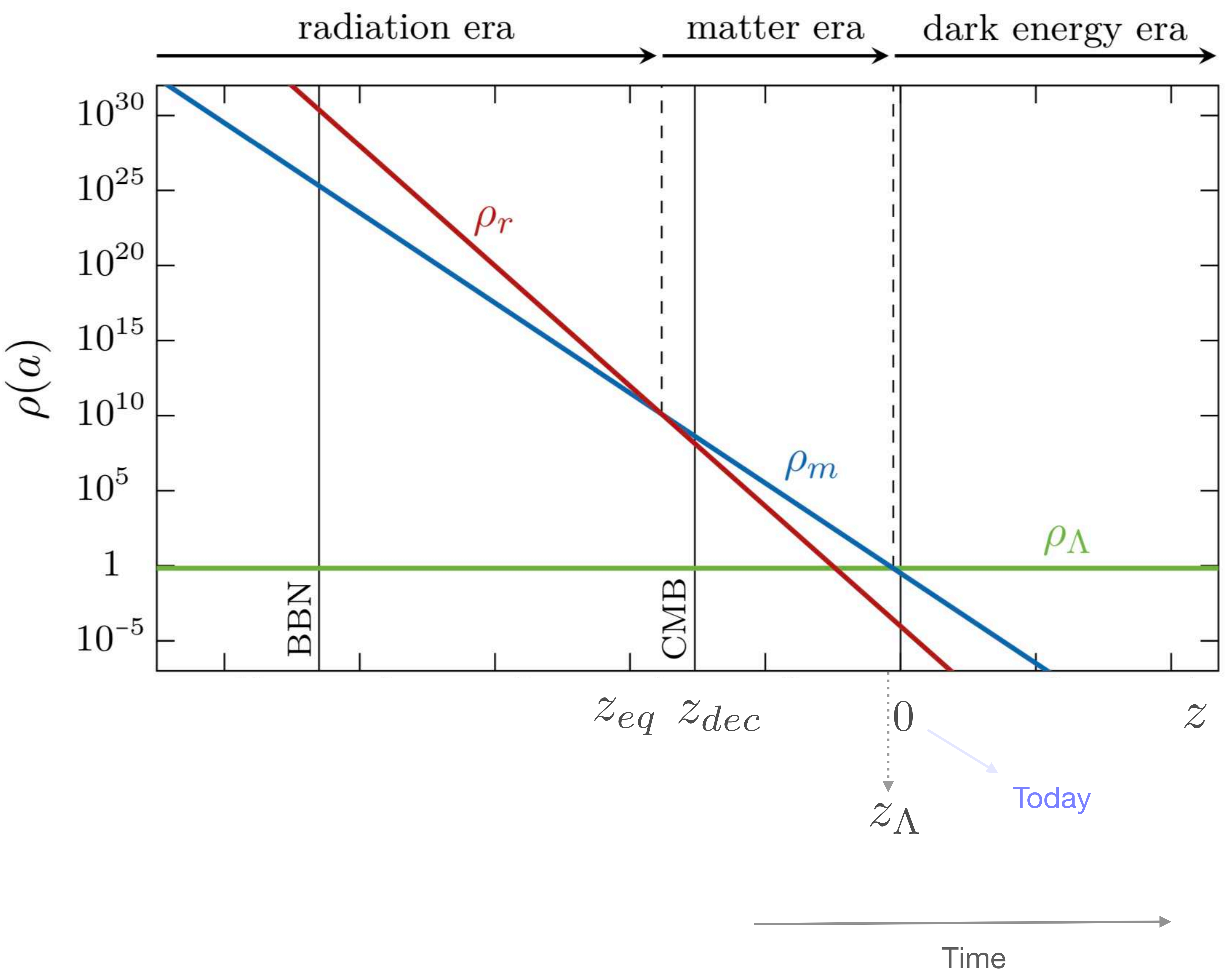
Standard cosmological model - Hot Big Bang model



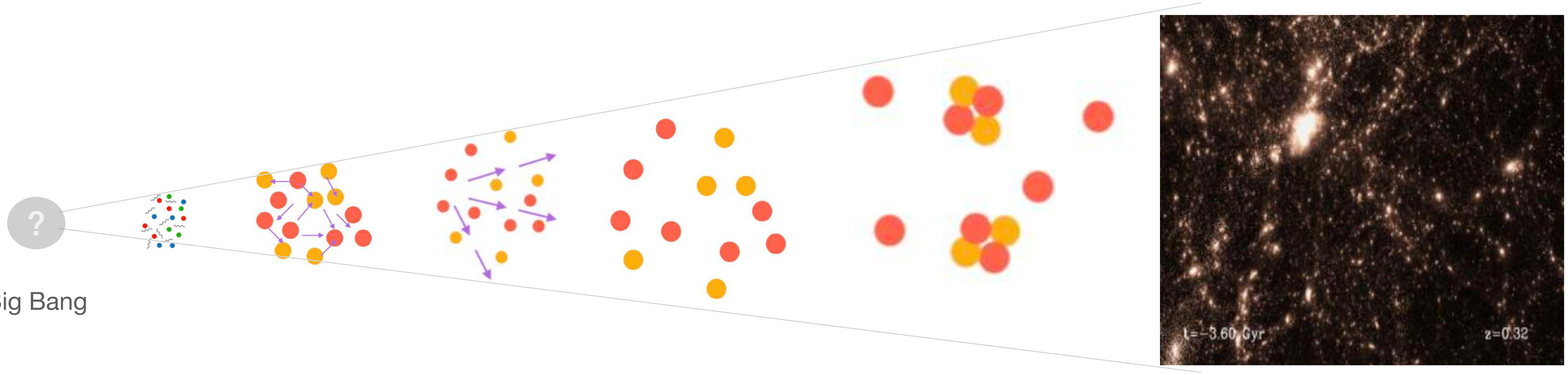
If the universe is expanding, this means that before its energy was contained in a small, hot and **dense** region.

Standard cosmological model

$$\begin{cases} \Omega_m = \Omega_m^0 \,(1+z)^3, & \text{matéria}, \\ \Omega_{rad} = \Omega_{rad}^0 \,(1+z)^4\,, & \text{radiação}, \\ \Omega_\Lambda = \Omega_\Lambda^0 \,(1+z)^{3(1+\omega)}\,, & \text{energia escura}\,, \end{cases}$$



Standard cosmological model - Hot Big Bang model

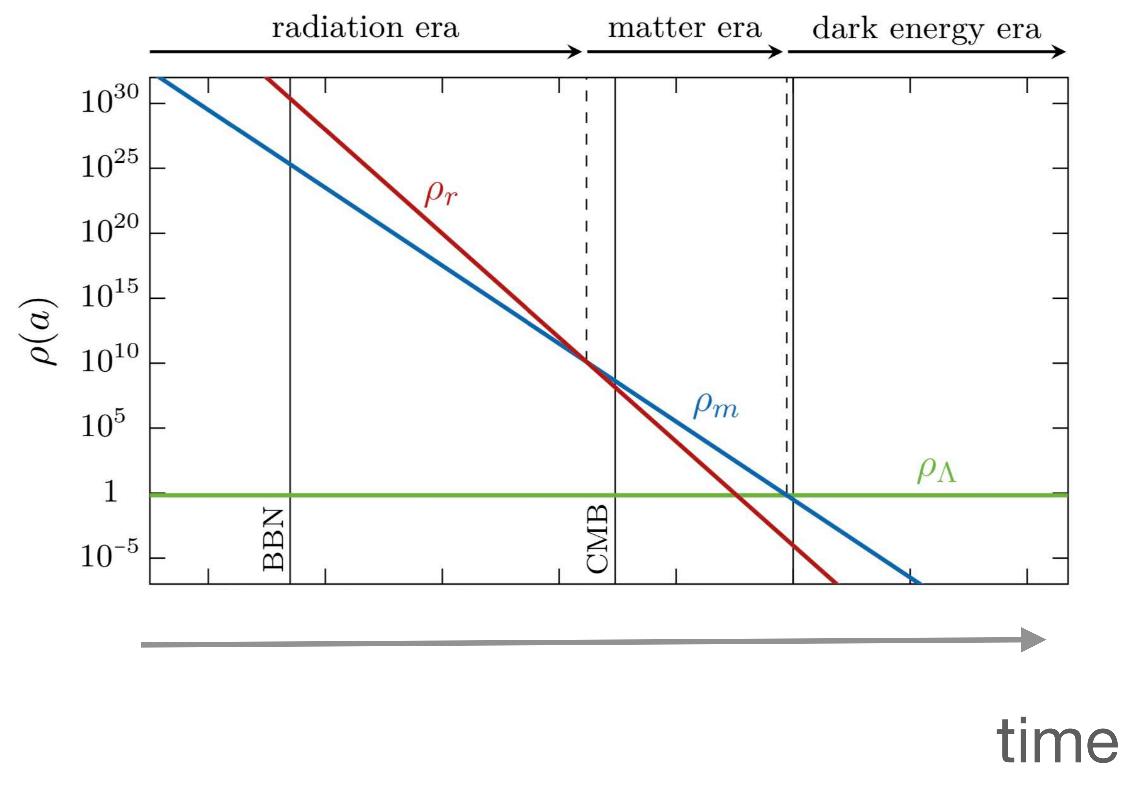


Big Bang

If the universe is expanding, this means that before its energy was contained in a small, hot and **dense** region.

$$\rho_{rad}(T) = \sum_{i=1}^{n} \alpha_i g_i \left(\frac{\pi^2}{30}\right) T^4 \longrightarrow T_{rad}(z) = T^0_{rad}(1+z)$$

$\downarrow t \quad \uparrow z \quad \uparrow T$



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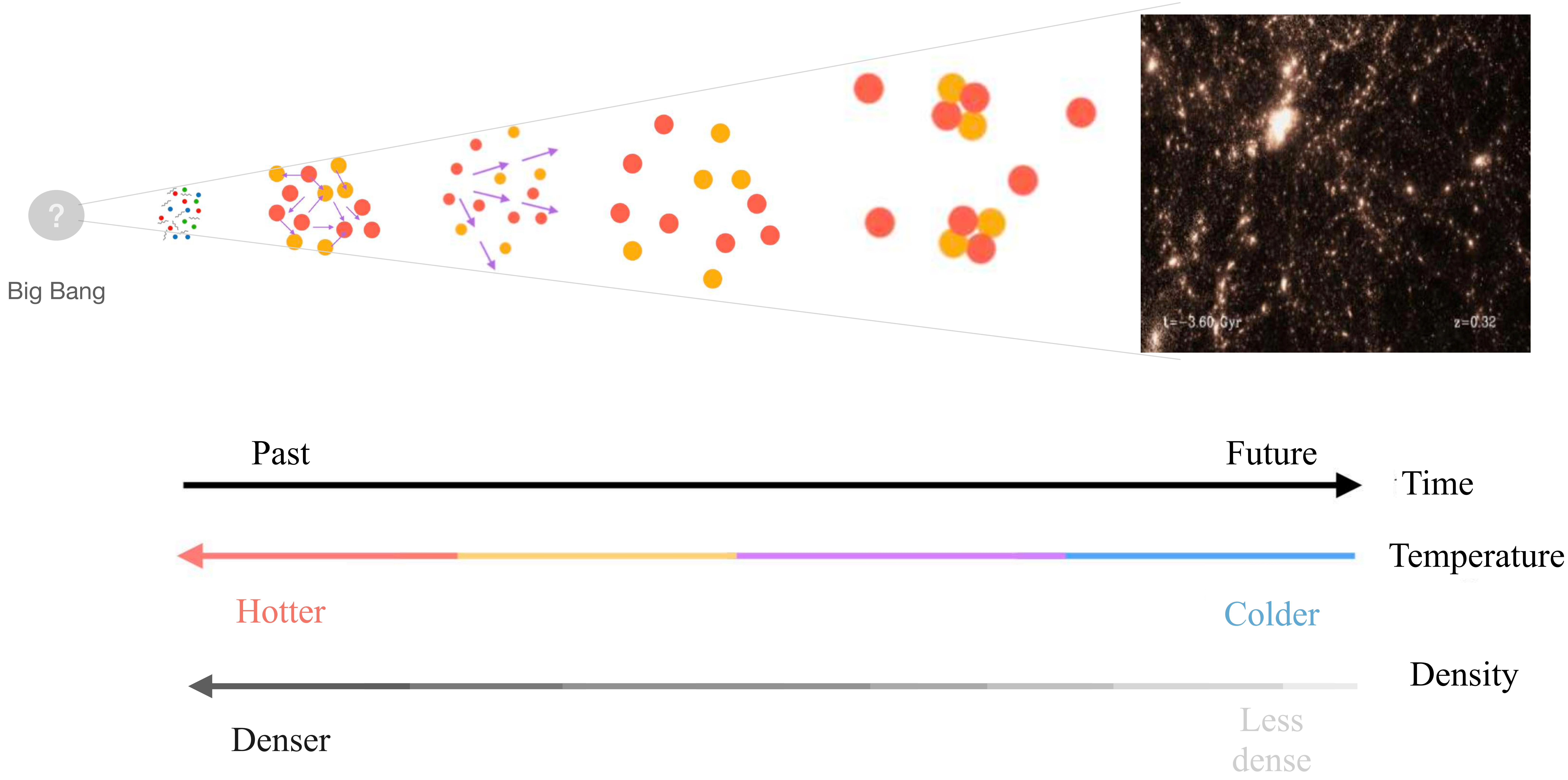
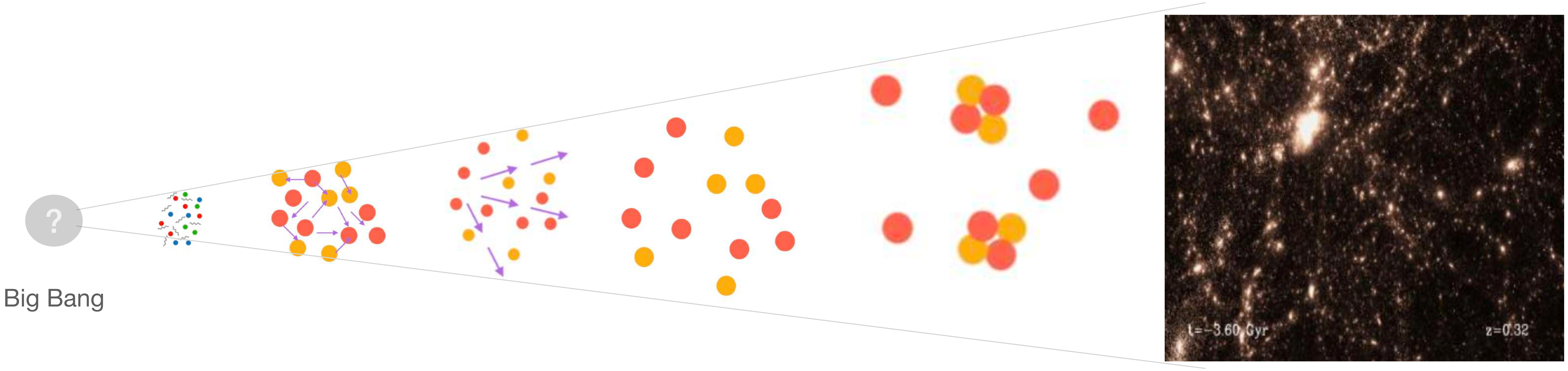


Standard cosmological model - Hot Big Bang model



Universo que começou de uma região quente e densa, Big Bang, e se expandindo e esfriando.

Ele é composto por radiação, neutrinos, matéria escura, bárions e energia escura.

Hoje em dia, é dominado pela energia escura, que leva a uma expansão acelerada.

Ele explica a evolução das estruturas do universo (a partir de pequenas perturbações iniciais) e a formação e abundância dos elementos presentes no nosso universo (MP das partículas elementares).

Origem dessas perturbações?
Aula 4

ΛCDM: parametriza esse modelo padrão usando 6 parâmetros!

Standard cosmological model

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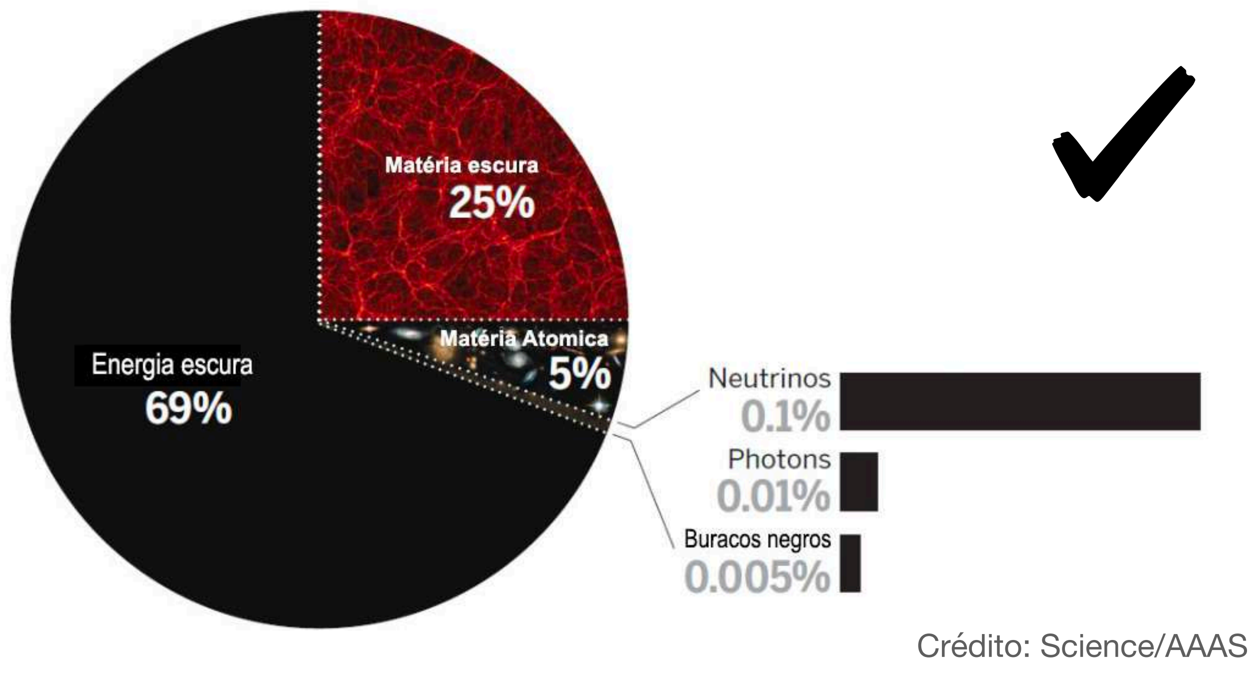

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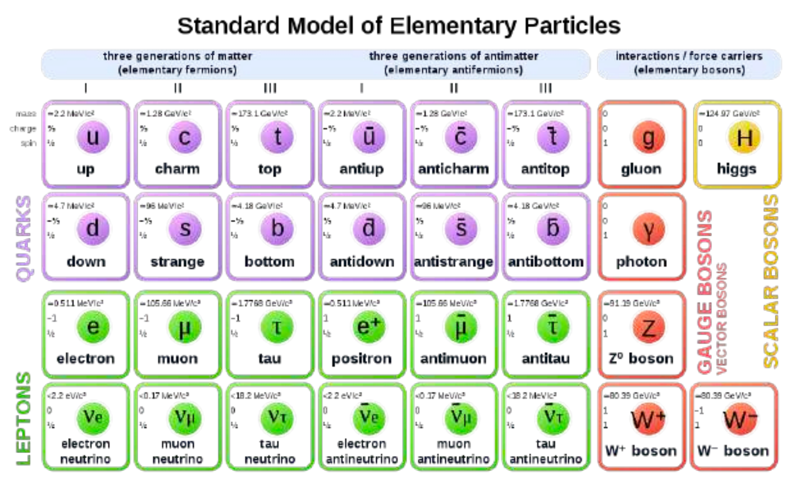
Standard model of elementary particles



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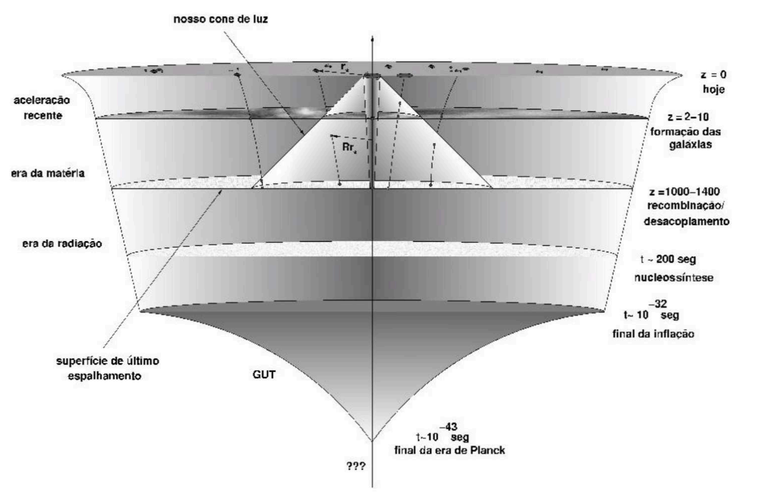
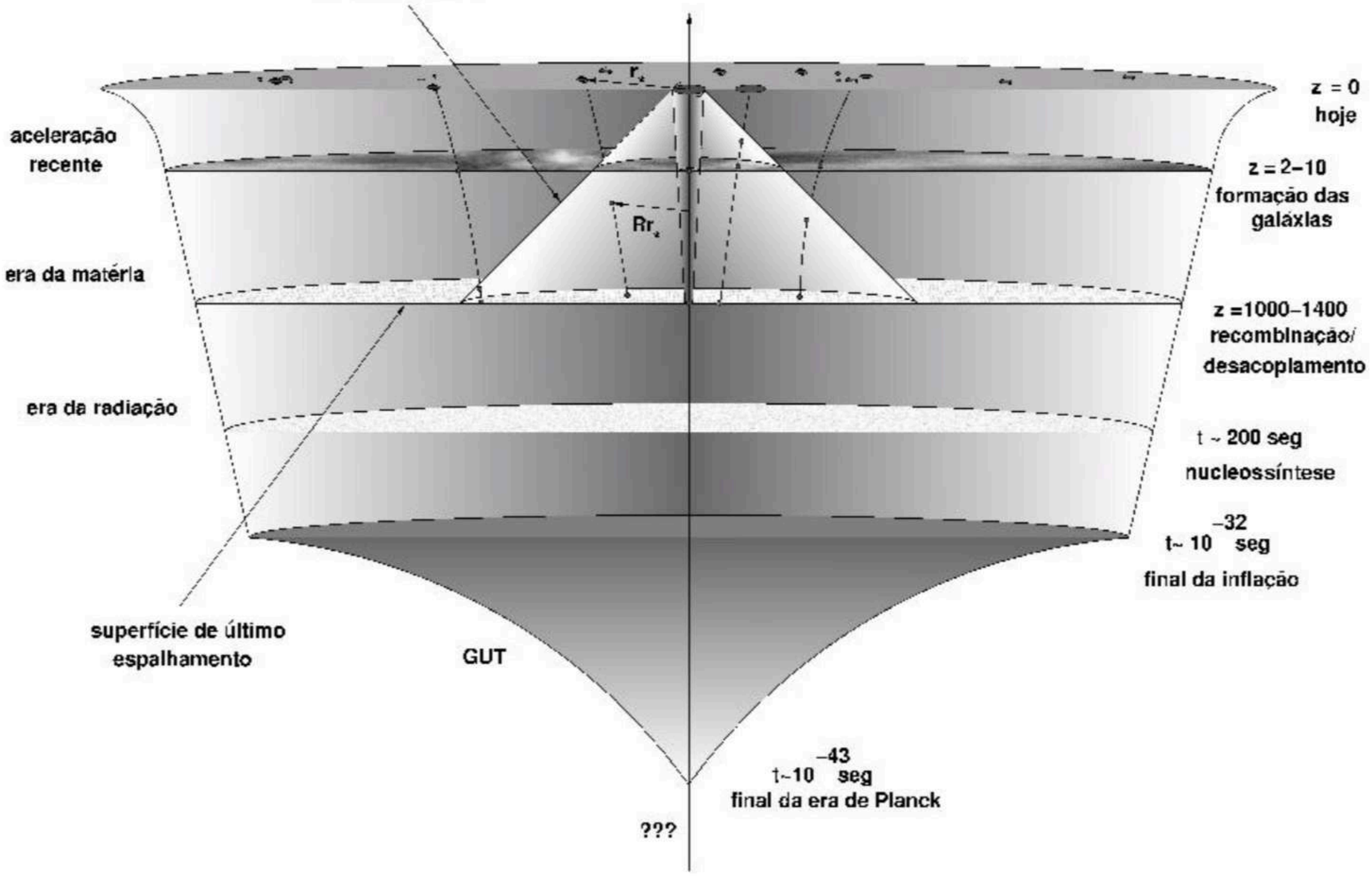
Thermal history



Thermal history of the universe

We want to describe here the main moments of the thermal history of our universe.

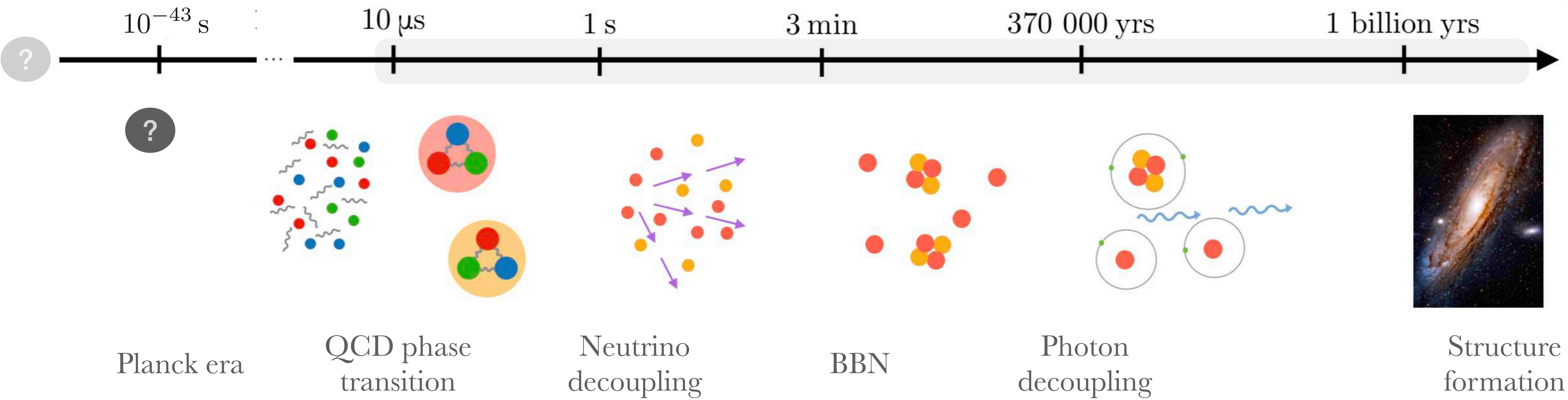


Crédito: Ronaldo de Souza

Thermal history of the universe

The universe“started" hot e **dense** → As it cools, the structures we know start to form

SCM



Crédito: D. Baumann

Thermal history of the universe

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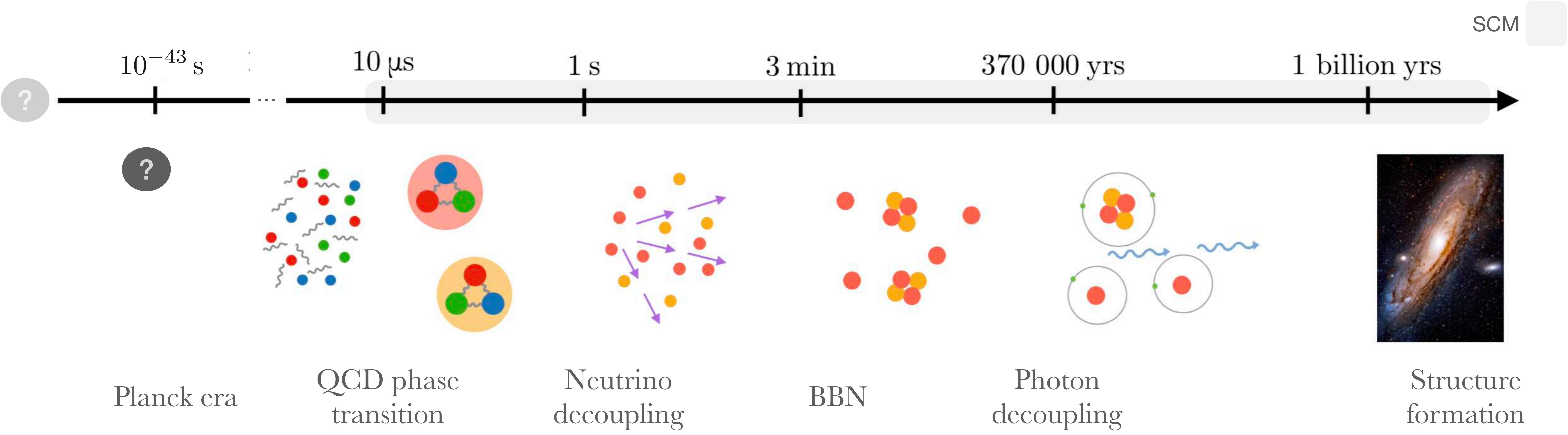


Crédito: D. Baumann

- (Most of them) observationally confirmed by independent measurements

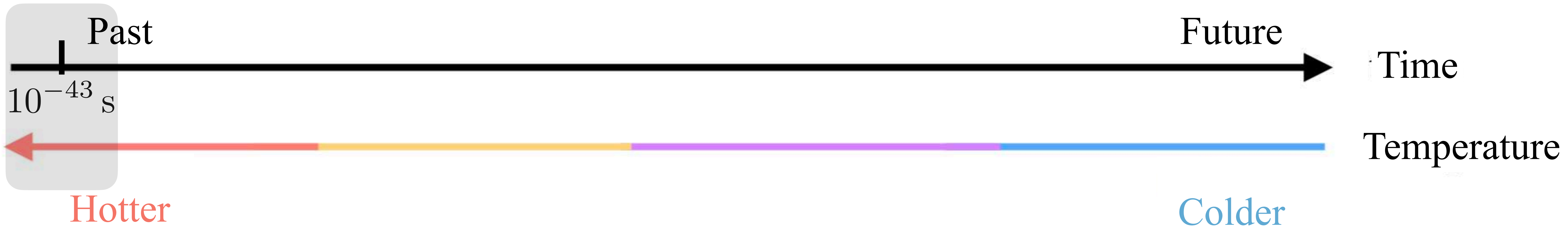
Planck era

$t < 10^{-43}\,\mathrm{s}$

Period from the "begining" of the universe until $t \sim 10^{-43}\,\mathrm{s}$, com $T \sim 10^{32}\,K \sim 10^{29}\,\mathrm{GeV}$

This period is NOT described by the SCM!

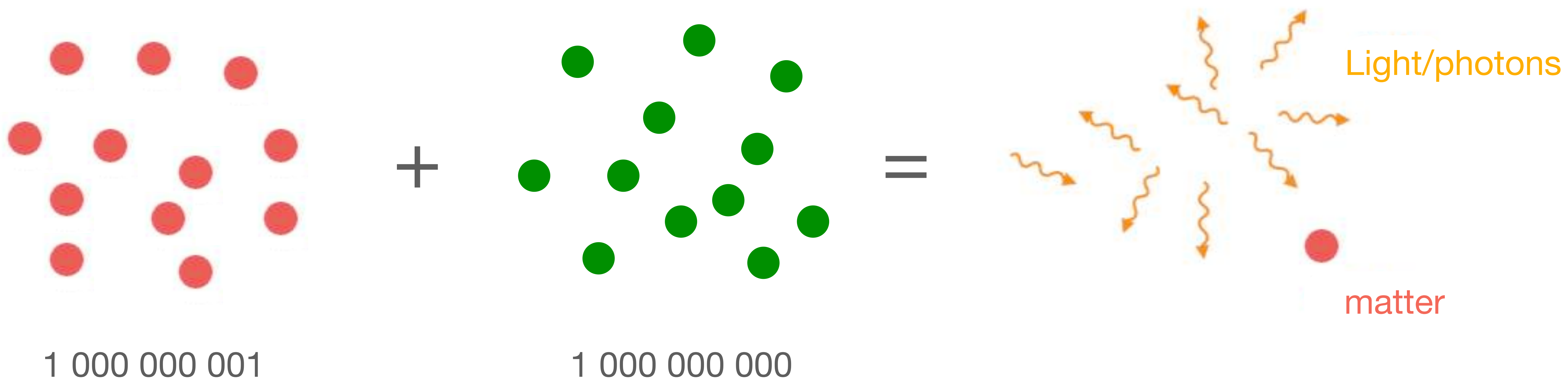
There is still not a lot of understanding about this period: GR is no longer valid and quantum effects start to dominate - quantum gravity (?)



Matter-antimatter asymmetry

$t \sim 10^{-19}\,\mathrm{s}$

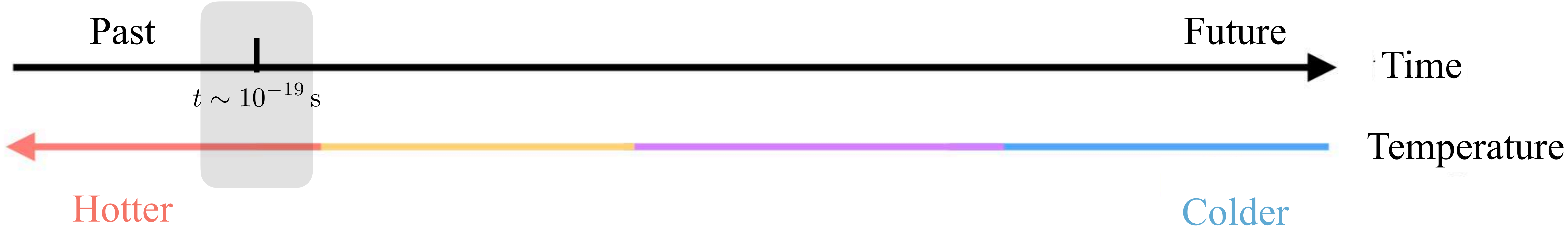
The universe has almost the same amount of matter and anti-matter



As the universe expands and
cools down,
Matter and anti-matter annihilate

Initially, more matter than anti-matter → Unknown explanation!
NOT part of the SCM!

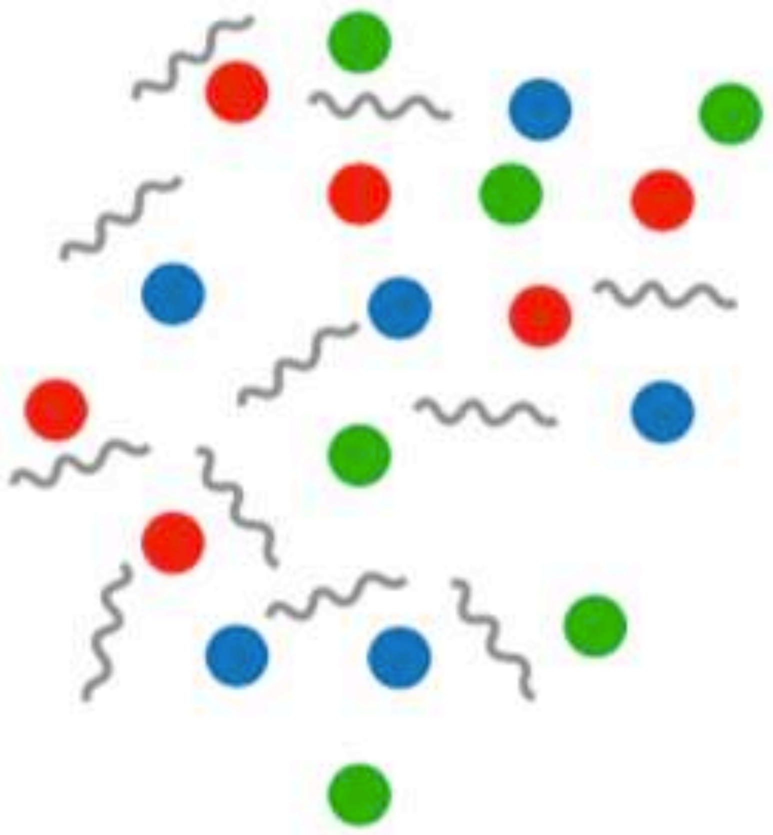
Assimetria necessária para existirmos!



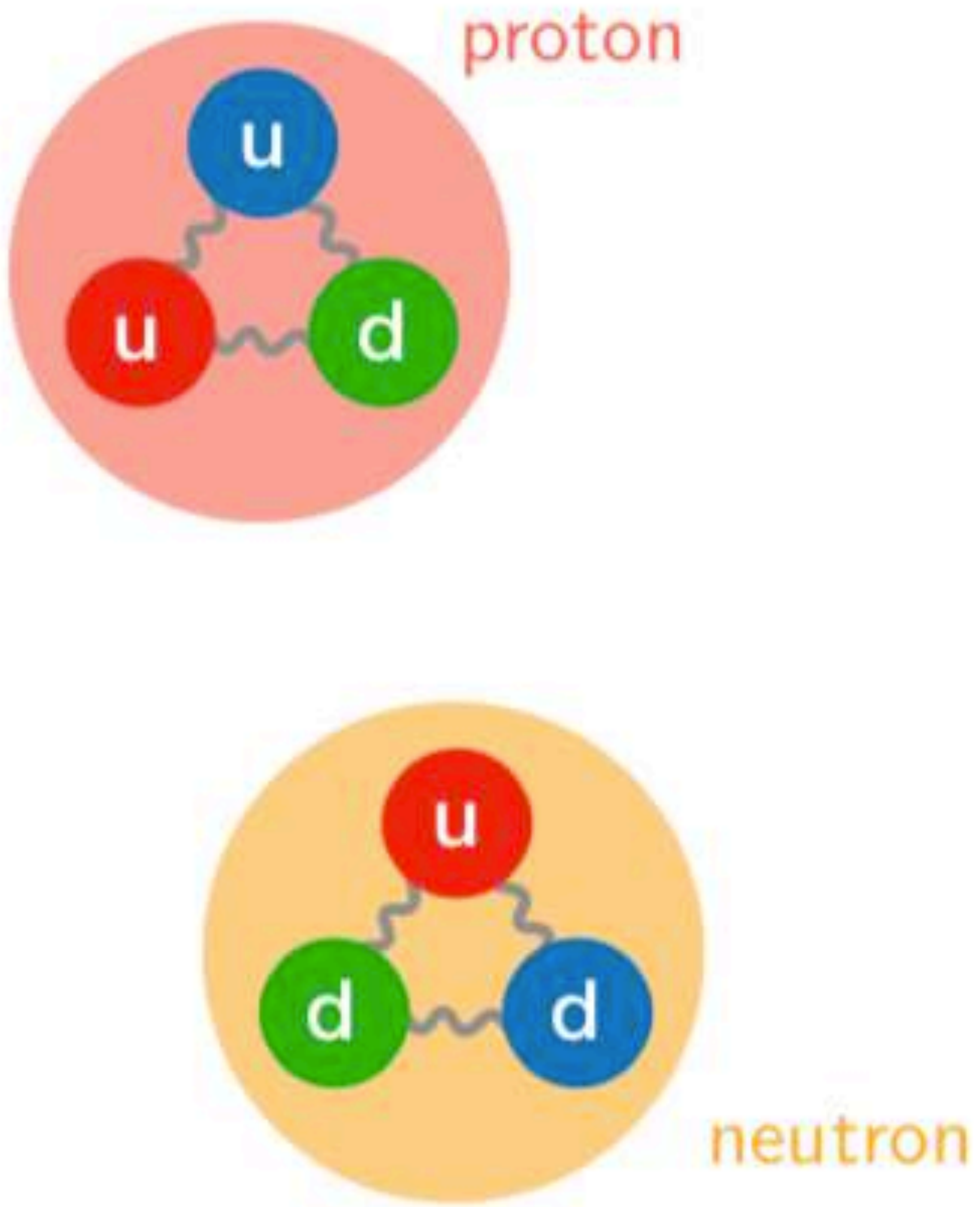
Baseado em: D. Baumann

Quarks and gluons condense inside the nucleus

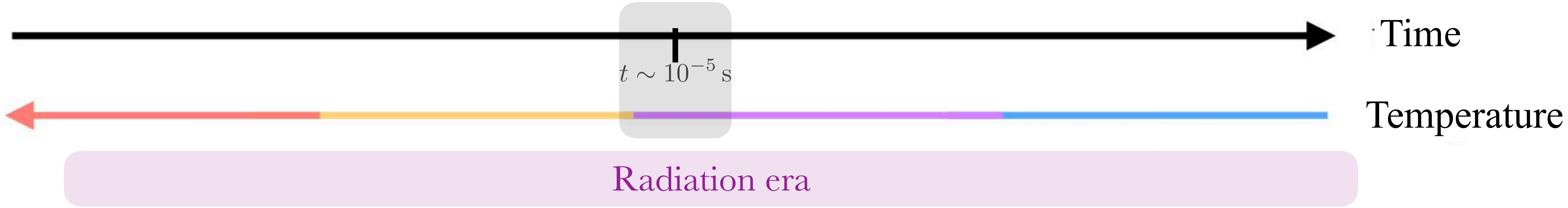
$t \sim 10^{-5}\,\mathrm{s}$



Primordial plasma ("soup") where quarks, gluons and radiation are coupled -
Thermal equilibrium



Protons and neutrons are formed



Crédito: D. Baumann

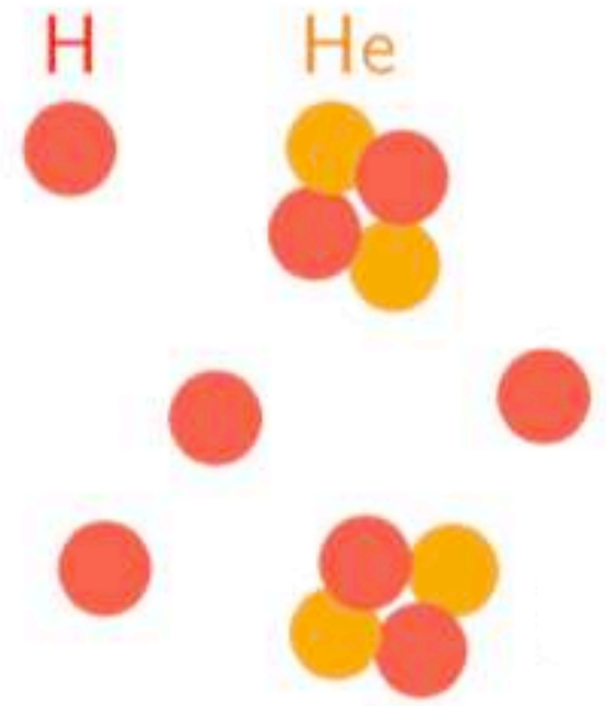
Big Bang nucleosynthesis (BBN)

$t \sim 3\,\mathrm{min}$

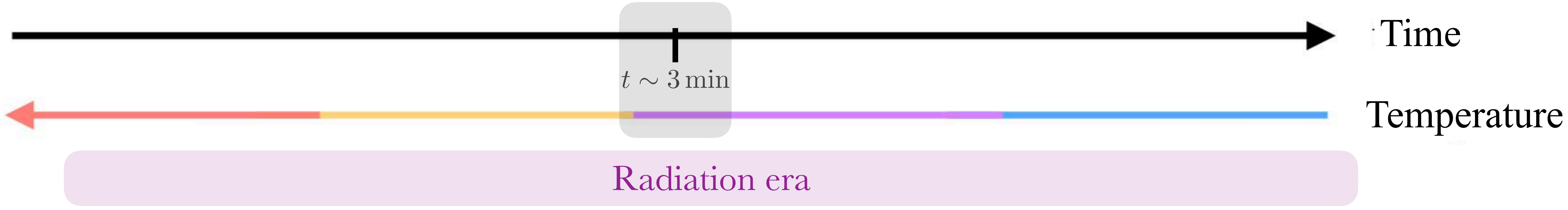
Light elements (H, He and Li) are formed

$T \sim 10^8 - 10^9\,\mathrm{K}$

Plasma ("soup") of coupled protons, neutrons and radiation



Protons and netutrons combine forming the nucleus of elements like H, He, deuterium and Li.



Crédito: D. Baumann

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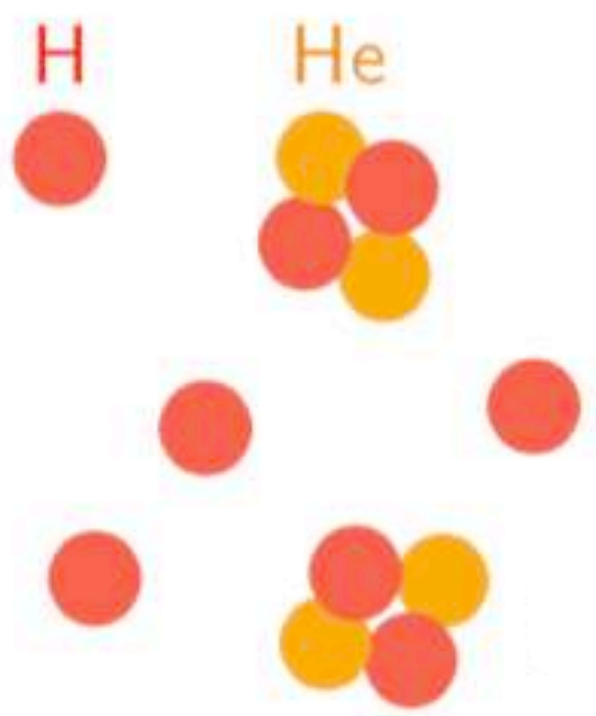
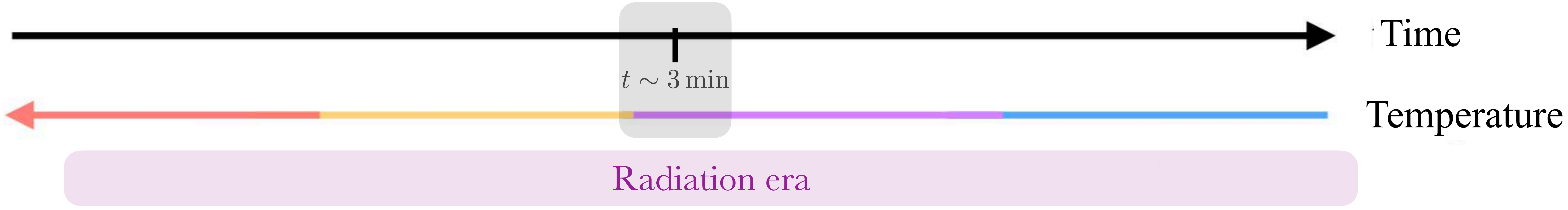


Plasma ("soup") of coupled protons, neutrons and radiation

- BBN predicts the correct abundance of light elements
- Heavy elements are formed in the stars

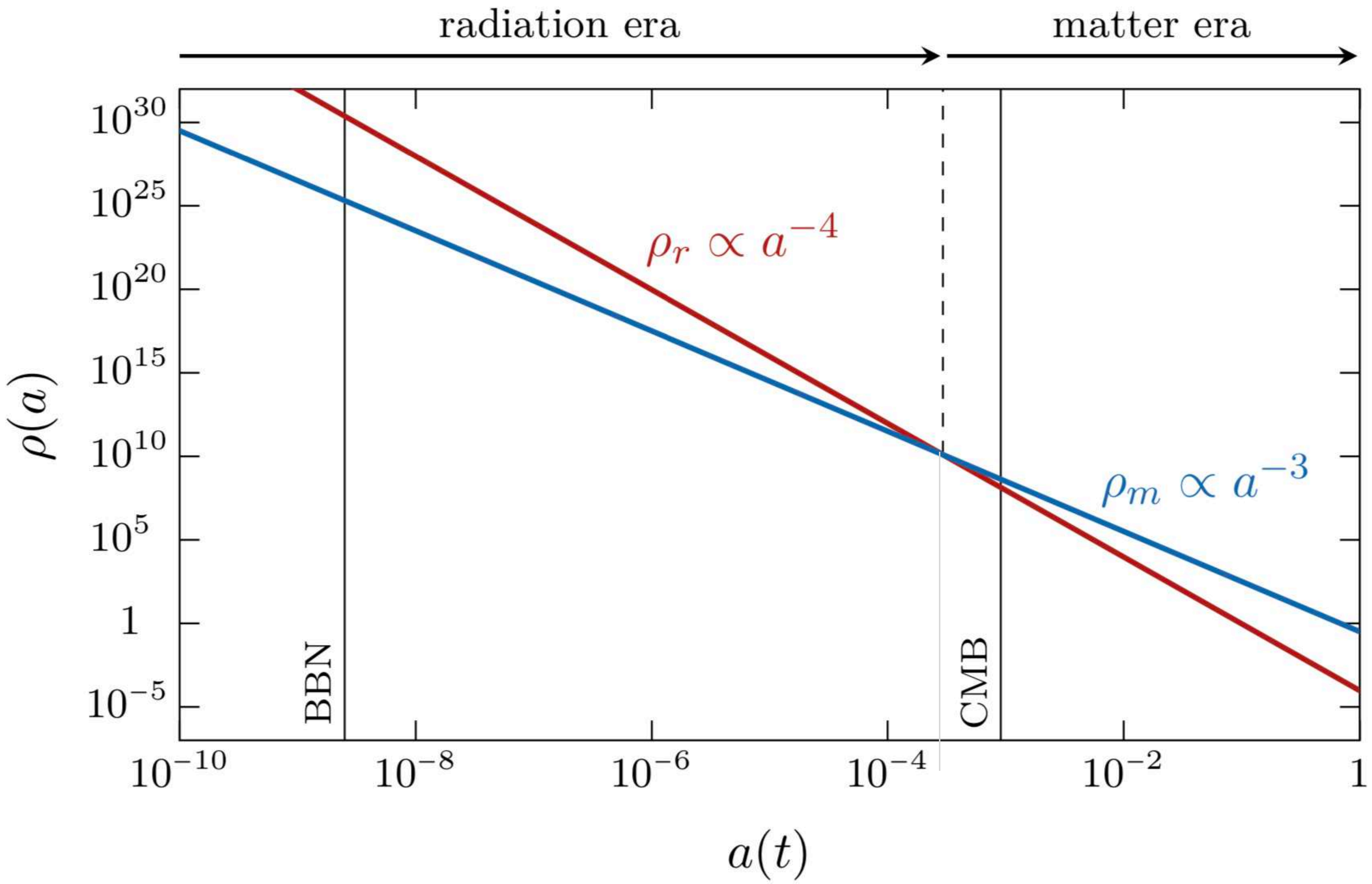


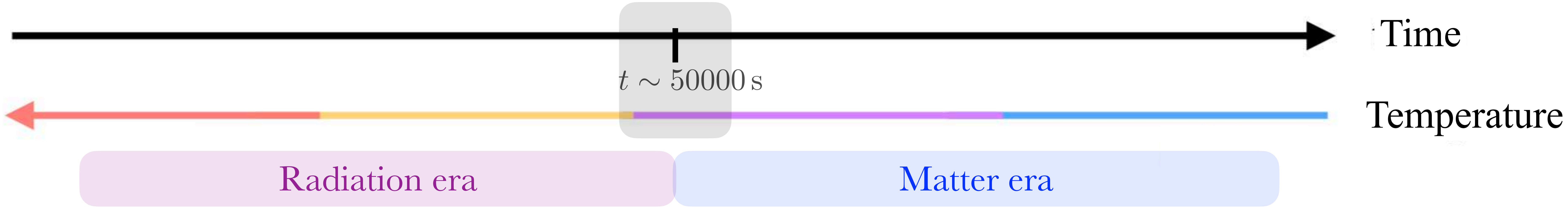
Crédito: D. Baumann

<u>Radiations ⟶ Matter</u>

$t \sim 50000\,\mathrm{s}$

$z_{eq} \sim 3500$



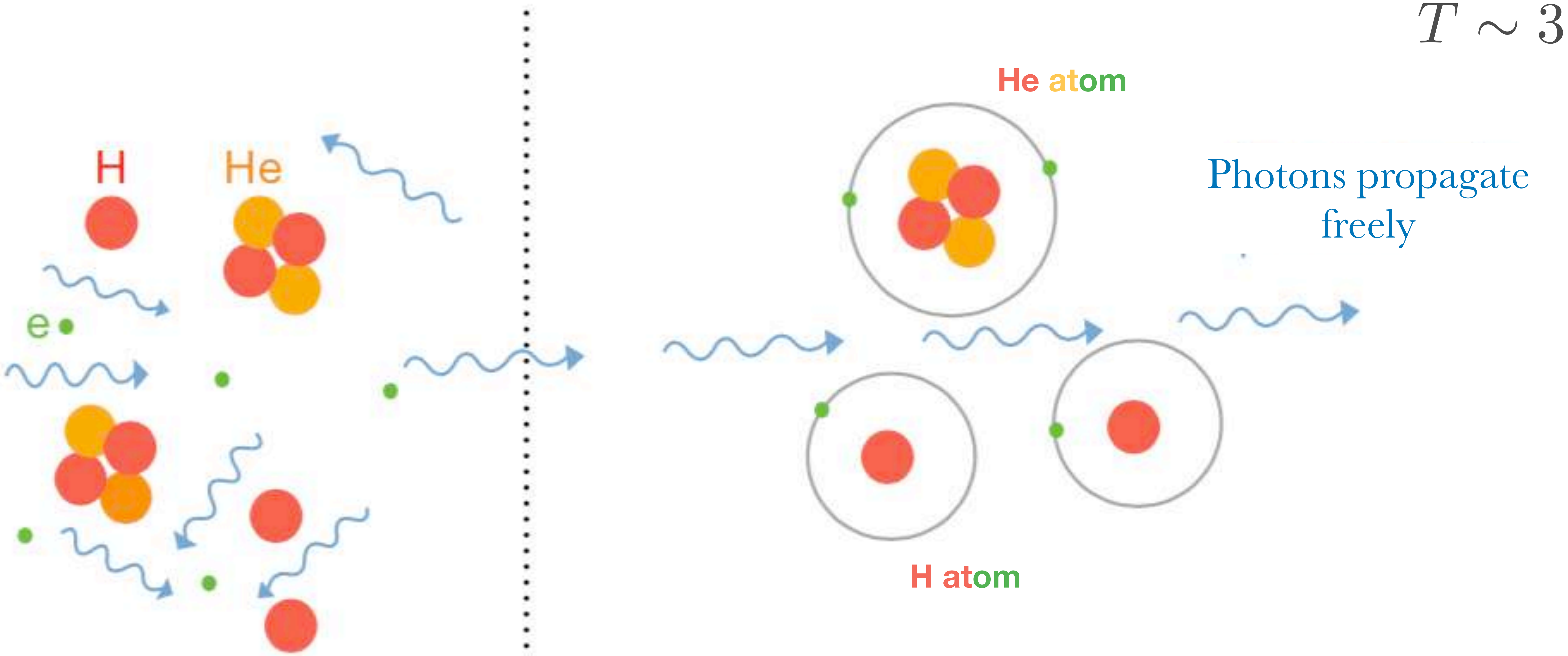


Crédito: D. Baumann

Recombination and photon decoupling

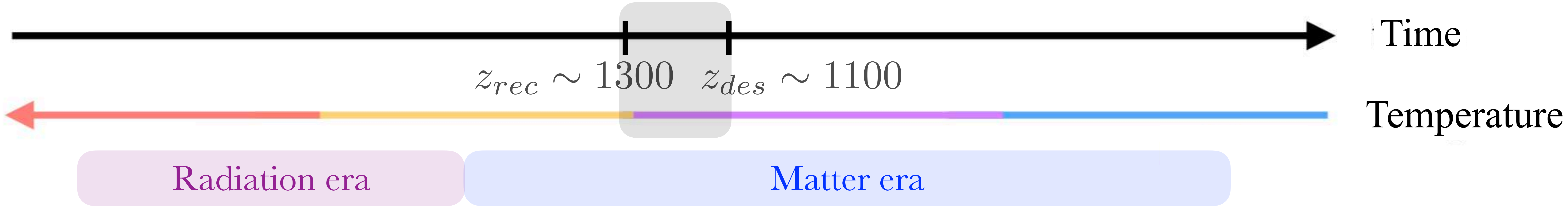
$t \sim 370000\,\text{yrs}$

$T \sim 3000\,\text{K}$



Plasma ("soup") of coupled H, He, elétrons and radiation - thermal equilibrium
- universe is opaque: radiation cannot scape!

Atoms are formed!
Charged electrons bound with n H and He nucleus



Crédito: D. Baumann

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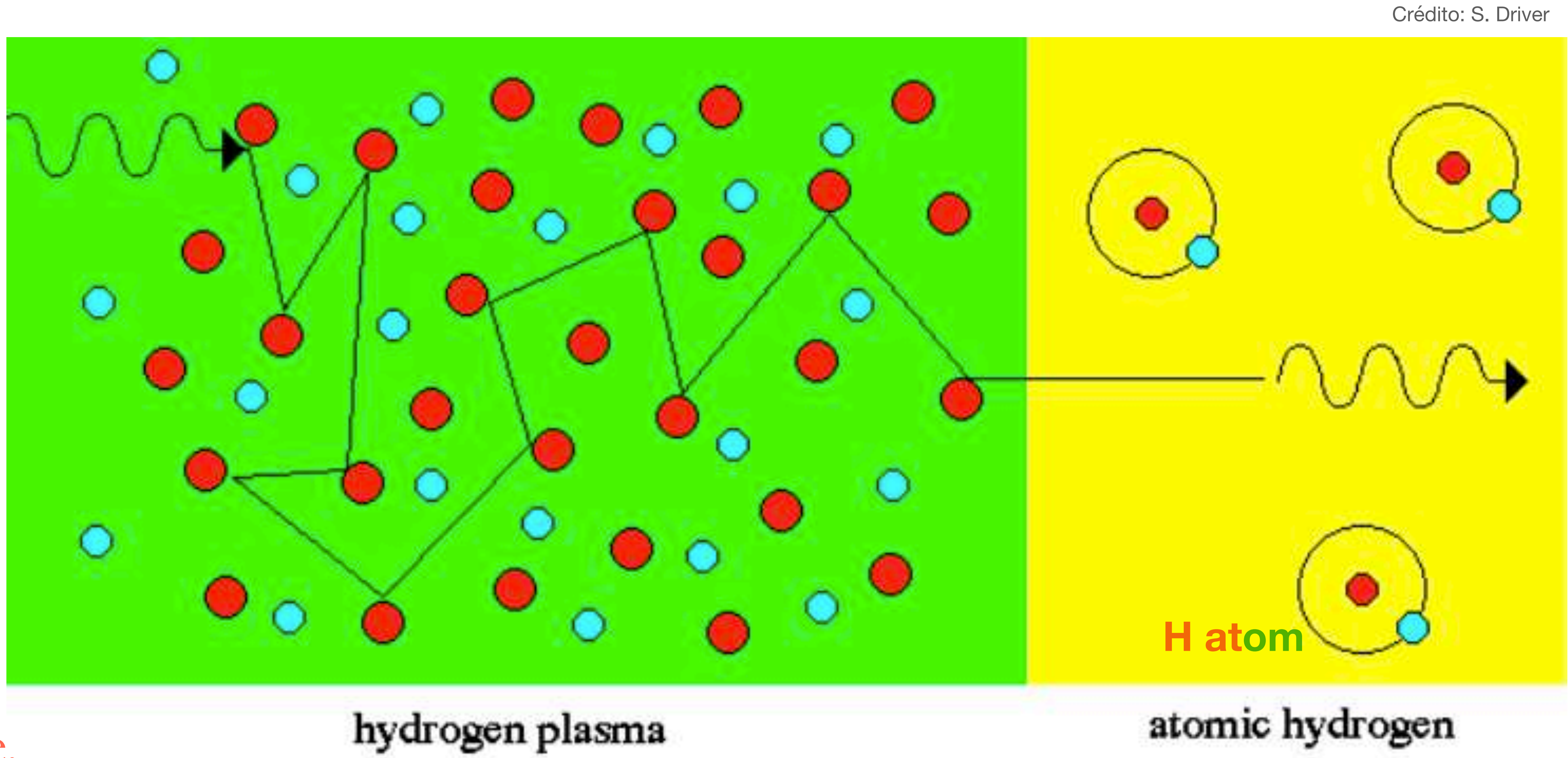

Photons propagate freely

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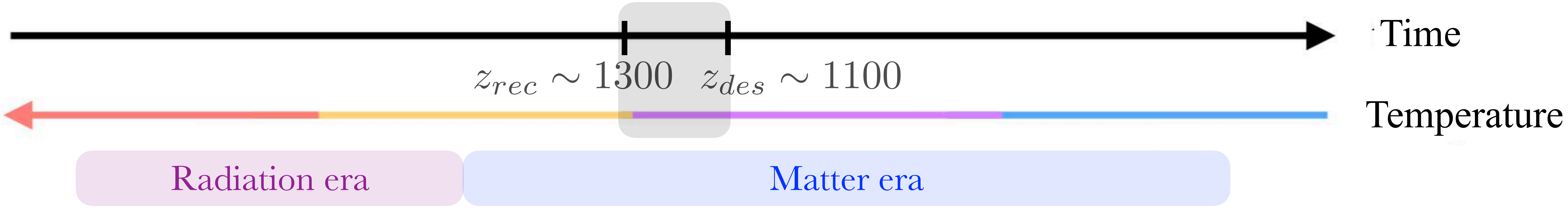

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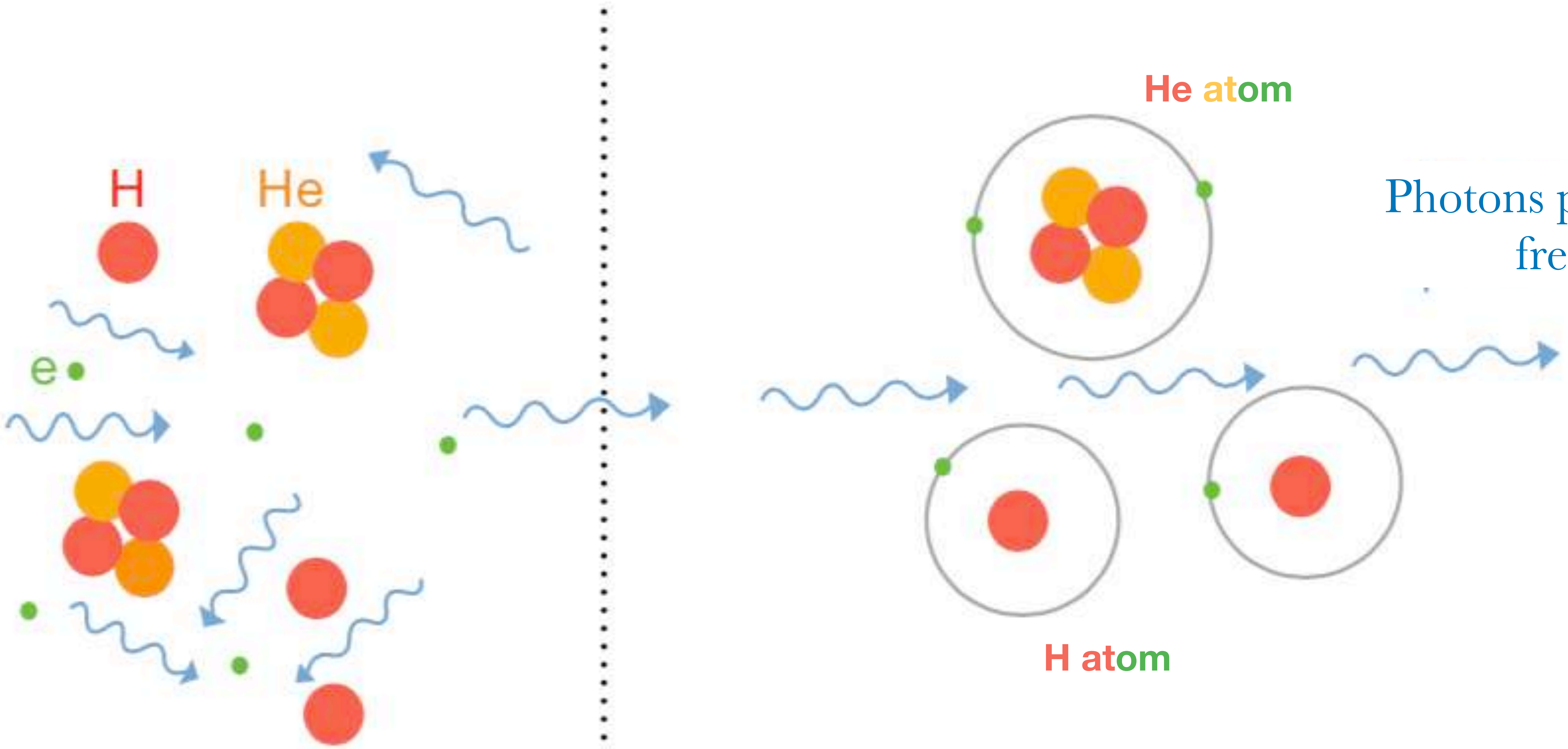

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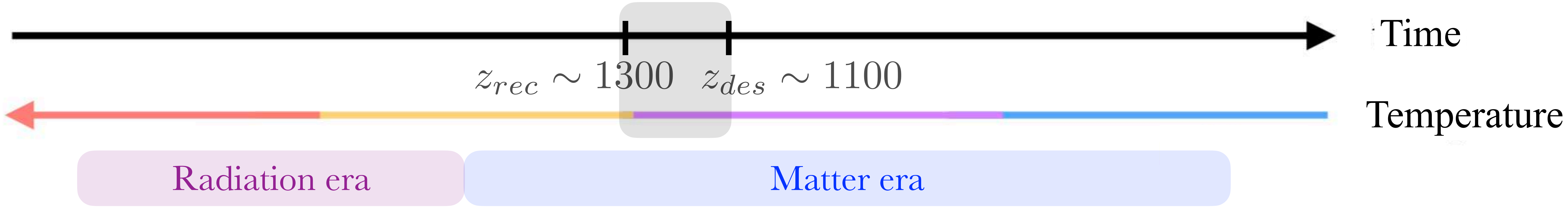
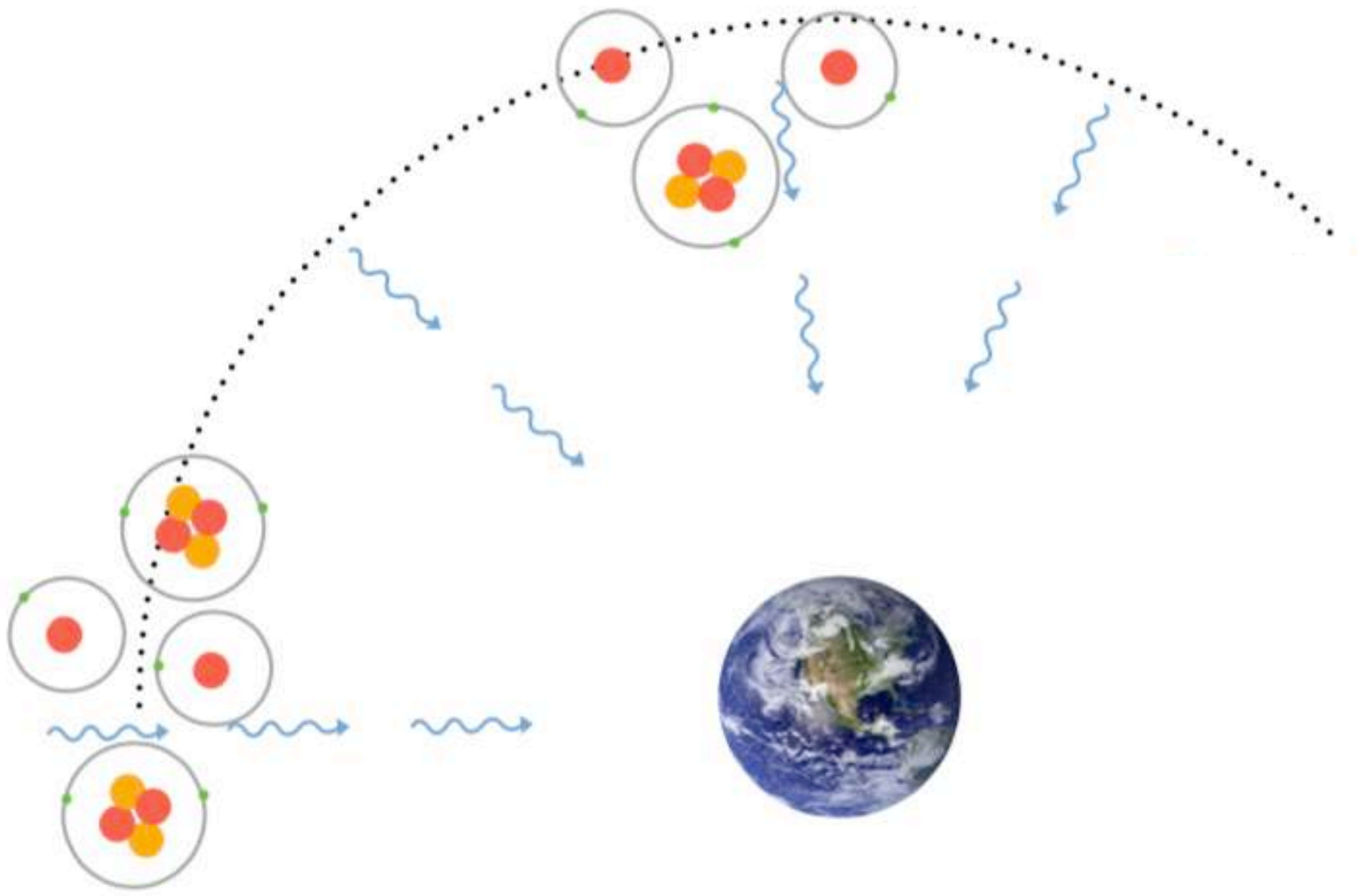


Crédito: D. Baumann

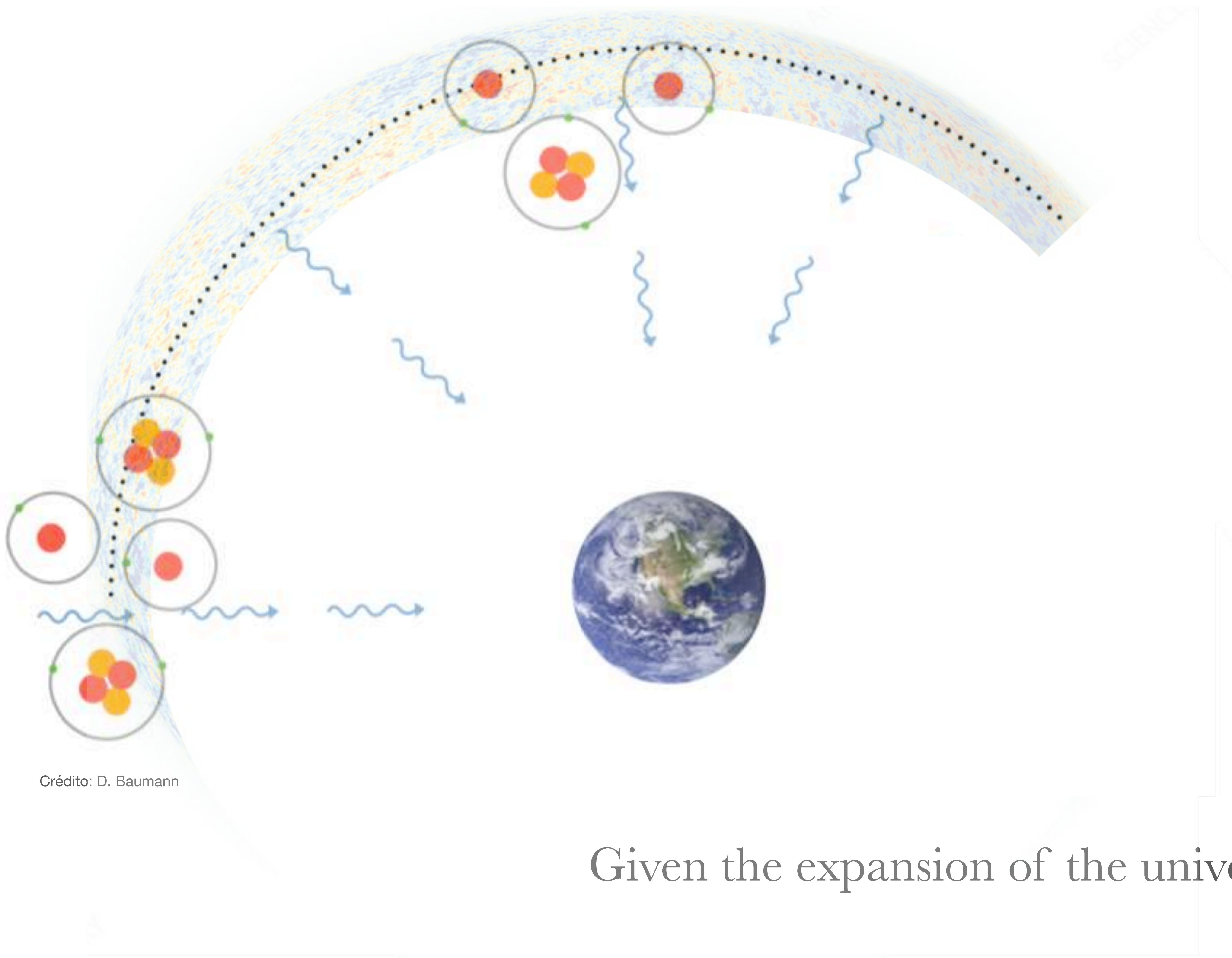
These photons are the first light of our universe…



Crédito: D. Baumann

… e tell us how the universe was at early times.

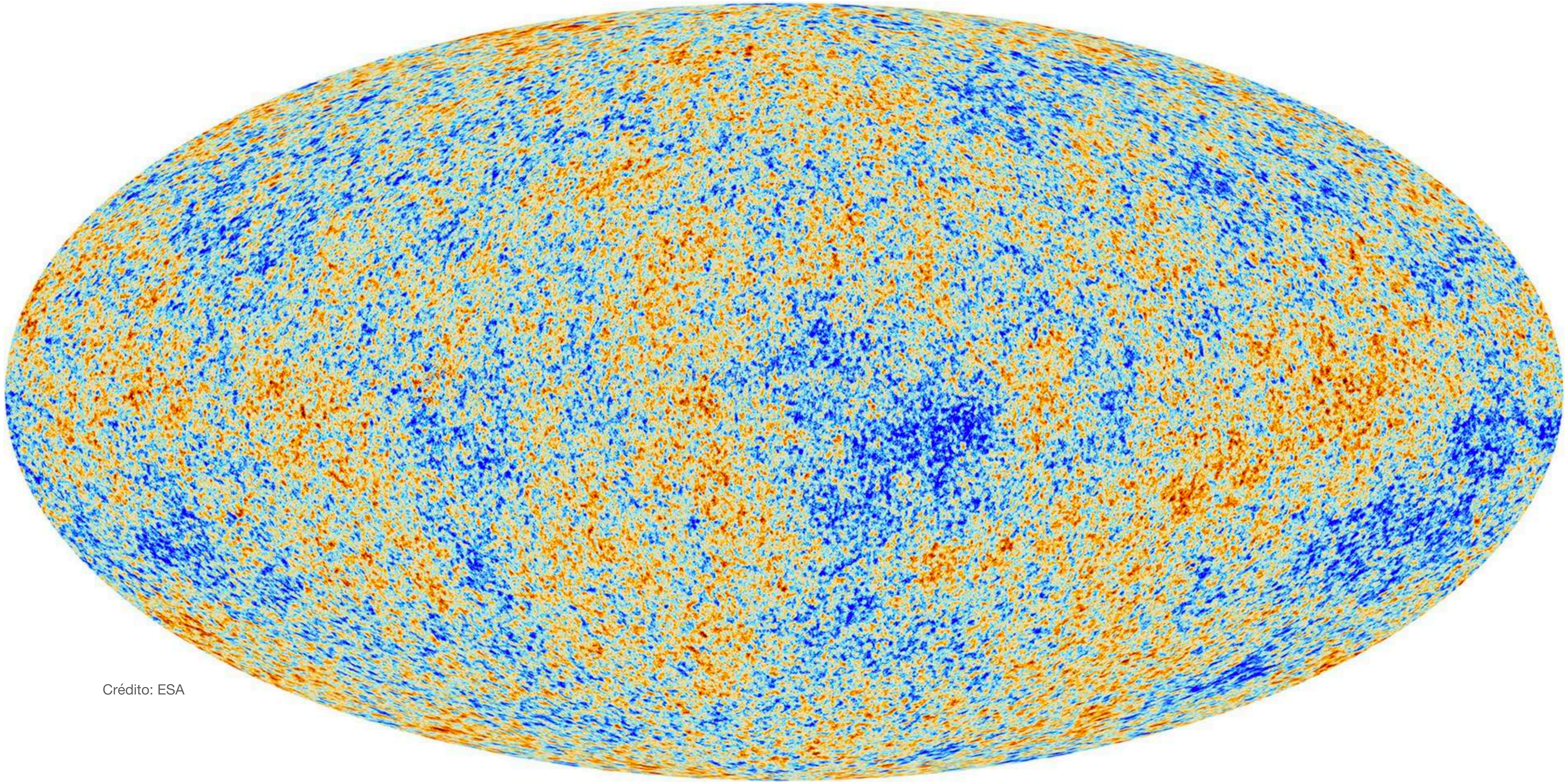
Cosmic **M**icrowave **B**ackground *(CMB)*



Crédito: D. Baumann

Given the expansion of the universe, we observe these photons in microwave.

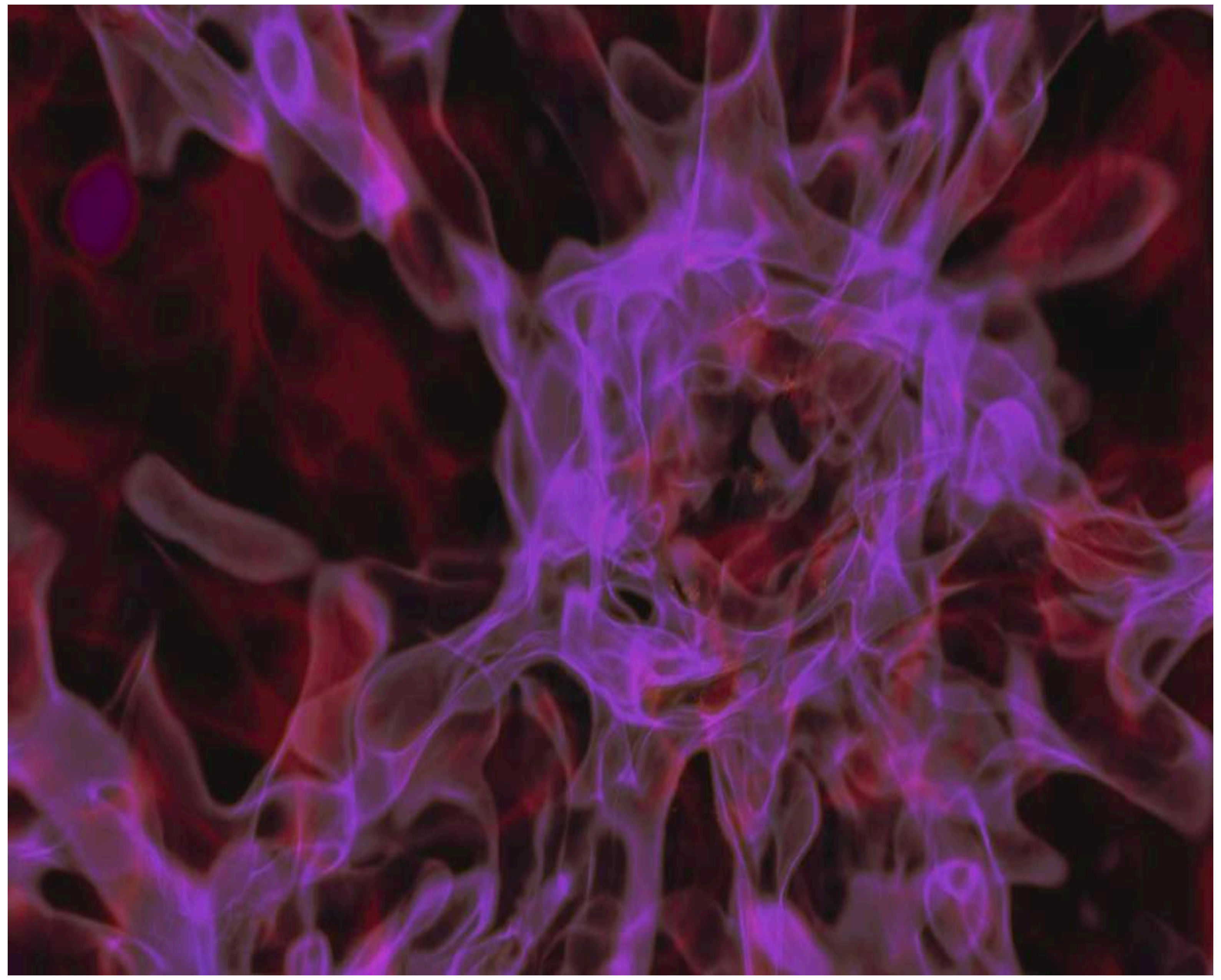
Cosmic **M**icrowave **B**ackground *(CMB)*



Temperature 2.7 K. Small fluctuations - initial condition for the structures of our universe

Formation of stars and galaxies

$t \gtrsim 1$ billion yrs



Credit: AARON SMITH/TACC/UT-AUSTIN

Credit: NASA and ESA

Time

$t \gtrsim 1$ billion yrs

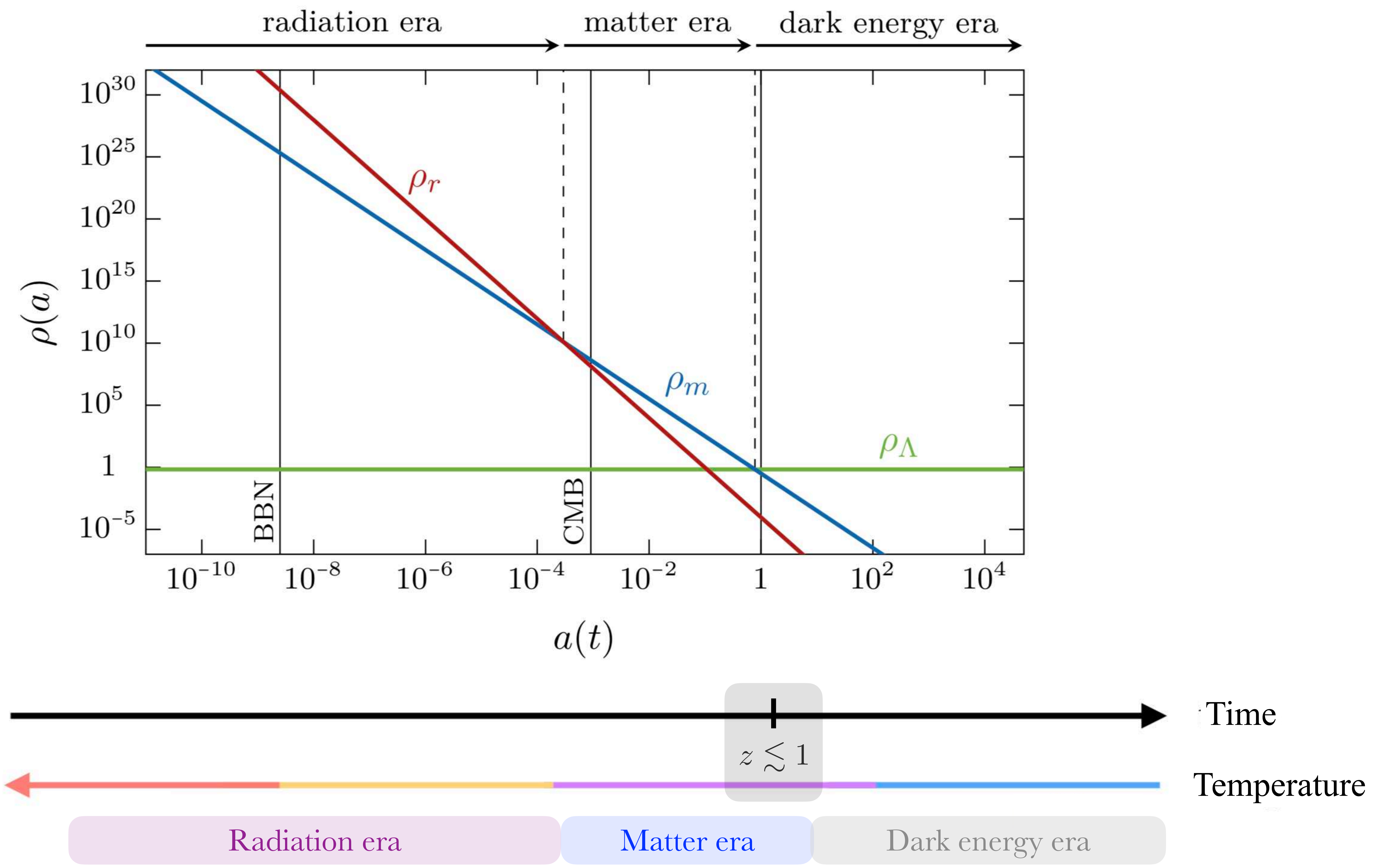
Temperature

Matter era

Crédito: D. Baumann

Dark Energy Era

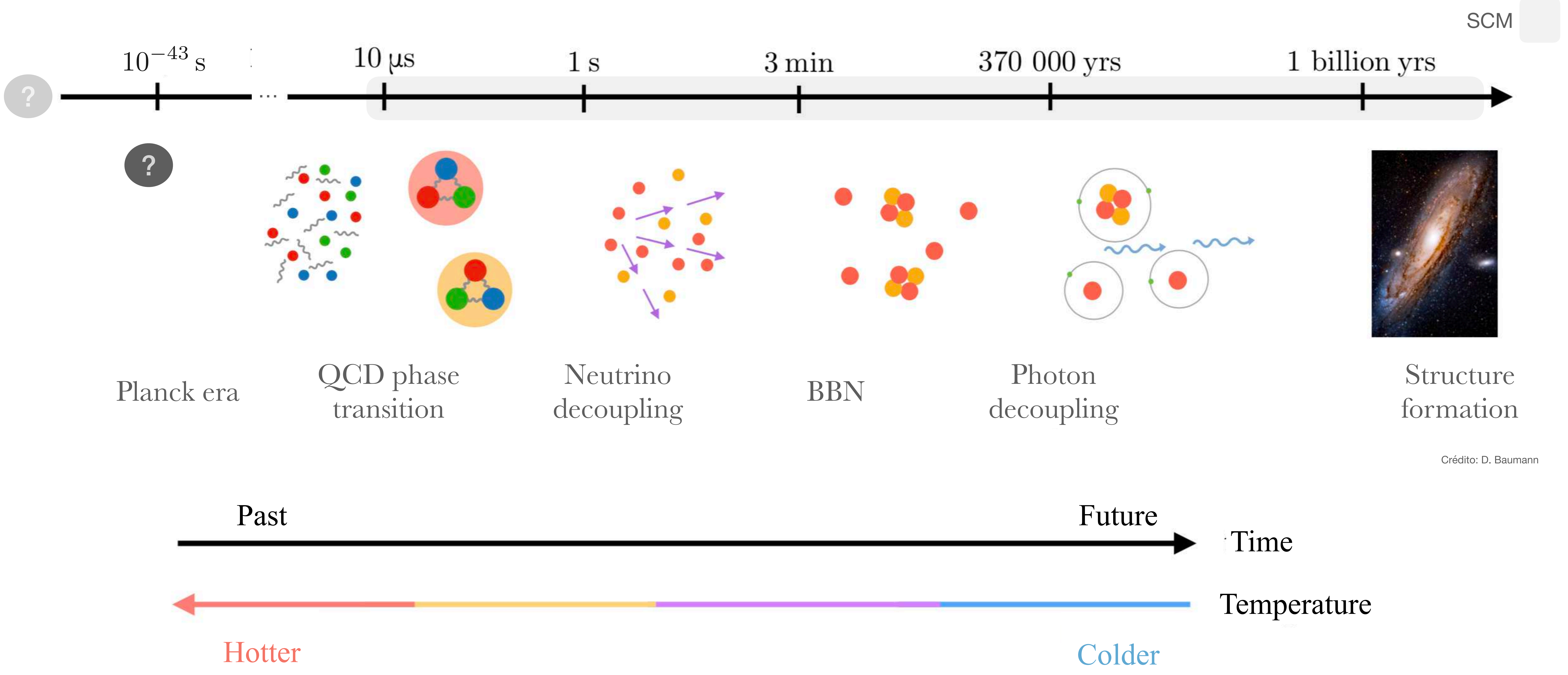
$z \lesssim 1$



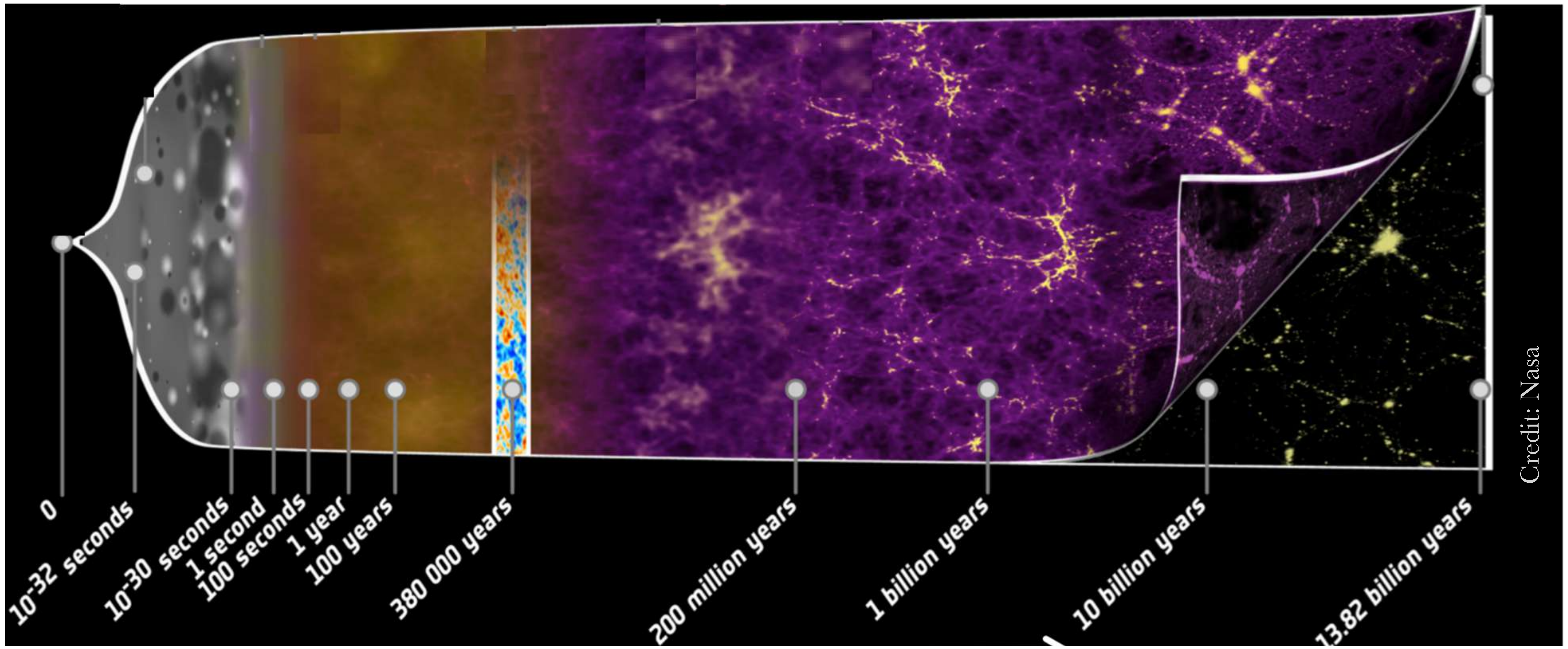
Crédito: D. Baumann

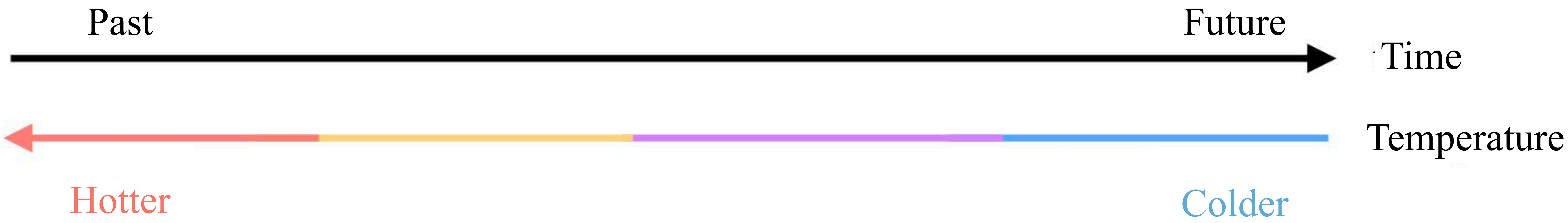
Thermal history of the universe

The universe"started" hot e **dense** → As it cools, the structures we know start to form

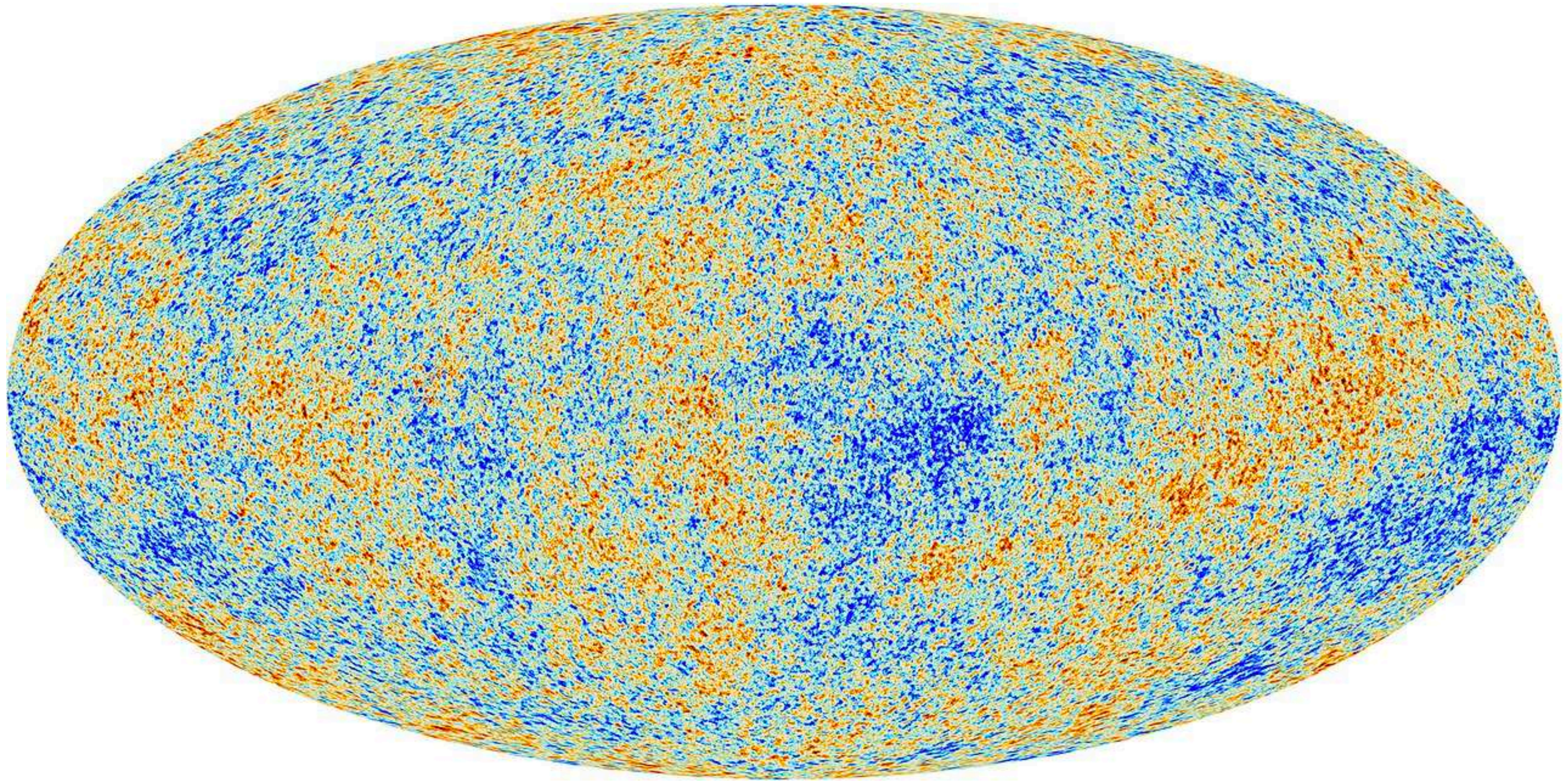


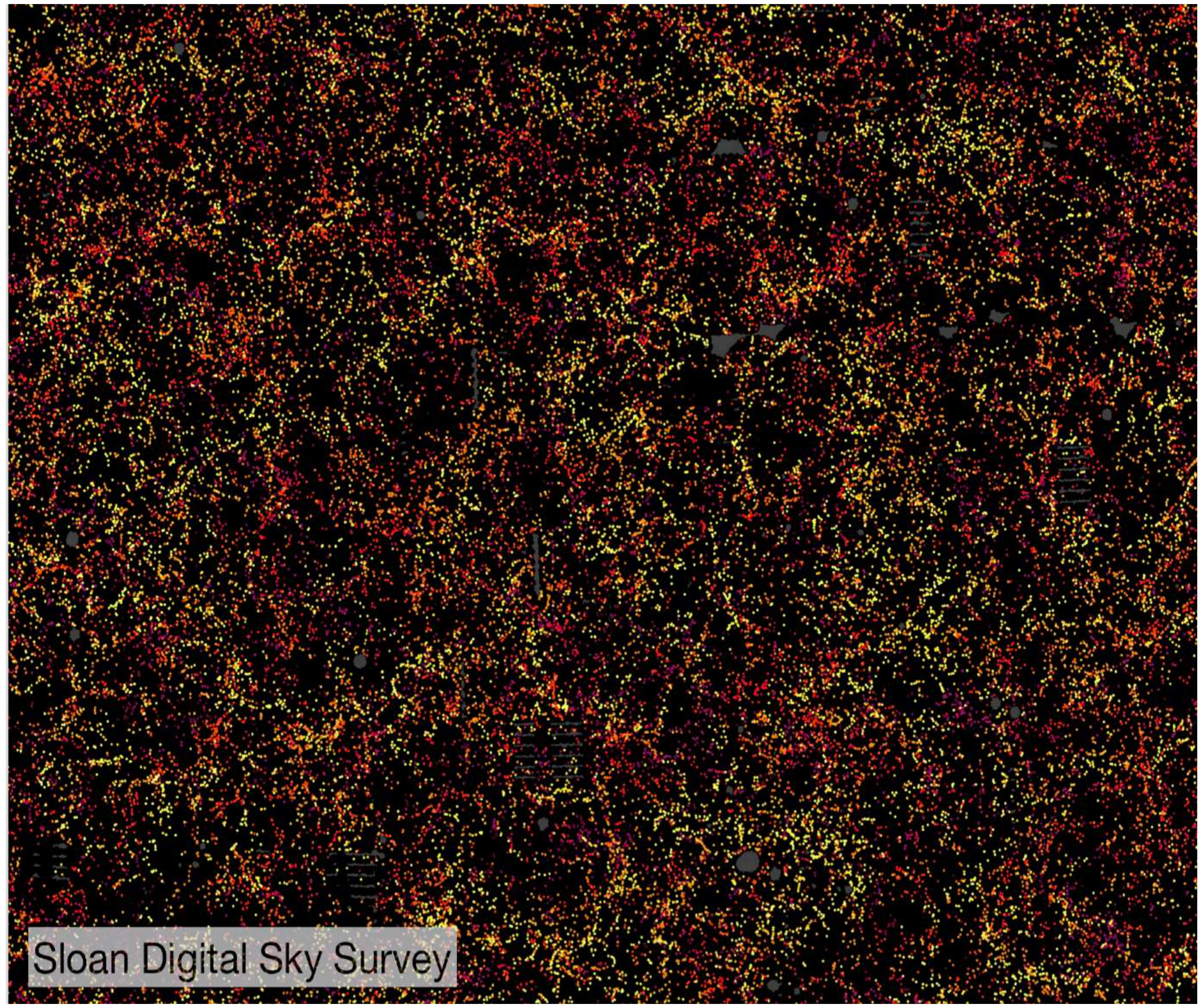
Thermal history of the universe





Large scale structure





Cosmological parameters

Standard cosmological model - LCDM model

$$\{\Omega_b, \, \Omega_m \, , \Omega_\Lambda \, , n_s \, , A_s \, , \tau\}$$

Using CMB and other LSS probes, can constraint the parameters with incredible precision.

Planck 2018

$$\Omega_b = 0.0484 \pm 0.0003 \quad \longrightarrow \quad \text{Quantidade de matéria visível/usual}$$
$$\Omega_m = 0.308 \pm 0.012 \quad \longrightarrow \quad \text{Quantidade de matéria escura}$$
$$\Omega_\Lambda = 0.692 \pm 0.012 \quad \longrightarrow \quad \text{Quantidade de energia escura}$$
$$\mathrm{n}_s = 0.9626 \pm 0.0057 \quad \longrightarrow \quad \text{Dependência de escala das flutuações iniciais}$$
$$10^9 \, A_s = 2.092 \pm 0.034 \quad \longrightarrow \quad \text{Amplitude das flutuações iniciais}$$
$$\tau = 0.0522 \pm 0.0080 \quad \longrightarrow \quad \text{Profundidade óptica}$$

How opaque the universe is to photons that travel in it

Summary

Standard cosmological model - Hot Big Bang model

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3 obs pilars:

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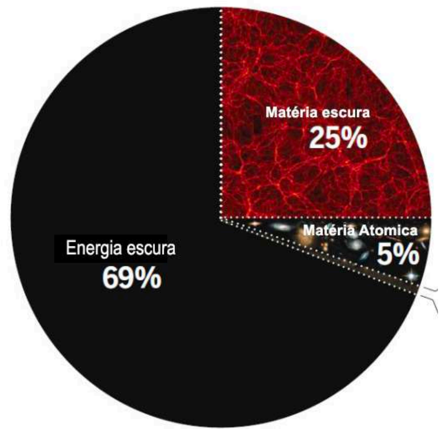
- BBN
- CMB

\+



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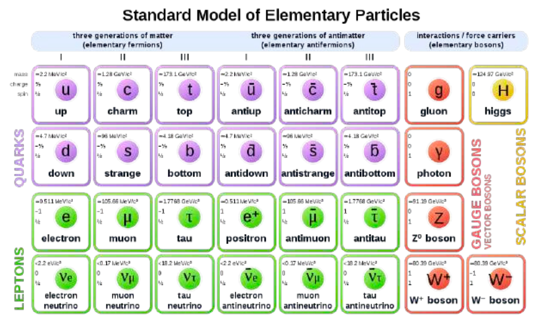
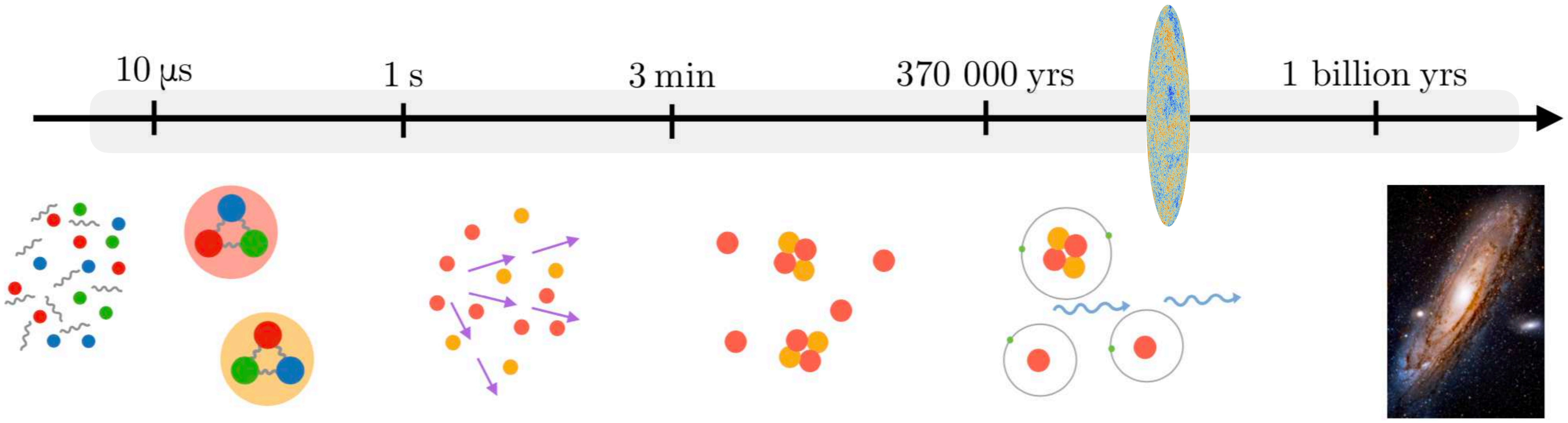
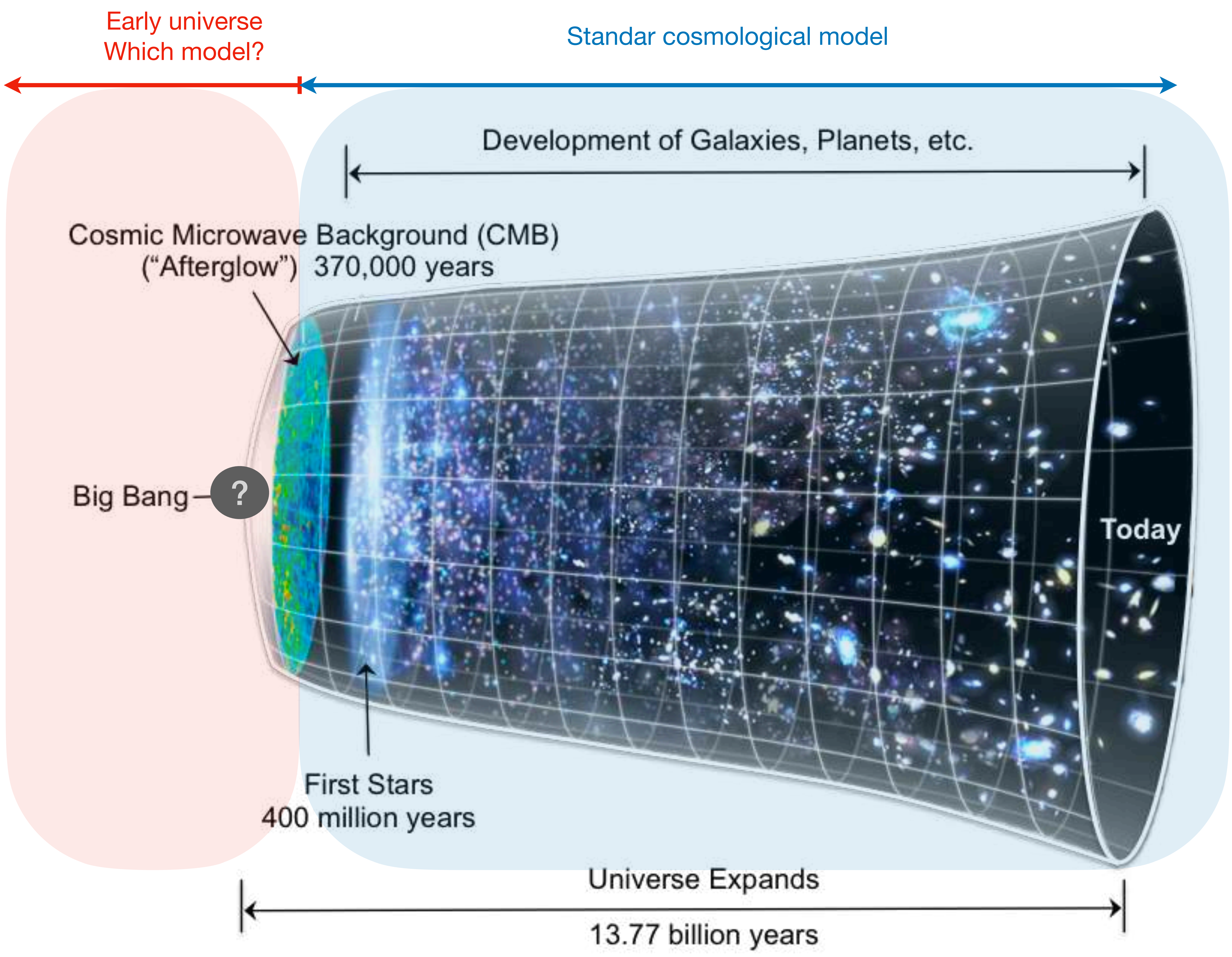
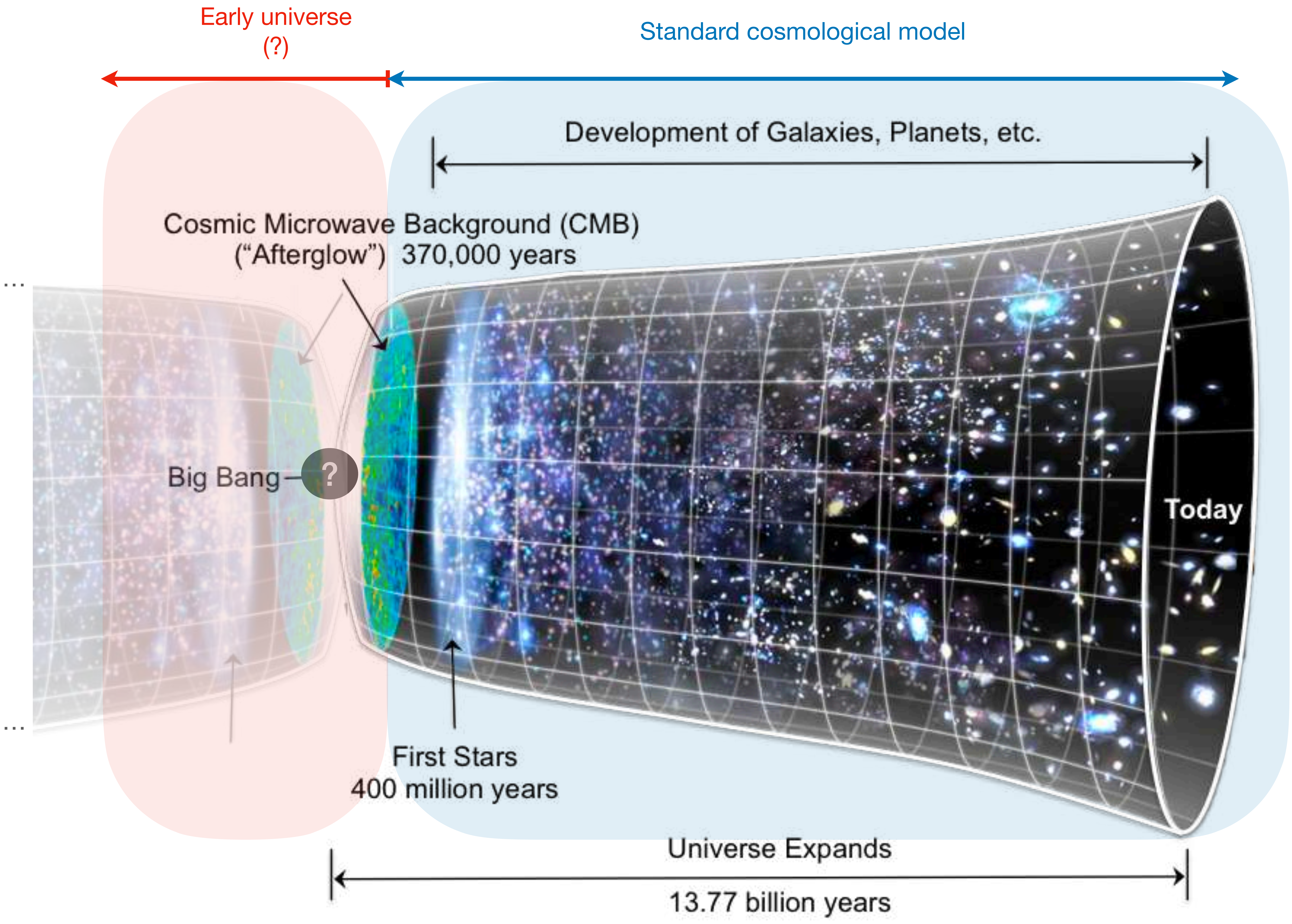


* Also presents problems!

Thermal history of the universe




Early universe
Which model?
Standar cosmological model
Development of Galaxies, Planets, etc.
Cosmic Microwave Background (CMB)
(“Afterglow”) 370,000 years
Big Bang
?
Today
First Stars
400 million years
Universe Expands
13.77 billion years


Early universe
(?)
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("Afterglow") 370,000 years
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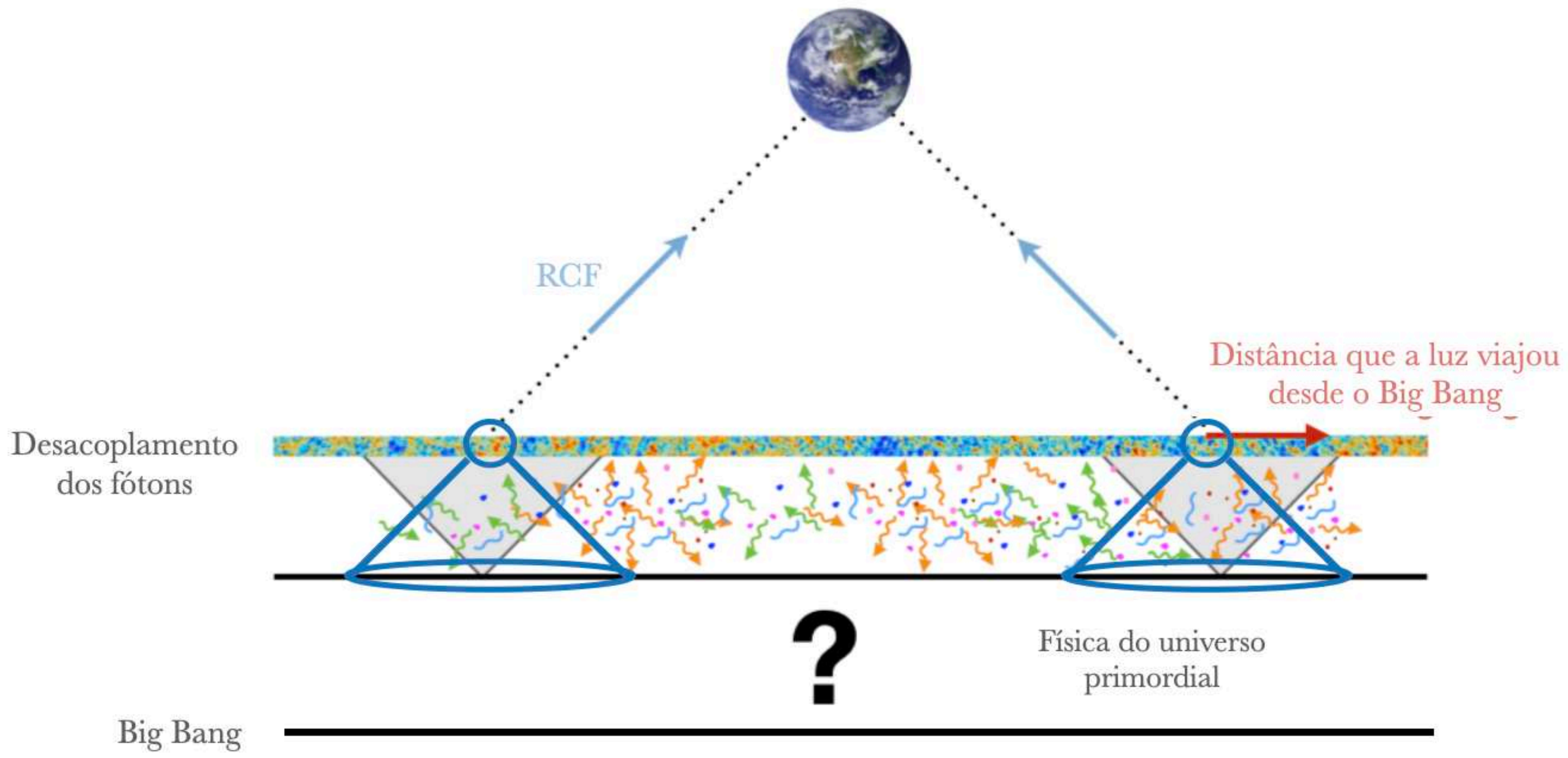
Next class

Problems with the SCM!



Possible solution: inflation!