Lesson 3:
Inflation (cont.)
Theory of cosmological perturbations

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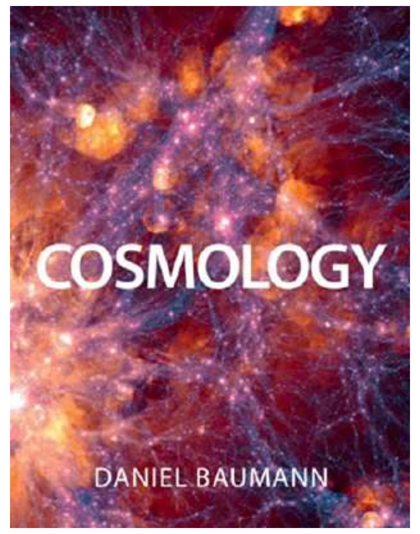
Early universe cosmology, USP
24/Nov/2022

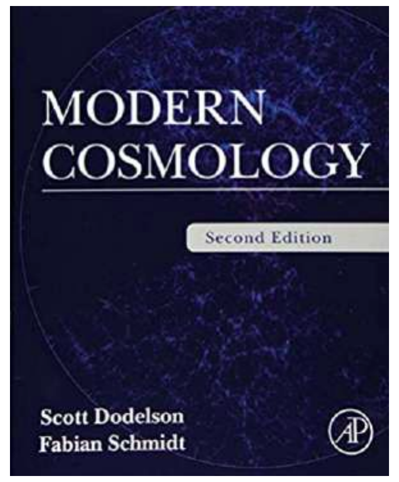
Image: Nasa,ESA, CSA, and STScI

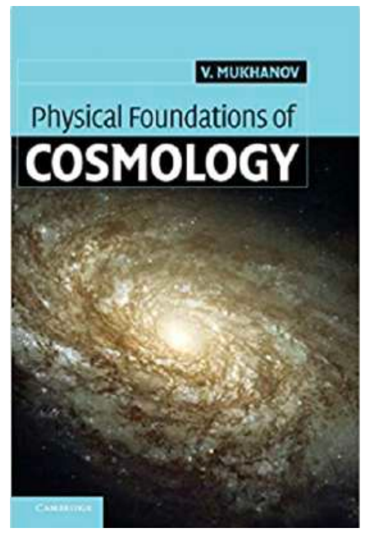
Early universe cosmology

References:

- Daniel Baumann, Cosmology, Cambridge University Press, 2022.
- Viatcheslav Mukhanov, Physical Foundations of Cosmology, Cambridge University Press, 2005
- Daniel Baumann, TASI lectures on inflation
- (Recurso em português) Tese de mestrado Elisa G. M. Ferreira (capítulos 2 e 3)







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Review - lesson 2

Inflation

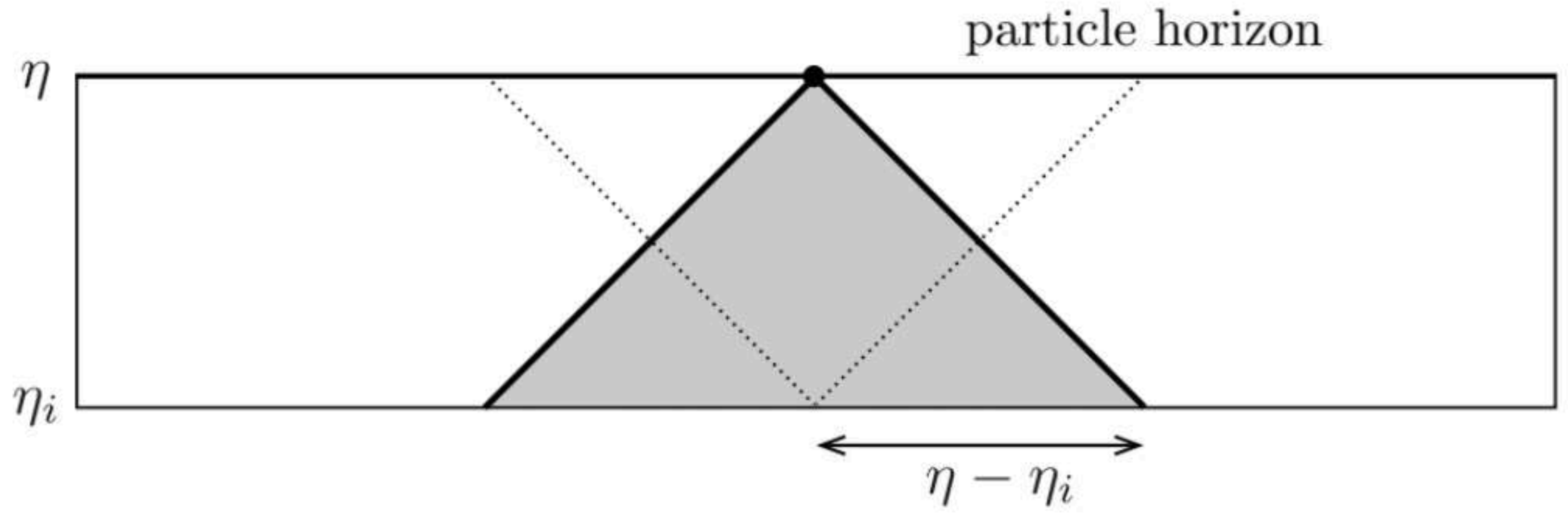
Problems of the standard cosmological model

- Horizon problem
- Problem of the origin of structures
- Flatness problem
- Problem of the magnetic monopoles
- Initial singularity
- DM and DE

Horizons in cosmology

Since the speed of light is constant and the universe is expanding, there is a limit for what is accessible to an observer in the universe.

Particle horizon: distance that the light travelled since the Big BangParticle horizon



(comoving) Hubble radius

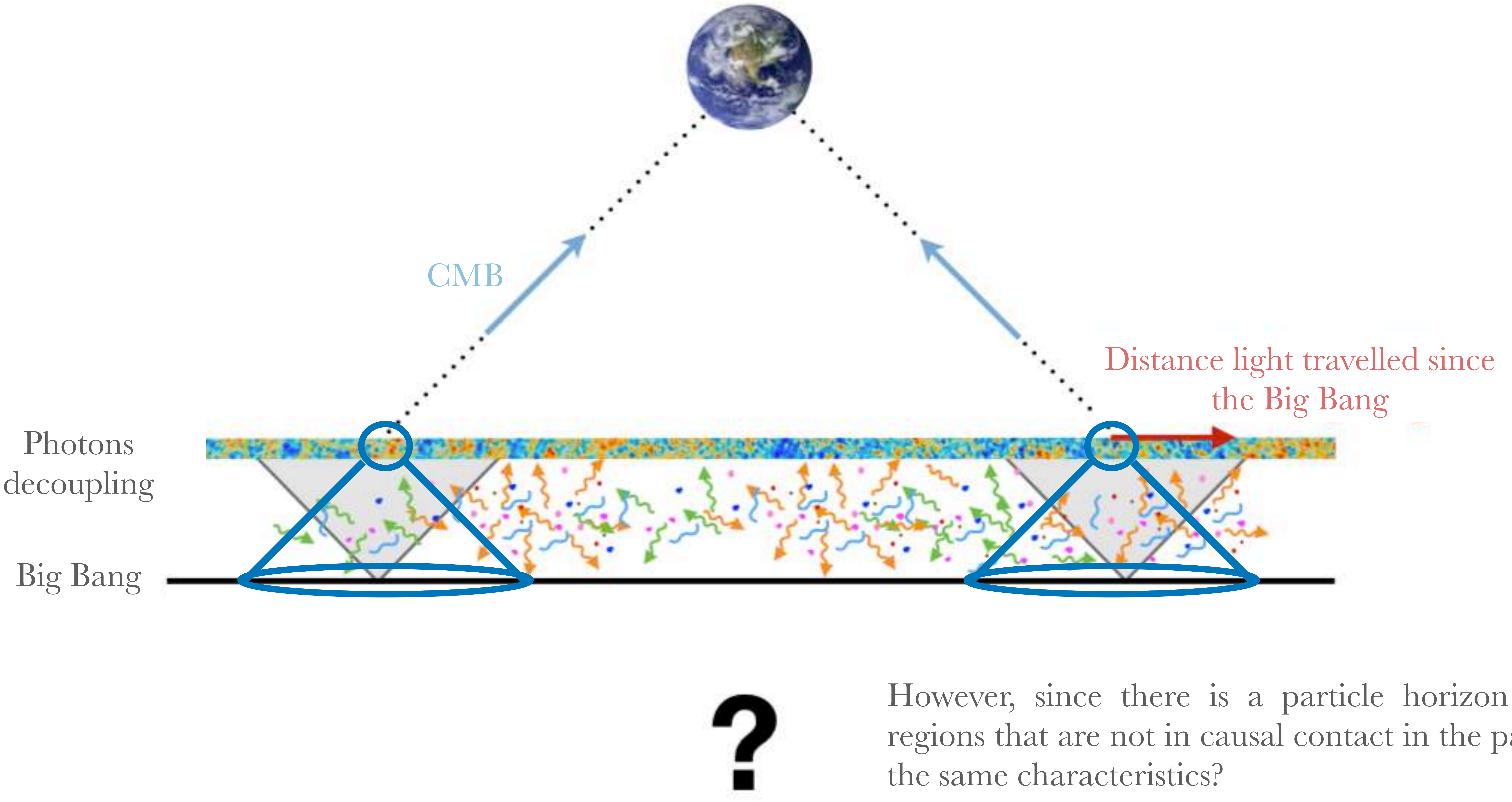
$$\chi_p(\eta) = \eta - \eta_i = \int_{t_i}^{t} \frac{dt}{a(t)} = \int_{\ln a_i}^{\ln a} \frac{d\ln a}{\dot{a}} = \int_{\ln a_i}^{\ln a} (aH)^{-1}\, d\ln a$$

Conformal time $d\eta = dt/a(t)$

The size of the particle horizon at η is the intersection of the past light cone of an observer O with the spacelike surface $\eta = \eta_i$

Horizon problem

As we saw, the CMB presents the same temperature in every point of the observable universe, except from small deviations



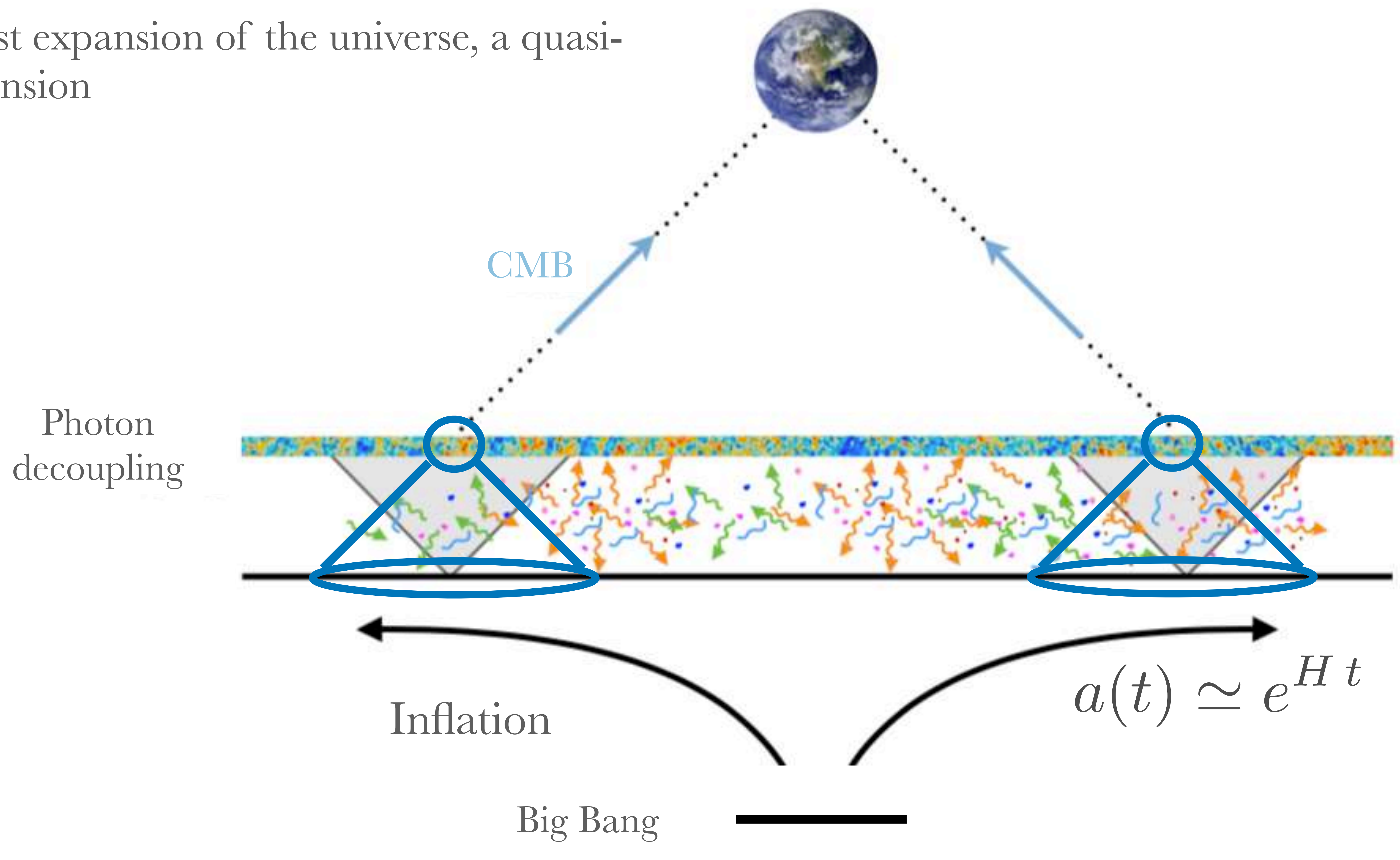
However, since there is a particle horizon today, HOW regions that are not in causal contact in the past can present the same characteristics?

Crédito: D. Baumann

Inflation

Guth (1980)
Linde(1982)
Albrecht e Steinhardt (1982)

Period of very fast expansion of the universe, a quasi-exponential expansion



Originally (Guth 1980) - to solve the magnetic monopoles problem

Crédito: D. Baumann

Constructing *inflation*

$N = \ln a$

e-fold

1. Decreasing radius / slowly-varying Hubble parameter

1st slow-roll parameter $\epsilon \equiv -\frac{\dot{H}}{H^2} < 1$ Small

2. Inflation persists for long enough (ϵ small for enough time)

2nd slow-roll parameter $\eta \equiv \frac{d \ln \epsilon}{dN} = \frac{\dot{\epsilon}}{H\epsilon}$, $|\eta| < 1$

$$\ddot{a} > 0 \leftrightarrow p \sim -\rho \leftrightarrow H = \text{const.} \leftrightarrow \rho \sim \text{const.} \leftrightarrow \epsilon < 1 \leftrightarrow a(t) \simeq \exp(Ht)$$

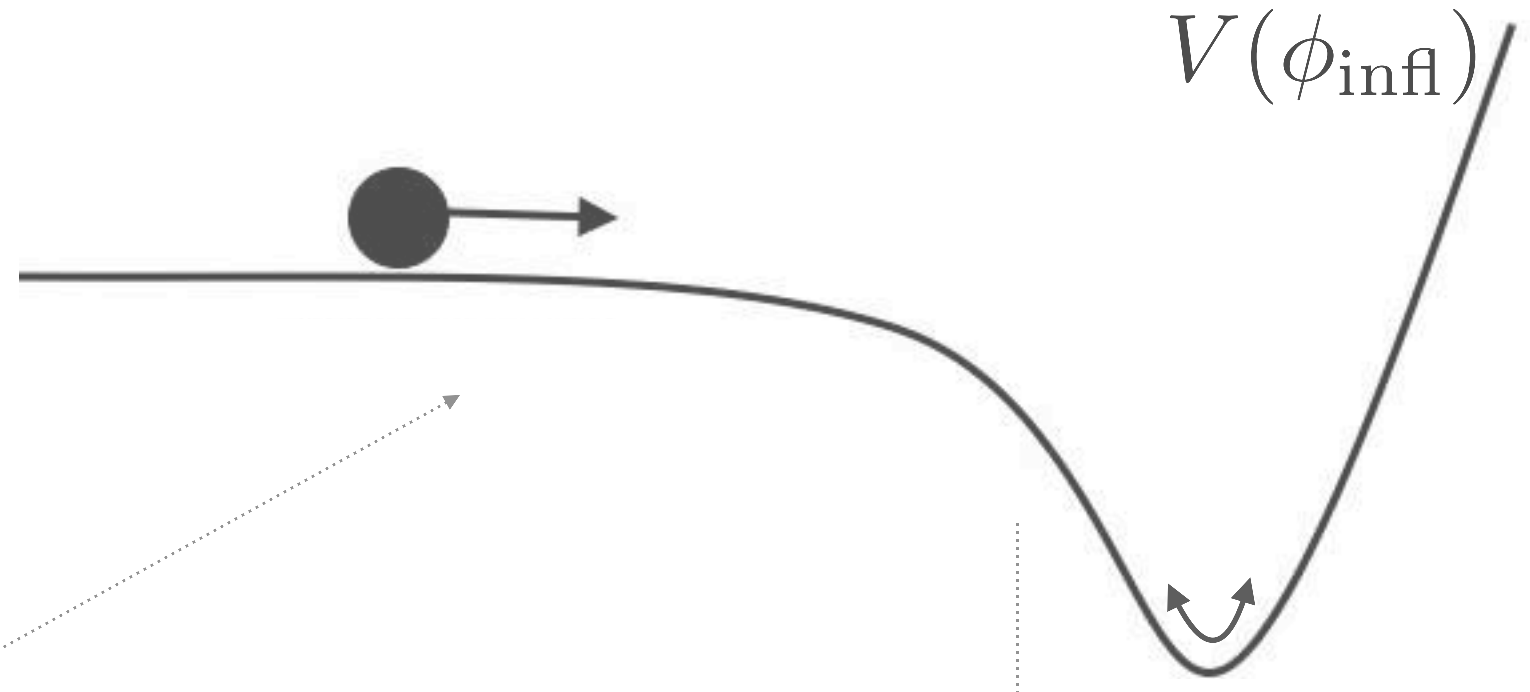
Inflation

What microphysics leads/gives $\{\epsilon, \eta\} \ll 1$

TOY MODEL:

Scalar field (*inflaton*) $\phi(t)$ in a FRW background

To cause the acceleration, the potential has to have the form:



Causes the accelerated expansion - quasi-exponential

However, inflation has to end, so the era of radiation begins - ***graceful exit***

$$\epsilon \sim 1$$

Inflation

Single scalar field inflation

$$S = -\int d^4x\sqrt{-g}\,\mathcal{L} = -\int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)\right]$$

Equation of motion:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Slow-roll parameters:

1st slow-roll parameter

$$\epsilon = \frac{\dot{\phi}^2/2}{M_{pl}^2 H^2}$$

2nd slow-roll parameter

$$\eta = 2\left(\epsilon + \frac{\ddot{\phi}}{H\dot{\phi}}\right)$$

Inflation models

Many models!!!

Name	Parameters	Sub-models	$V(\phi)$
HI	0	1	$M^4\left(1-e^{-\sqrt{2/3}\phi/M_{\rm Pl}}\right)$
RCHI	1	1	$M^4\left(1-2e^{-\sqrt{2/3}\phi/M_{\rm Pl}}+\frac{A_{\rm I}}{16\pi^2}\frac{\phi}{\sqrt{6}M_{\rm Pl}}\right)$
LFI	1	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^p$
MLFI	1	1	$M^4\frac{\phi^2}{M_{\rm Pl}^2}\left[1+\alpha\frac{\phi^2}{M_{\rm Pl}^2}\right]$
RCMI	1	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^2\left[1-2\alpha\frac{\phi^2}{M_{\rm Pl}^2}\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]$
RCQI	1	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^4\left[1-\alpha\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]$
NI	1	1	$M^4\left[1+\cos\left(\frac{\phi}{f}\right)\right]$
ESI	1	1	$M^4\left(1-e^{-q\phi/M_{\rm Pl}}\right)$
PLI	1	1	$M^4e^{-\alpha\phi/M_{\rm Pl}}$
KMII	1	2	$M^4\left(1-\alpha\frac{\phi}{M_{\rm Pl}}e^{-\phi/M_{\rm Pl}}\right)$
HF1I	1	1	$M^4\left(1+A_1\frac{\phi}{M_{\rm Pl}}\right)^2\left[1-\frac{2}{3}\left(\frac{A_1}{1+A_1\phi/M_{\rm Pl}}\right)^2\right]$
CWI	1	1	$M^4\left[1+\alpha\left(\frac{\phi}{Q}\right)^4\ln\left(\frac{\phi}{Q}\right)\right]$
LI	1	2	$M^4\left[1+\alpha\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]$
RpI	1	3	$M^4e^{-2\sqrt{2/3}\phi/M_{\rm Pl}}\left\|e^{\sqrt{2/3}\phi/M_{\rm Pl}}-1\right\|^{2p/(2p-1)}$
DWI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-1\right]^2$
MHI	1	1	$M^4\left[1-\mathrm{sech}\left(\frac{\phi}{\mu}\right)\right]$
RGI	1	1	$M^4\frac{(\phi/M_{\rm Pl})^2}{\alpha+(\phi/M_{\rm Pl})^2}$
MSSMI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{2}{3}\left(\frac{\phi}{\phi_0}\right)^6+\frac{1}{5}\left(\frac{\phi}{\phi_0}\right)^{10}\right]$
RIPI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{4}{3}\left(\frac{\phi}{\phi_0}\right)^3+\frac{1}{2}\left(\frac{\phi}{\phi_0}\right)^4\right]$

AI	1	1	$M^4\left[1-\frac{2}{\pi}\arctan\left(\frac{\phi}{\mu}\right)\right]$
CNAI	1	1	$M^4\left[3-(3+\alpha^2)\tanh^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\rm Pl}}\right)\right]$
CNBI	1	1	$M^4\left[(3-\alpha^2)\tan^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\rm Pl}}\right)-3\right]$
OSTI	1	1	$-M^4\left(\frac{\phi}{\phi_0}\right)^2\ln\left[\left(\frac{\phi}{\phi_0}\right)^2\right]$
WRI	1	1	$M^4\ln\left(\frac{\phi}{\phi_0}\right)^2$
SFI	2	1	$M^4\left[1-\left(\frac{\phi}{\mu}\right)^p\right]$
II	2	1	$M^4\left(\frac{\phi-\phi_0}{M_{\rm Pl}}\right)^{-\beta}-M^4\frac{\beta^2}{6}\left(\frac{\phi-\phi_0}{M_{\rm Pl}}\right)^{-\beta-2}$
KMIII	2	1	$M^4\left[1-\alpha\frac{\phi}{M_{\rm Pl}}\exp\left(-\beta\frac{\phi}{M_{\rm Pl}}\right)\right]$
LMI	2	2	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^\alpha\exp\left[-\beta(\phi/M_{\rm Pl})^\gamma\right]$
TWI	2	1	$M^4\left[1-A\left(\frac{\phi}{\phi_0}\right)^2e^{-\phi/\phi_0}\right]$
GMSSMI	2	2	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{2}{3}\alpha\left(\frac{\phi}{\phi_0}\right)^6+\frac{\alpha}{5}\left(\frac{\phi}{\phi_0}\right)^{10}\right]$
GRIPI	2	2	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{4}{3}\alpha\left(\frac{\phi}{\phi_0}\right)^3+\frac{\alpha}{2}\left(\frac{\phi}{\phi_0}\right)^4\right]$
BSUSYBI	2	1	$M^4\left(e^{\sqrt{6}\frac{\phi}{M_{\rm Pl}}}+e^{\sqrt{6}\gamma\frac{\phi}{M_{\rm Pl}}}\right)$
TI	2	3	$M^4\left(1+\cos\frac{\phi}{\mu}+\alpha\sin^2\frac{\phi}{\mu}\right)$
BEI	2	1	$M^4\exp_{1-\beta}\left(-\lambda\frac{\phi}{M_{\rm Pl}}\right)$
PSNI	2	1	$M^4\left[1+\alpha\ln\left(\cos\frac{\phi}{f}\right)\right]$
NCKI	2	2	$M^4\left[1+\alpha\ln\left(\frac{\phi}{M_{\rm Pl}}\right)+\beta\left(\frac{\phi}{M_{\rm Pl}}\right)^2\right]$
CSI	2	1	$\frac{M^4}{\left(1-\alpha\frac{\phi}{M_{\rm Pl}}\right)^2}$
OI	2	1	$M^4\left(\frac{\phi}{\phi_0}\right)^4\left[\left(\ln\frac{\phi}{\phi_0}\right)^2-\alpha\right]$
CNCI	2	1	$M^4\left[(3+\alpha^2)\coth^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\rm Pl}}\right)-3\right]$
SBI	2	2	$M^4\left\{1+\left[-\alpha+\beta\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]\left(\frac{\phi}{M_{\rm Pl}}\right)^4\right\}$
SSBI	2	6	$M^4\left[1+\alpha\left(\frac{\phi}{M_{\rm Pl}}\right)^2+\beta\left(\frac{\phi}{M_{\rm Pl}}\right)^4\right]$
IMI	2	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^{-p}$
BI	2	2	$M^4\left[1-\left(\frac{\phi}{\mu}\right)^{-p}\right]$

RMI	3	4	$M^4\left[1-\frac{c}{2}\left(-\frac{1}{2}+\ln\frac{\phi}{\phi_0}\right)\frac{\phi^2}{M_{\rm Pl}^2}\right]$
VHI	3	1	$M^4\left[1+\left(\frac{\phi}{\mu}\right)^p\right]$
DSI	3	1	$M^4\left[1+\left(\frac{\phi}{\mu}\right)^{-p}\right]$
GMLFI	3	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^p\left[1+\alpha\left(\frac{\phi}{M_{\rm Pl}}\right)^q\right]$
LPI	3	3	$M^4\left(\frac{\phi}{\phi_0}\right)^p\left(\ln\frac{\phi}{\phi_0}\right)^q$
CNDI	3	3	$\frac{M^4}{\left\{1+\beta\cos\left[\alpha\left(\frac{\phi-\phi_0}{M_{\rm Pl}}\right)\right]\right\}^2}$

Encyclopædia Inflationaris J. Martin, et al (2013)

Case study

Inflation $\longrightarrow$ graceful exit $\longrightarrow$ SCM evolution

(quasi-exp. expansion)

Many models!!!

CHAOTIC INFLATION

Linde(1982)

Conditions

- Single field
- Potential that has a slow-roll region
- Graceful exit
- General initial conditions

$\Rightarrow \quad \phi_i \gg m_{pl}$

(Large field)

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Types of potential:

- Polinomial $V(\phi) \propto \phi^p$
- Power law $V(\phi) \propto 1 + \cos(\phi/f)$
- Intermediary inflation $V(\phi) \propto \phi^{-\beta}$
- Natural inflation $V(\phi) \propto \exp(\phi/m_{pl})$

Case study

CHAOTIC INFLATION

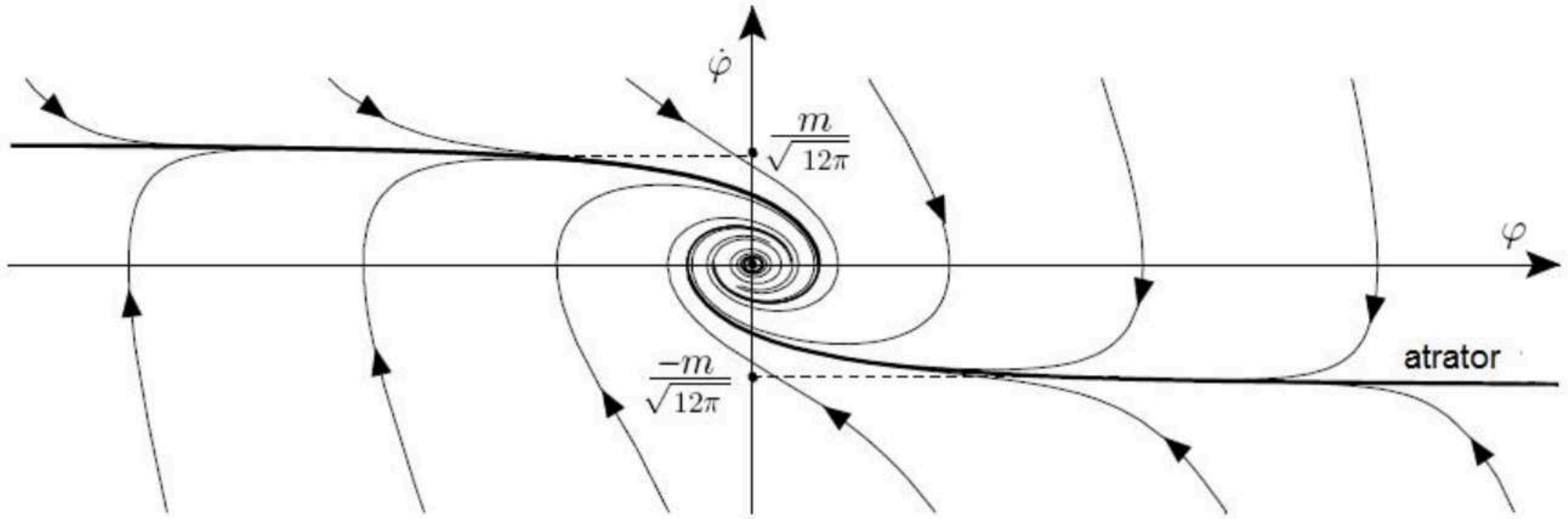
Simplest case: massive scalar field

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

EoM:

$$\ddot{\phi} + \sqrt{12\,\pi}(\dot{\phi} + m^2\phi^2)^{1/2}\dot{\phi} + m^2\phi = 0$$

attractor solution when $\dot{\phi} \ll m^2\phi^2$ (slow-roll)



Slow-roll is stablished ∀ initial conditions

Any initial condition converges eventually to a slow-roll solution

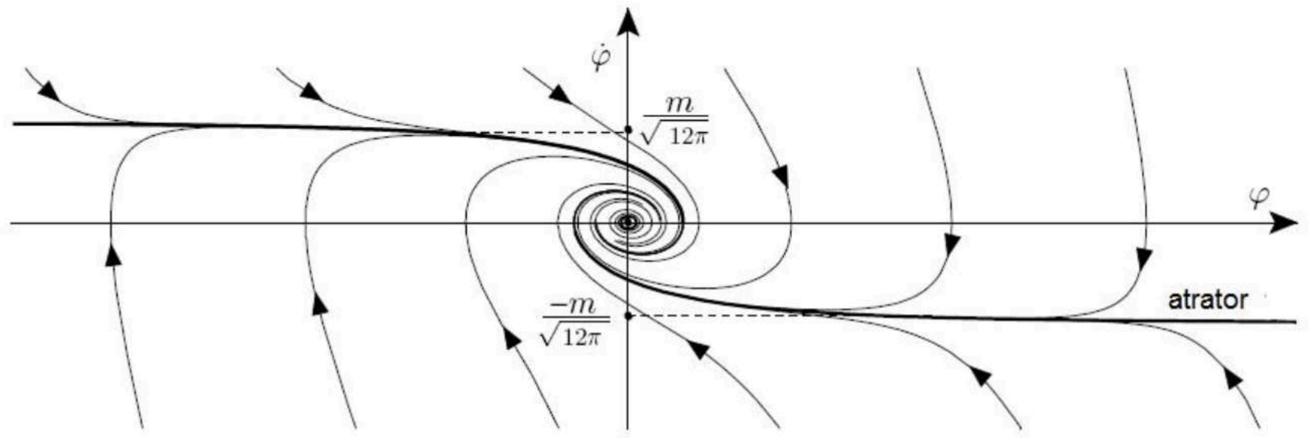
Case study

CHAOTIC INFLATION

Simplest case: massive scalar field

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$

EoM: (slow-roll)



$$3H\dot{\phi} + m^2\phi = 0$$

$$H^2 = \frac{1}{6}\frac{m^2}{m_{pl}^2}\phi^2$$

$$\Rightarrow \qquad \epsilon = \eta = \frac{2m_{pl}^2}{\phi^2} \xrightarrow{|\phi| \gg m_{pl}} \ll 1$$

solving these equations: $$N = \frac{1}{4}\frac{\phi_i^2}{m_{pl}^2} - \frac{1}{2}$$

Therefore, if $N \gtrsim 70\,\mathrm{e-folds} \;\; \Rightarrow \;\; \phi_i \gtrsim 17 m_{pl}$

Models of inflation

Inflation: a mechanism, not a unique theory!

SLOW-ROLL INFLATION

Large field inflation: $\Delta\phi \gg m_{pl}$

In large-field models the field moves over a large (super-Planckian) distances.
Predicts: amplitude of the gravitational waves produced during inflation is large (**Lecture 5**).

Small field inflation: $\Delta\phi \ll m_{pl}$

In small-field models the field moves over a small (sub-Planckian) distance.
Small-field models predicts: amplitude of the gravitational waves produced during inflation is too small to be detected (**Lecture 5**).

The potentials that give rise to such small-field evolution often arise in mechanisms of *spontaneous symmetry breaking*, e.g., Higgs-like potential

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{\mu}\right)^2\right]^2 (+\dots)$$

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BEYOND SLOW-ROLL INFLATION

- Non-minimal coupling to gravity
- Modified gravity
- Non-canonical kinetic term
- Multifield inflation

Inflation - solving the SCM problems

Amount of inflation:

The largest scales observed in the CMB are produced about 60e-folds before the end of inflation

$$N_{cmb} = \int_{\phi_{cmb}}^{\phi_f} \frac{1}{\sqrt{2\epsilon_V}} \frac{|d\phi|}{M_{pl}} d\phi \approx 60$$

HORIZON PROBLEM

To solve the horizon problem we want that our observable universe was inside the Hubble radius at the begining of inflation

$$H_0^{-1} \lesssim \frac{a_0}{a_i} H_I^{-1} = \frac{a_0}{a_f} \frac{a_f}{a_i} H_I^{-1} = \frac{a_0}{a_f} e^N H_I^{-1}$$

where

$$N \gtrsim \ln\left(\frac{a_f H_I}{a_0 H_0}\right) = \ln\left(\frac{H_I}{T_f H_0}\right)$$

Assuming inflation coincides with the end of the GUT era

$$T_{\rm GUT} \sim 10^{29}\, K$$

$$H_I \sim T_{\rm GUT}^2/m_{pl} \sim 10^{56}\,{\rm km/s/Mpc}$$

$$\Rightarrow \quad N \gtrsim 60\,{\rm e-folds}$$

Inflation - slow roll approximation

Amount of inflation:

slow-roll

$$N_{tot} \equiv \int_{t_i}^{t_f} H\, dt = \int_{a_i}^{a_f} d\ln a \simeq \int_{\phi_i}^{\phi_f} \frac{1}{\sqrt{2\epsilon_V}} \frac{|d\phi|}{M_{pl}}\, d\phi$$

$\epsilon_V < 1$ for $t \in [t_i, t_f]$

The largest scales observed in the CMB are produced about 60e-folds before the end of inflation

$$N_{cmb} = \int_{\phi_{cmb}}^{\phi_f} \frac{1}{\sqrt{2\epsilon_V}} \frac{|d\phi|}{M_{pl}}\, d\phi \approx 60$$

Successful solution to the horizon problem requires

$$N_{tot} > N_{cmb}$$

Inflation - solving the SCM problems

Amount of inflation:

The largest scales observed in the CMB are produced about 60e-folds before the end of inflation

$$N_{cmb} = \int_{\phi_{cmb}}^{\phi_f} \frac{1}{\sqrt{2\epsilon_V}} \frac{|d\phi|}{M_{pl}} d\phi \approx 60$$

FLATNESS PROBLEM

$$|\Omega - 1|_{t=t_f} \approx \left(\frac{a_i}{a_f}\right)^2 |\Omega - 1|_{t=t_i} = e^{-2N} |\Omega - 1|_{t=t_i}$$

Even if it is far from flat initially, inflation inflates and makes $\Omega_0 \sim 1$

Amplification of the curvature radius

$$R = \frac{H^{-1}}{|\Omega - 1|^{1/2}} = \left(\frac{a^2}{k}\right)^{1/2}$$

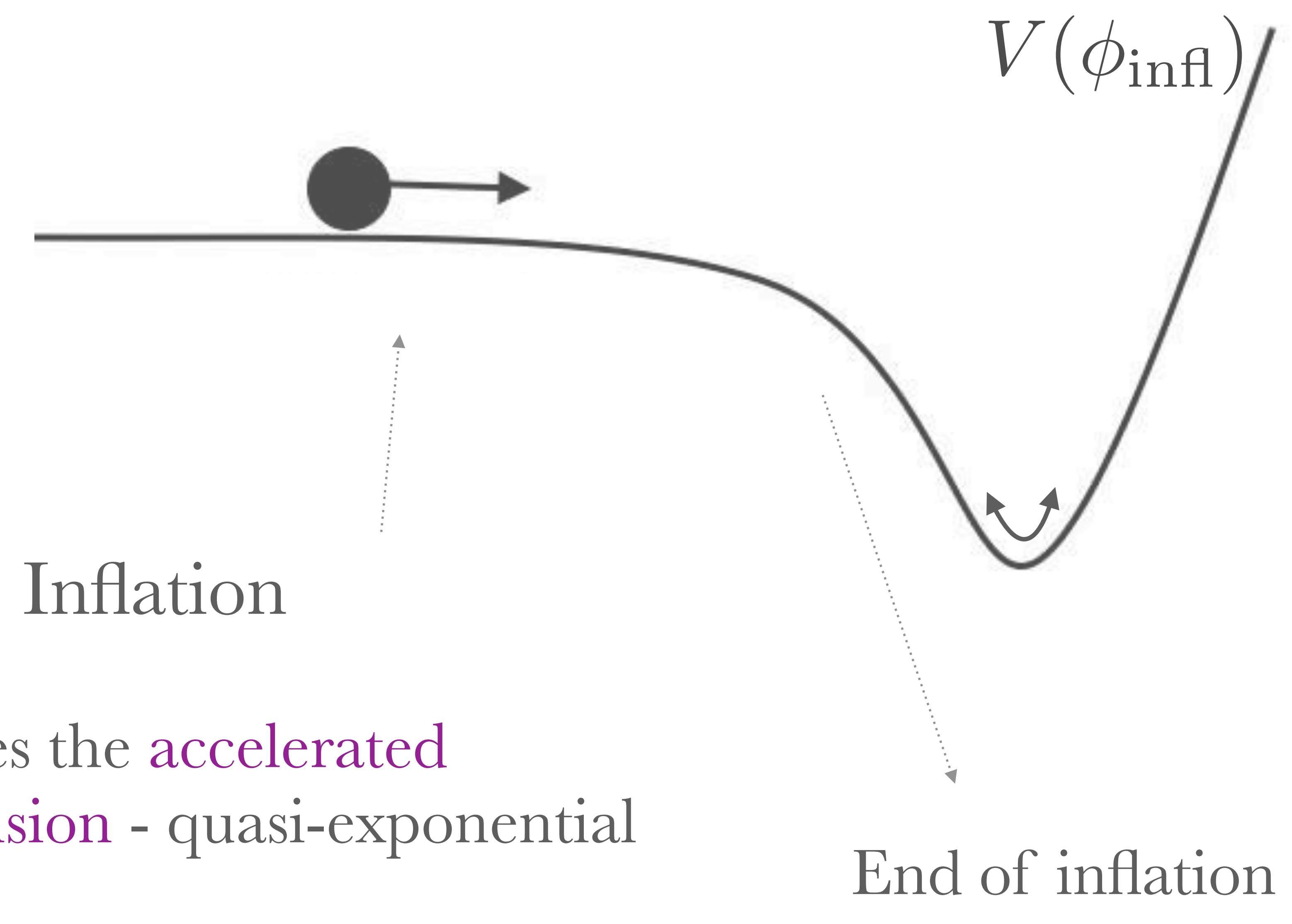
Any global geometry will look flat locally!

MAGNETIC MONOPOLES PROBLEM

Inflation dilutes the density of these monopoles to the observed quantity today

$$n(t_f) \sim \mathrm{e}^{-3N} 10^{82} \mathrm{m}^{-3} = \mathrm{e}^{-300} 10^{82} \mathrm{m}^{-3} \sim 5 \times 10^{-49} \mathrm{m}^{-3}\ 15\mathrm{pc}^{-3}$$

End of inflation



Causes the accelerated expansion - quasi-exponential

$$\epsilon \ll 1$$

$$|\delta| \ll 1$$

$$\epsilon \sim 1$$

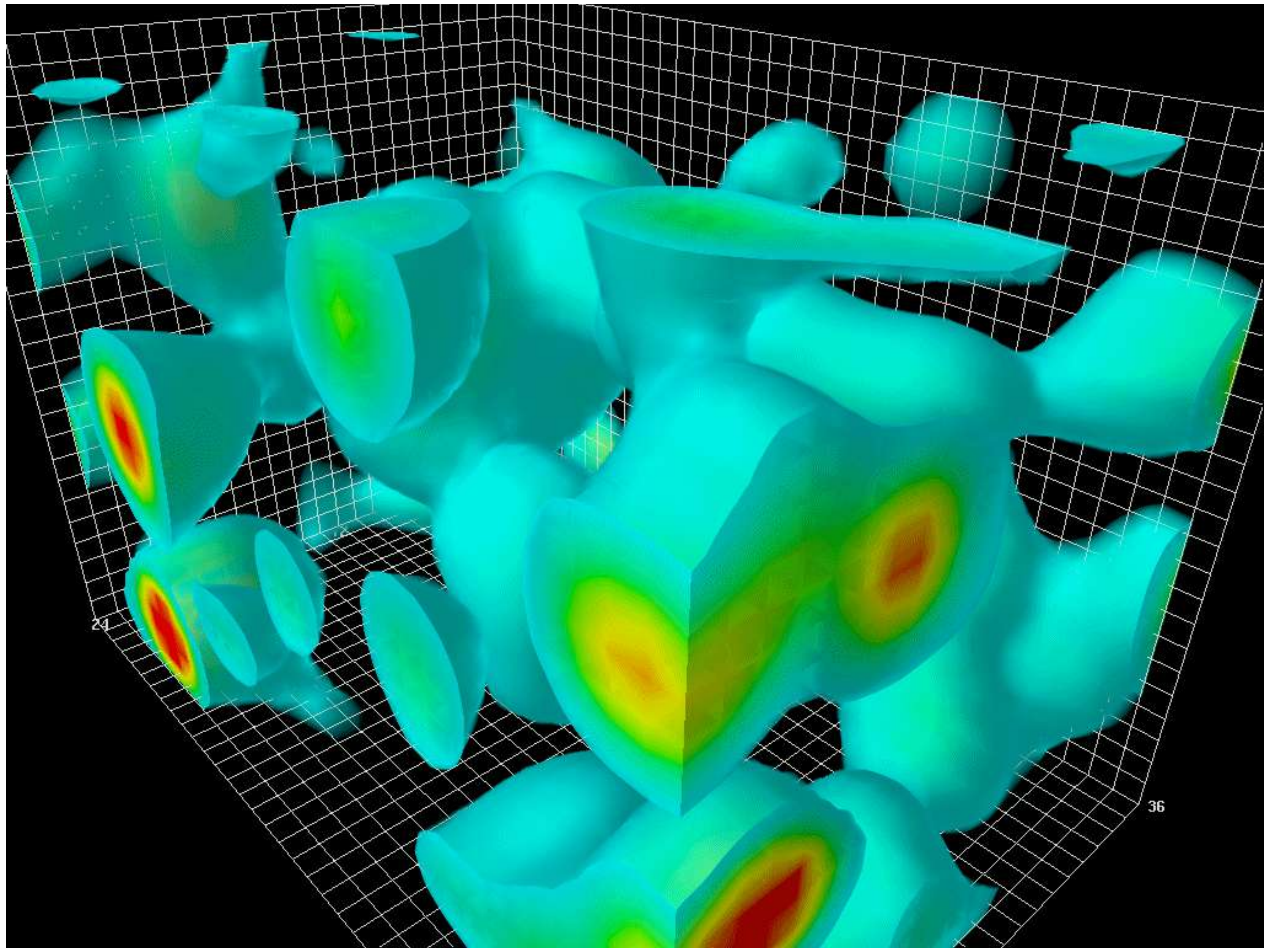
Potential slow-roll parameters

$$\epsilon_{\rm v} \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \quad , \quad |\eta_{\rm v}| \equiv M_{\rm pl}^2 \frac{|V''|}{V}$$

The inflationary mechanism has to have a period of quasi-accelerated expansion, BUT this has to end, to enter the radiation domination period → ***graceful exit***

Inflation - Origin of structures

In quantum mechanics the vacuum or empty space if full of fluctuations



Uncertainty principle

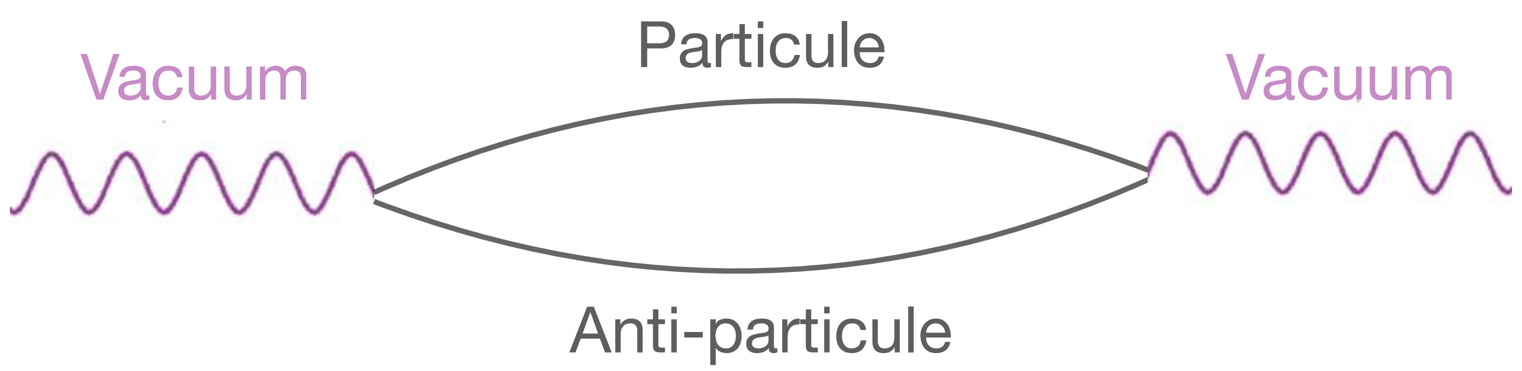
$$\Delta x \Delta p \geq \hbar$$

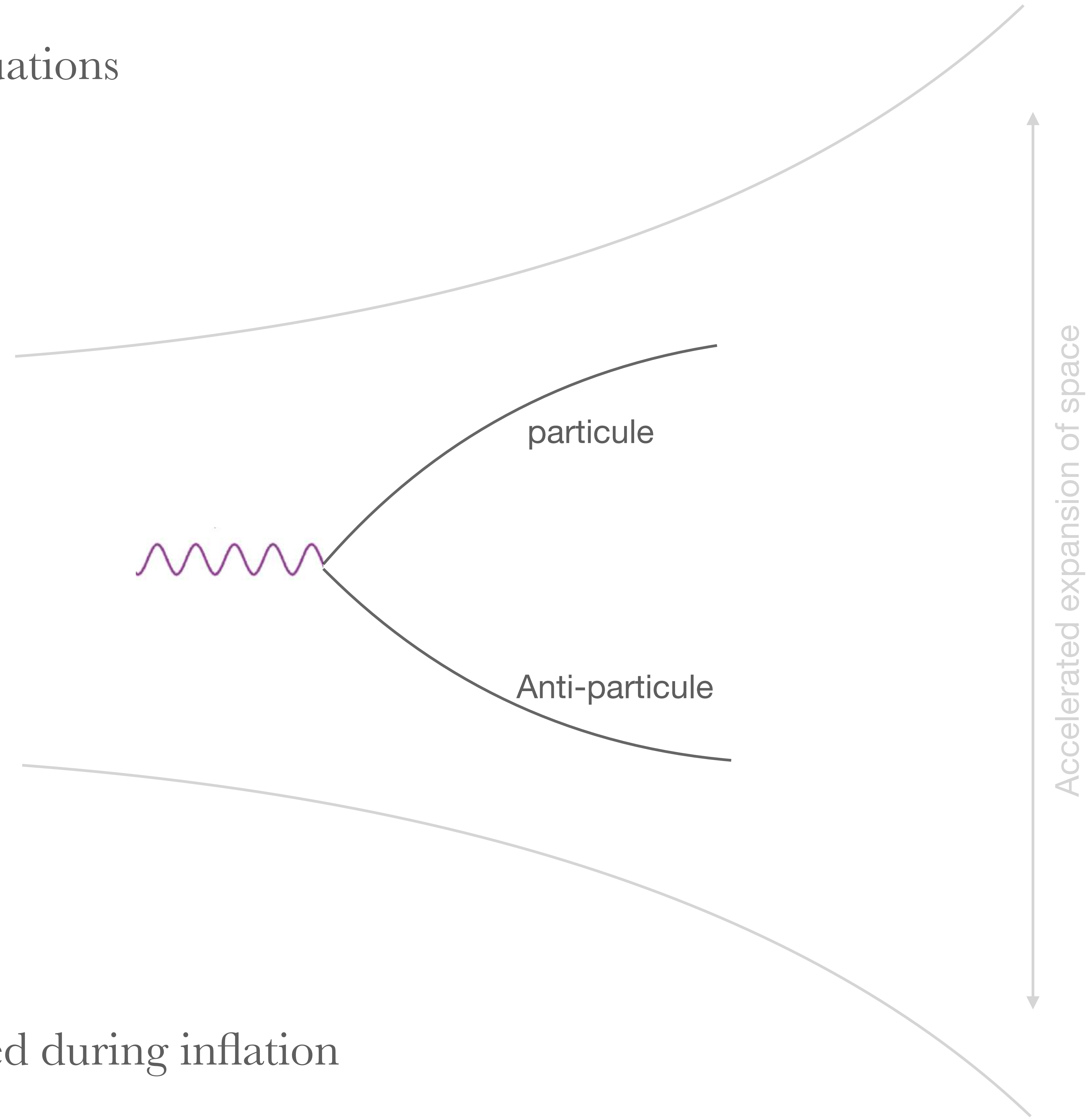
These fluctuations are real and present in our universe. But their effects are very small.

Inflation - Origin of structures

In quantum mechanics the vacuum or empty space if full of fluctuations

Pictorically:

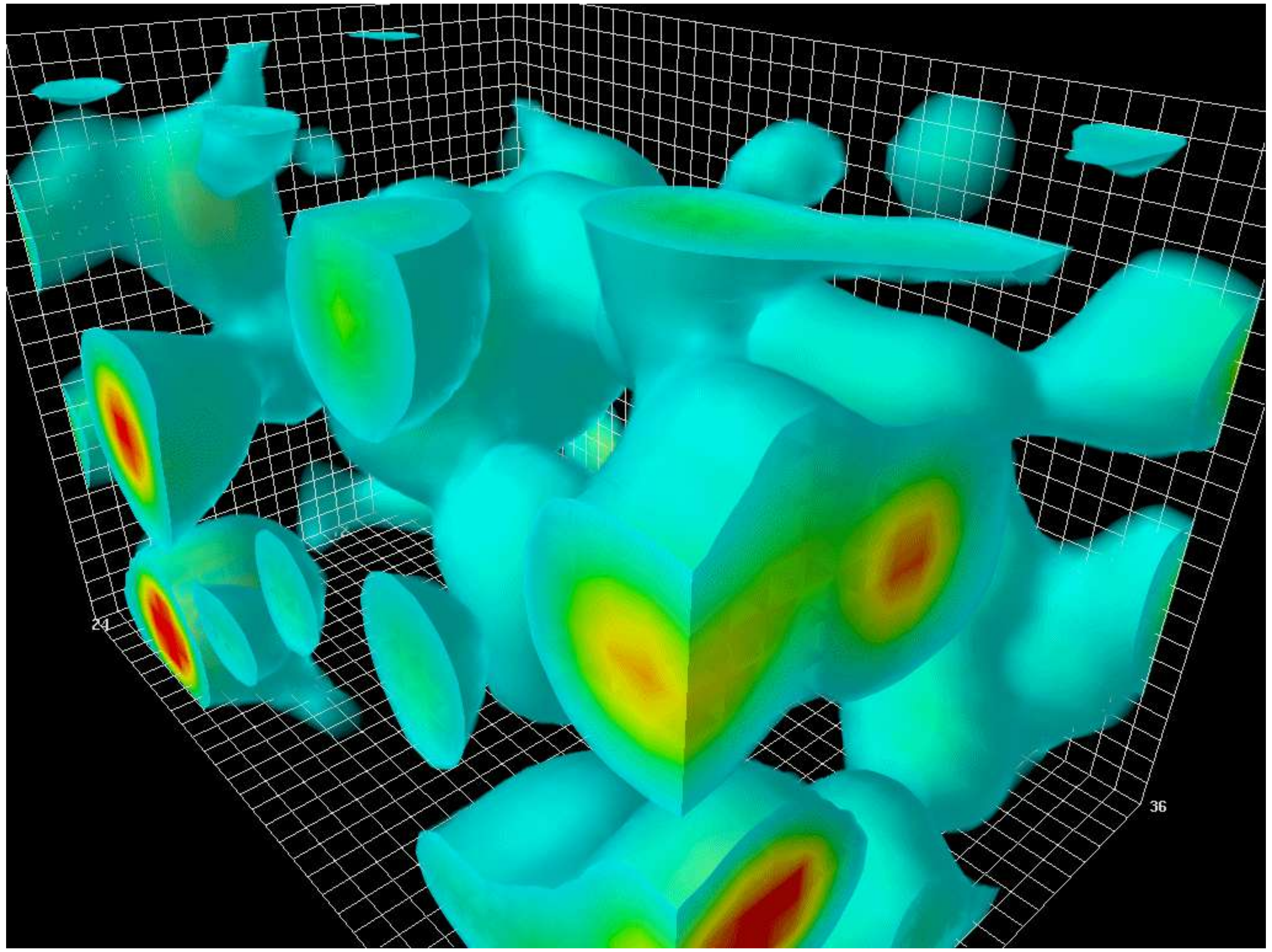




These particles describe the quantum perturbations that are generated during inflation

Inflation - Origin of structures

The initial perturbation have a quantum origin in the beginning of the universe, during inflation *

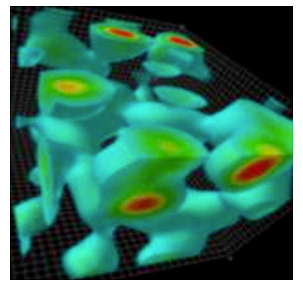


Possible since inflation puts all the regions in the universe in causal contact

Inflation - Origin of structures

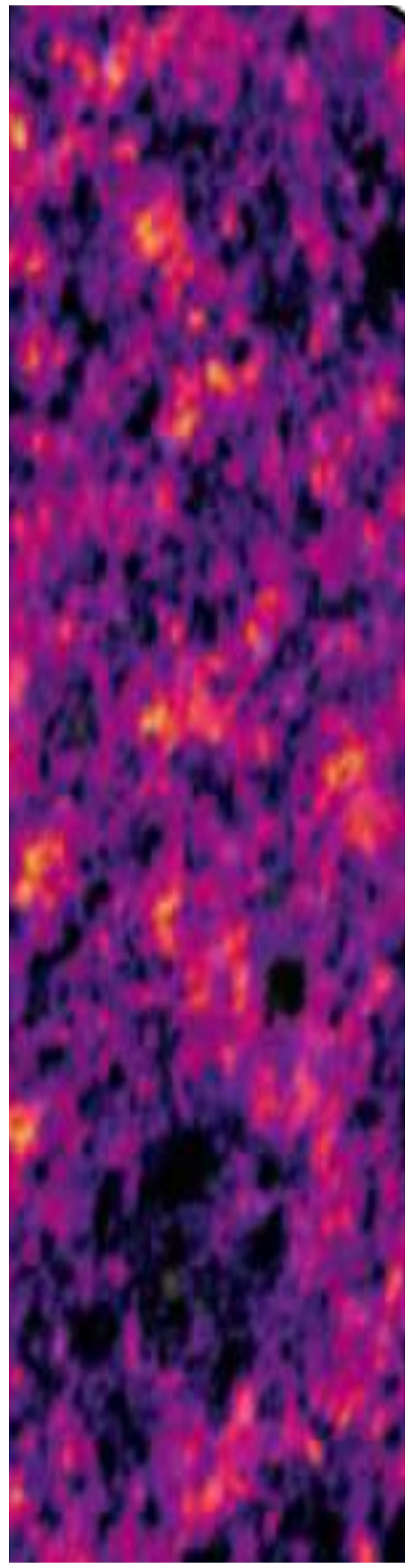
In $\sim 10^{-32}$ seconds, the universe grew exponentially in size

Perturbations are amplified
and stretched during *inflation*



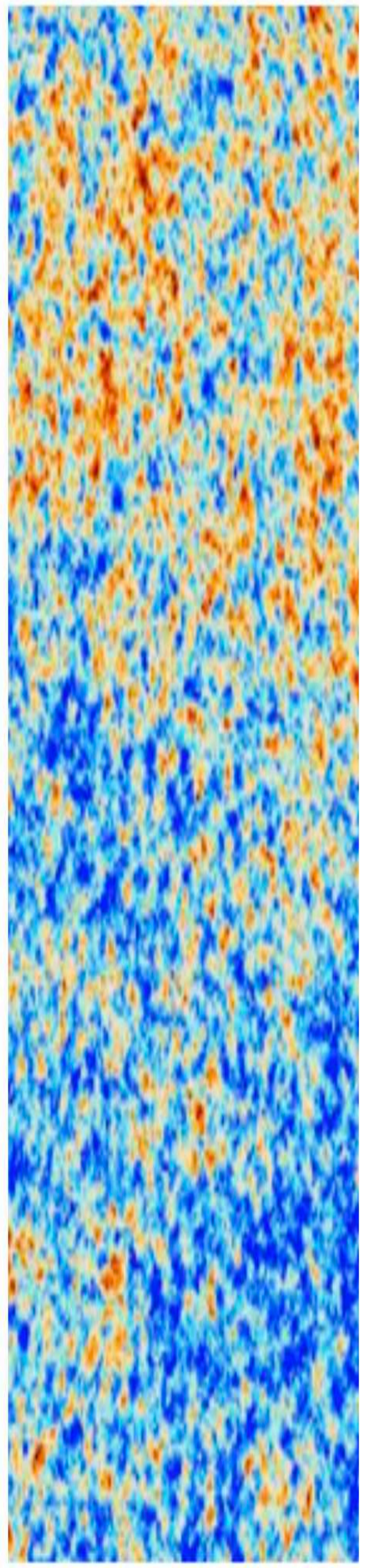
Microscopic initial
perturbations

$$a(t) \simeq e^{H\,t}$$



Classical and
macroscopic
perturbations

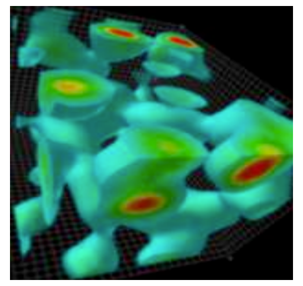
Radiation
era



CMB

Inflation - *Origin of structures*

In $\sim 10^{-32}$ seconds, the universe grew exponentially in size



Microscopic initial perturbations

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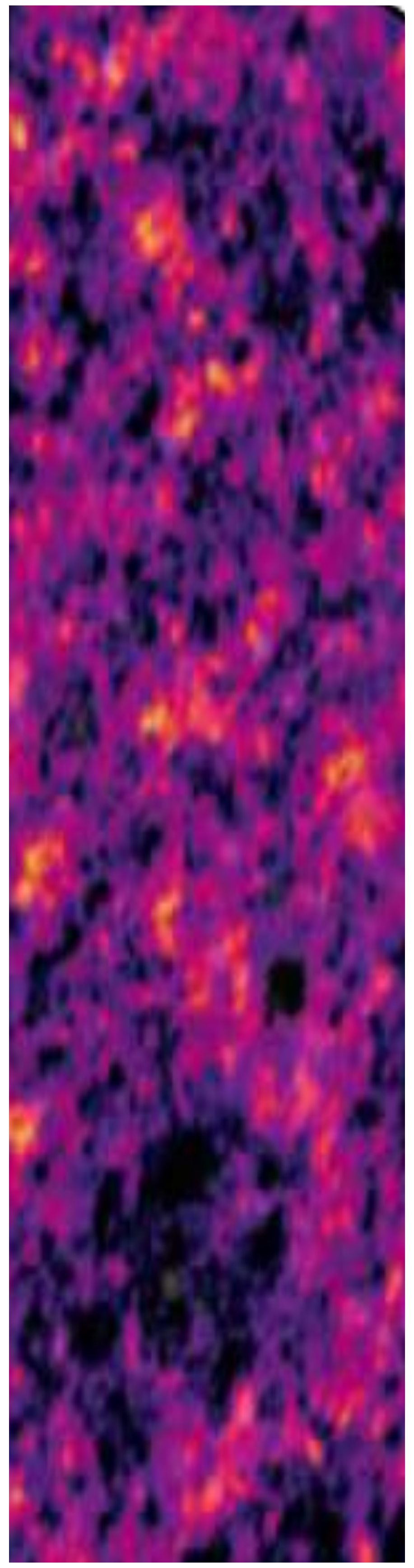
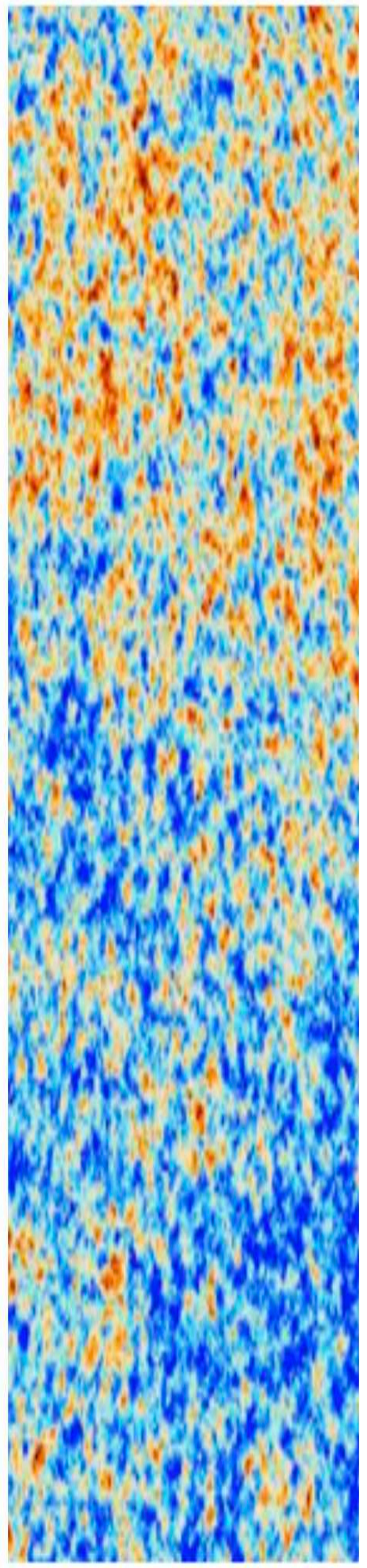


Classical and macroscopic perturbations

Radiation era

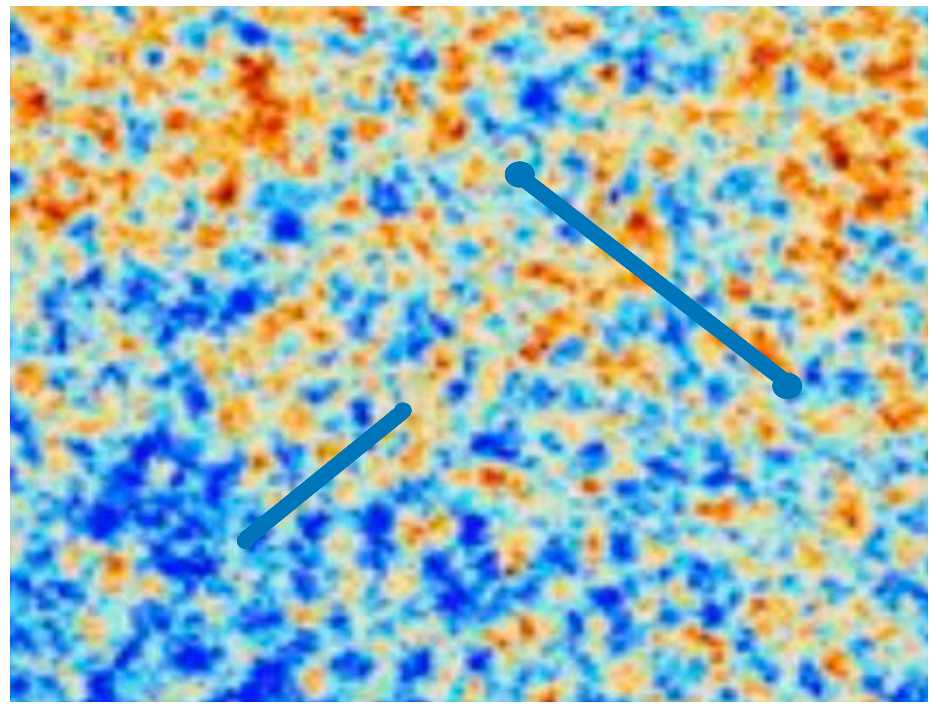


CMB

✓ Solves the formation of structure problem

Spectrum of the initial perturbations

The initial fluctuations created in inflation, because of the inflationary dynamics, lead to a almost scale invariant spectrum



$$P(k) = A_S \left(\frac{k}{k_*}\right)^{n_S - 1}$$

$n_s \longrightarrow$ Scale-dependency of the initial fluctuations
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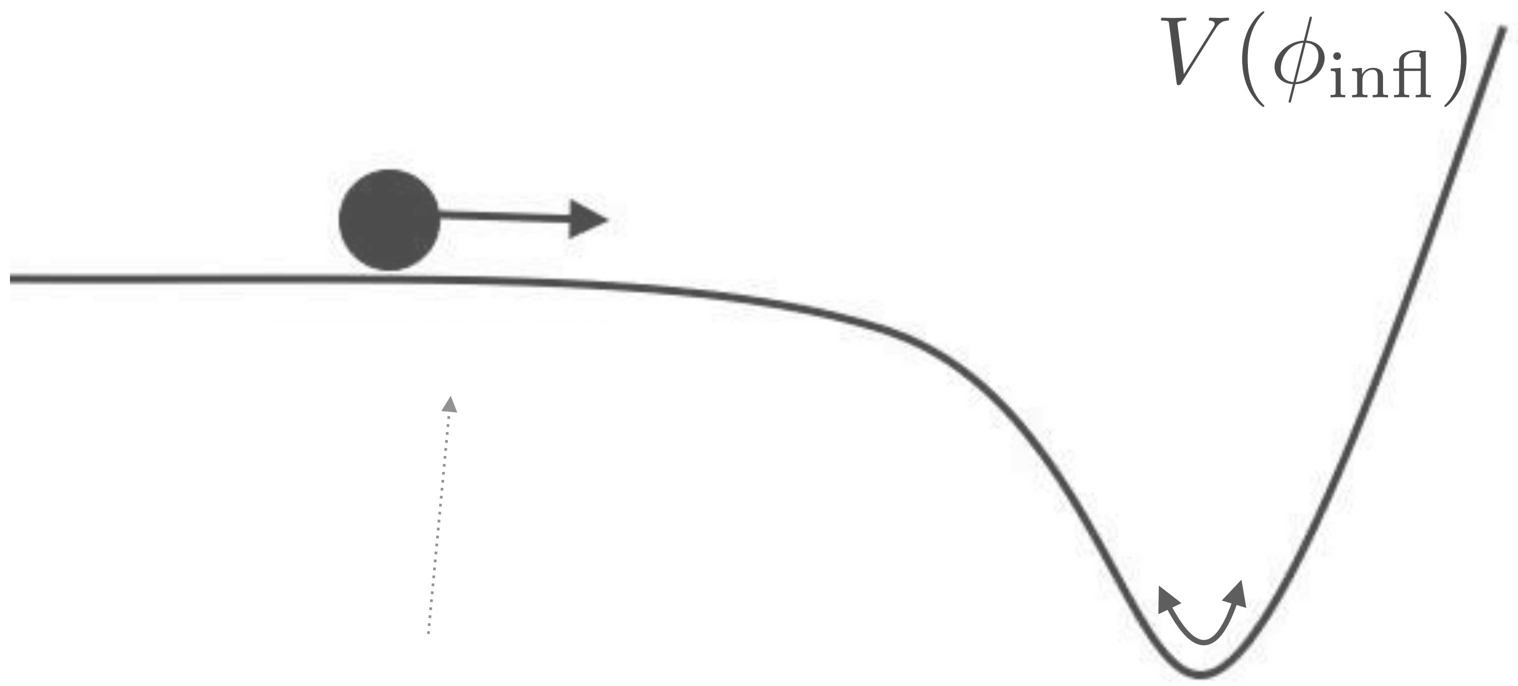
$10^9 A_s = 2.092 \pm 0.034$	$\longrightarrow$	Amplitude of the initial fluctuations
$\tau = 0.0522 \pm 0.0080$	$\longrightarrow$	Optical depth

After inflation - (p)reheating

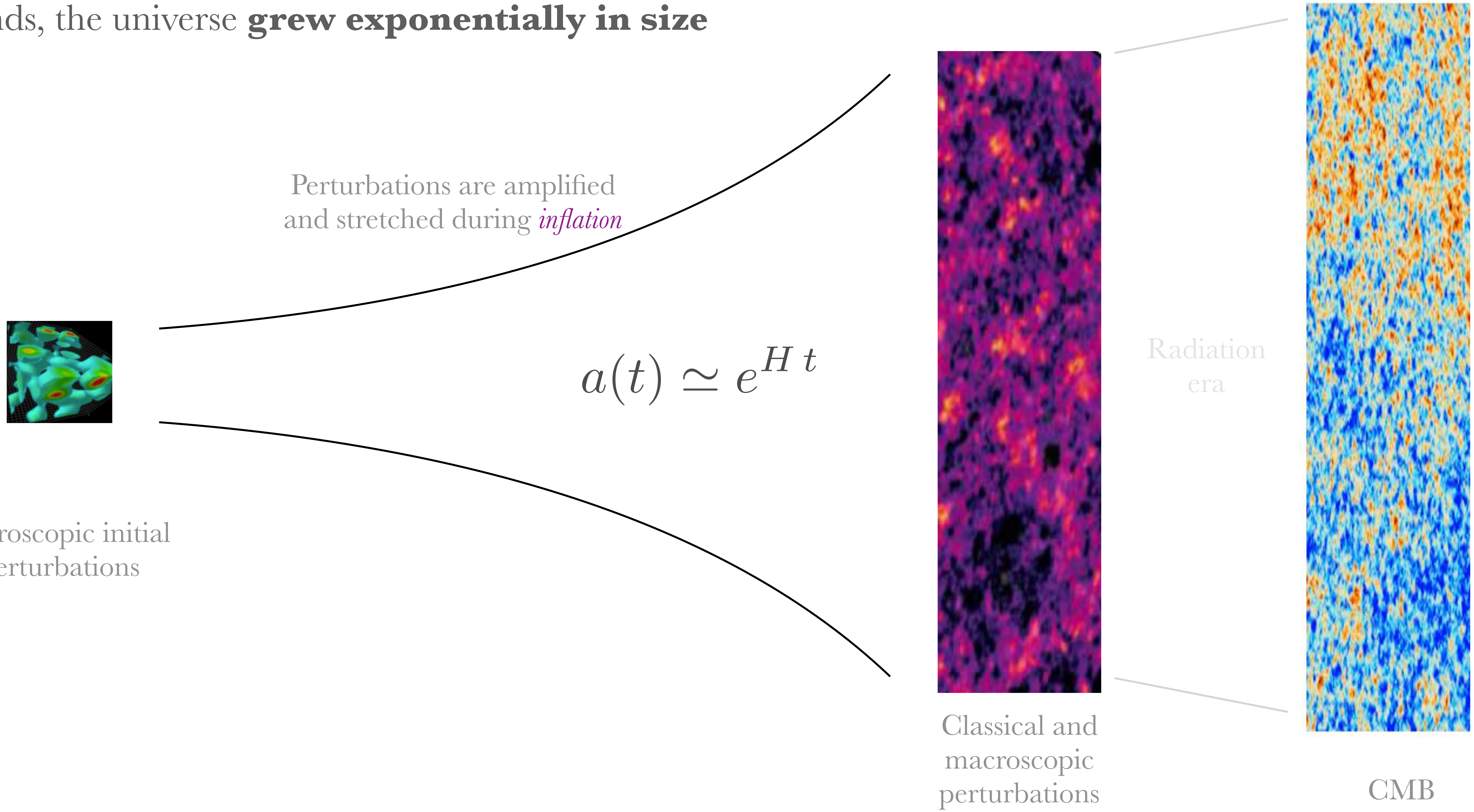


Causes the accelerated expansion - quasi-exponential

Leaves the universe empty!!

Inflation - inflates the universe

In $\sim 10^{-32}$ seconds, the universe **grew exponentially in size**



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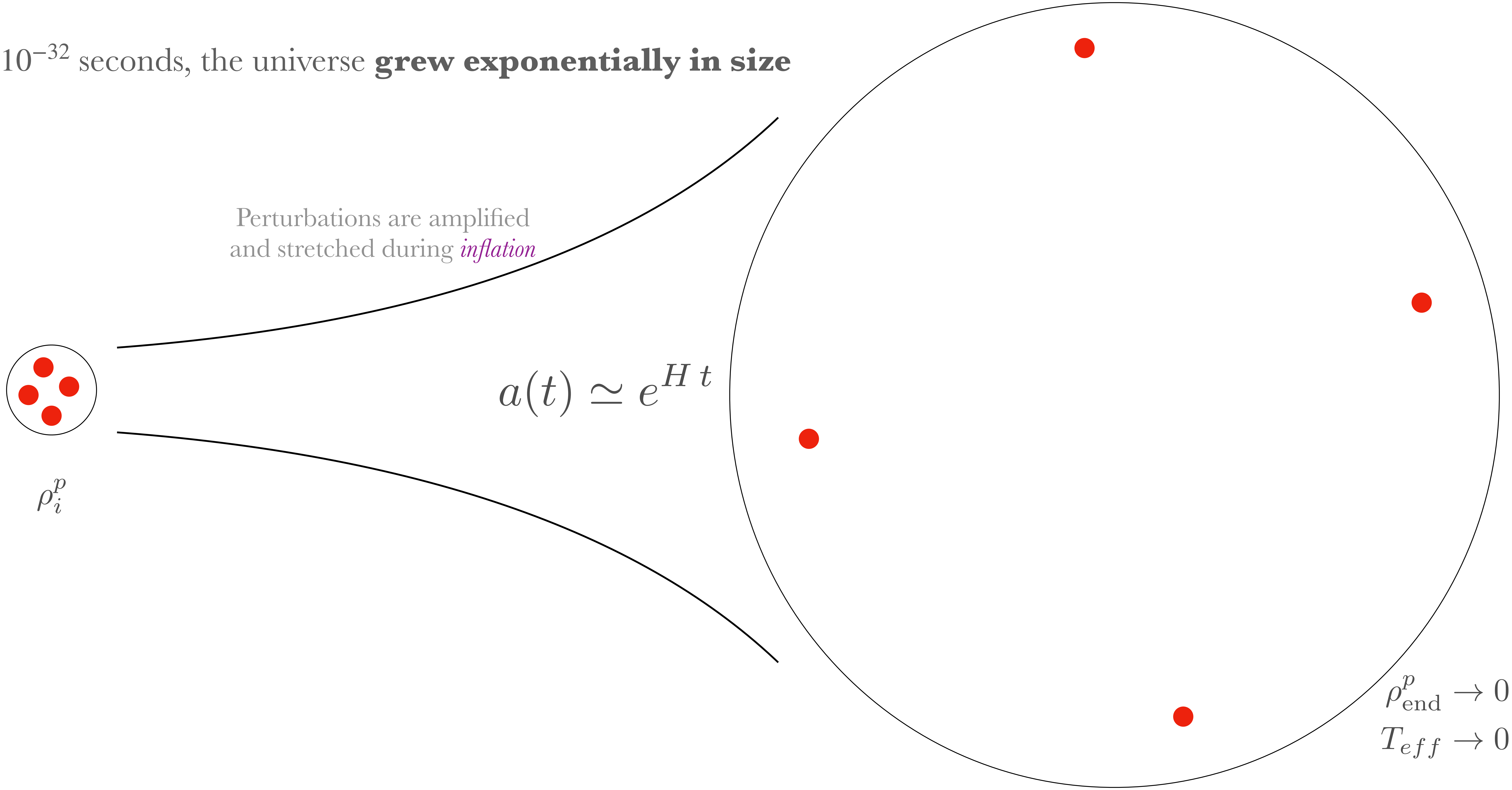

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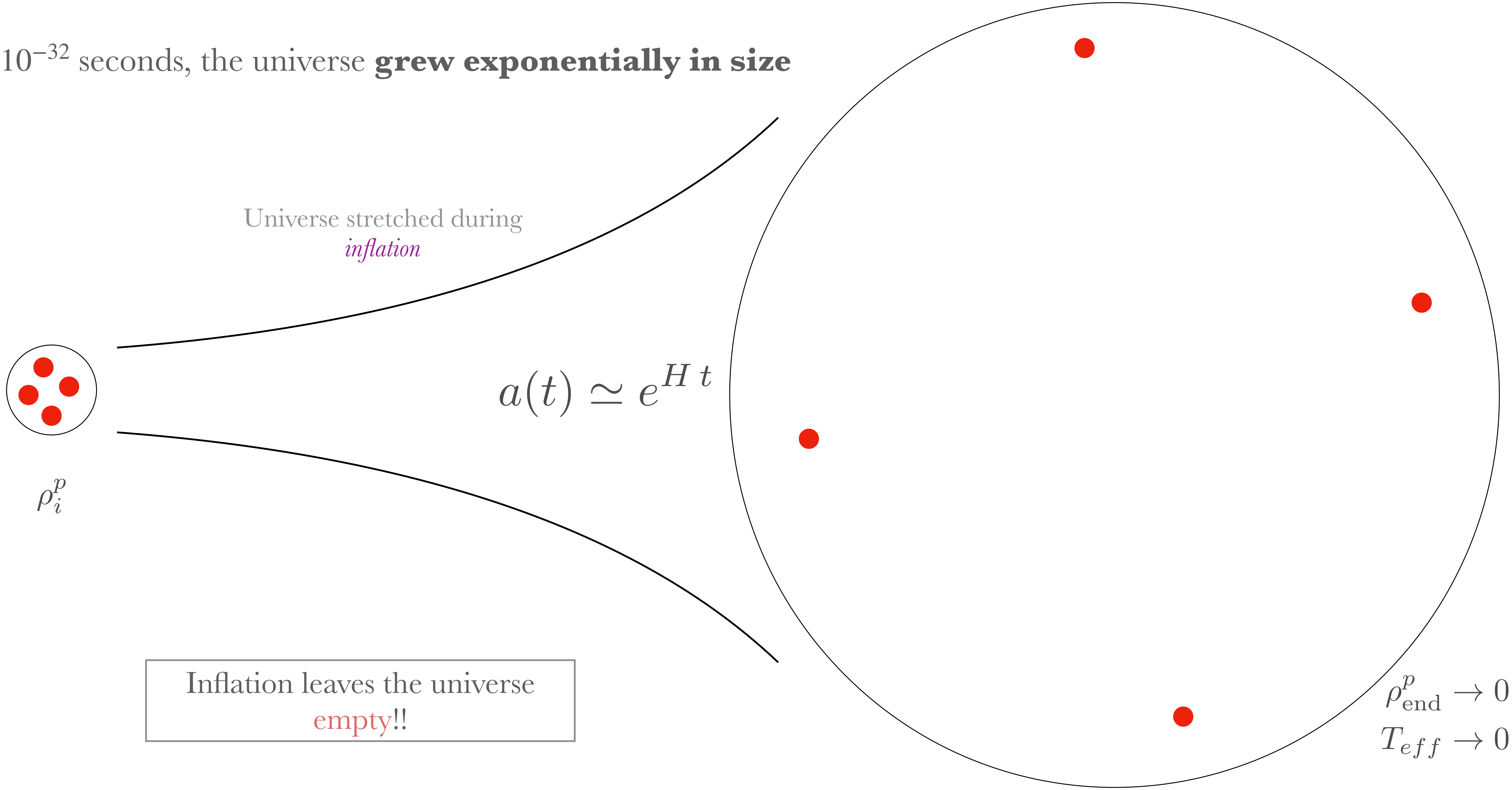

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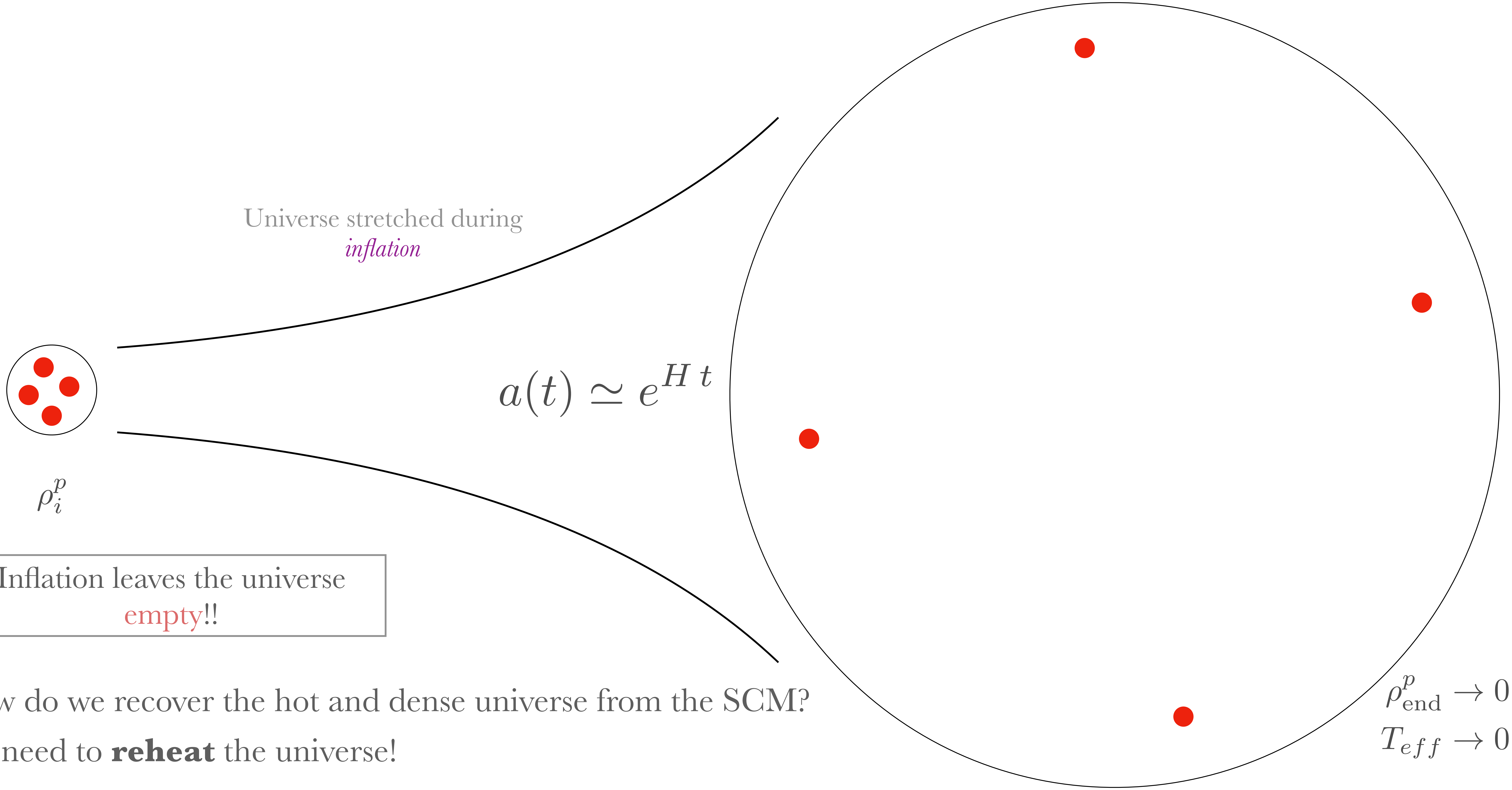
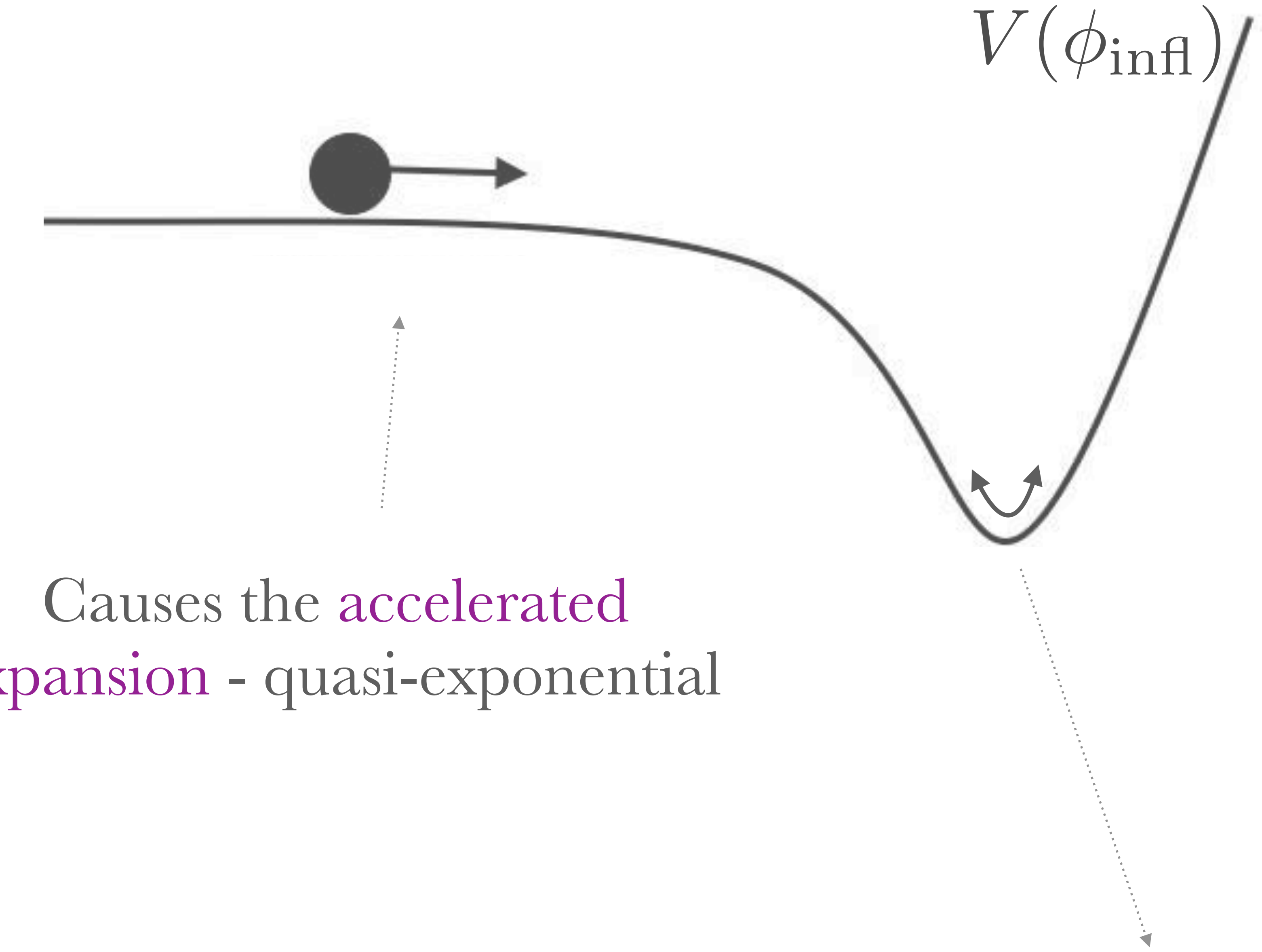


Inflation leaves the universe empty!!

How do we recover the hot and dense universe from the SCM?

We need to **reheat** the universe!

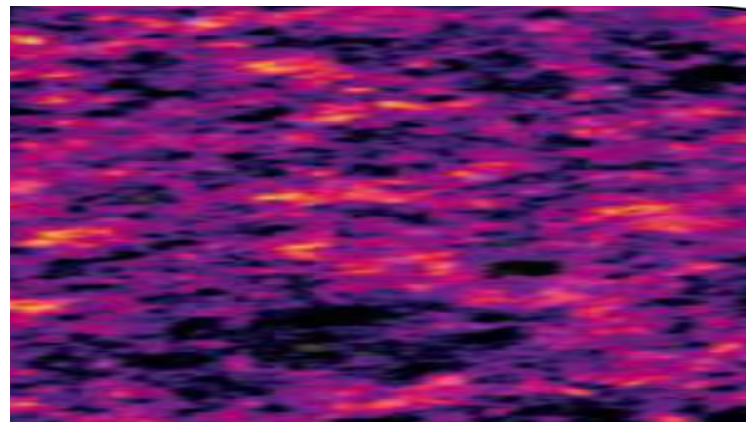
After inflation - (p)reheating

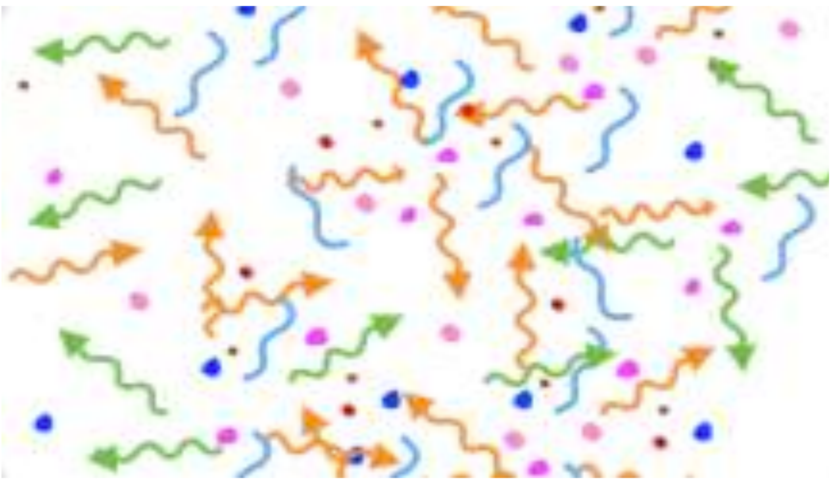


Causes the accelerated expansion - quasi-exponential

Leaves the universe empty!!

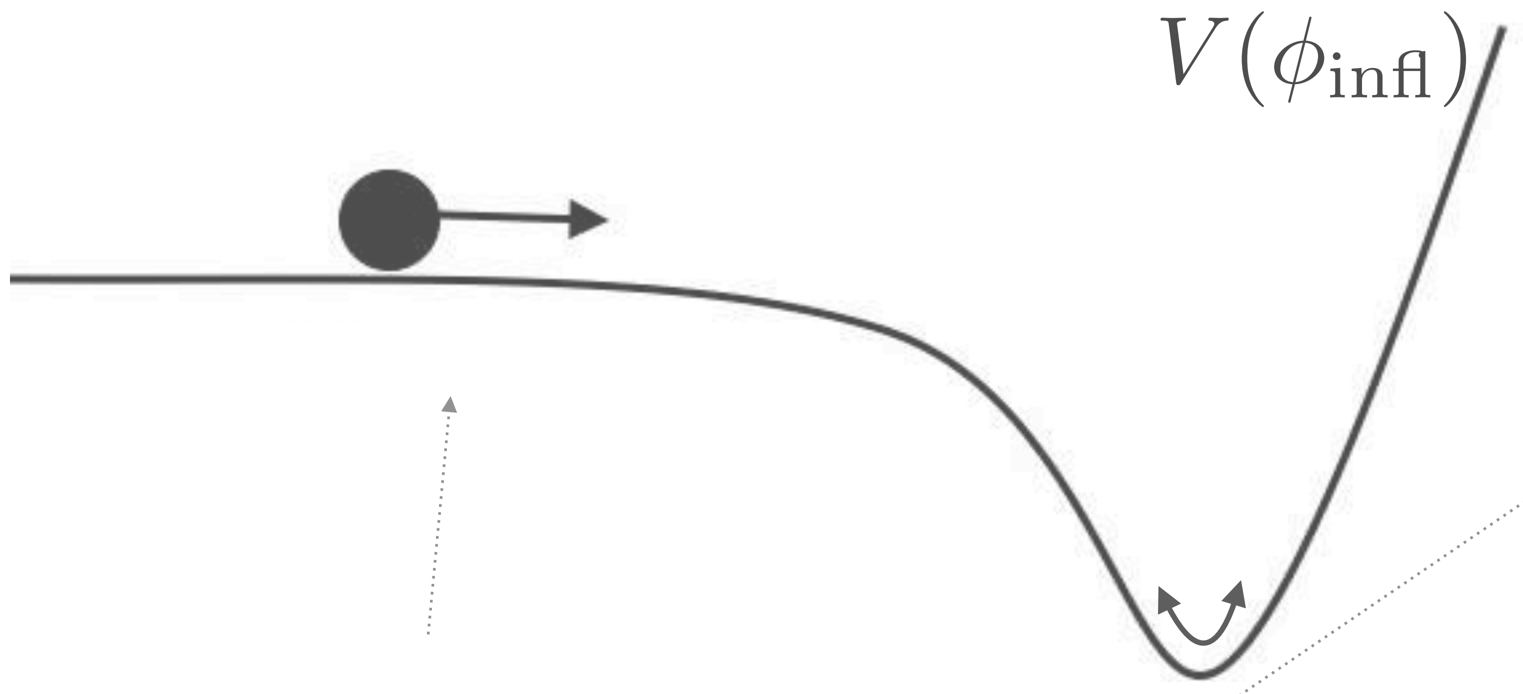
By the end of inflation, the *inflaton* decays, and the product of this decay are the particles of the standard model → starts the SCM





(P)reheating: populates the universe with particles. Creates all the elementary particles (or its precursors) that we have today.

After inflation - (p)reheating



Causes the accelerated expansion - quasi-exponential

PREHEATING

Non-perturbative

Initial stage of reheating

After inflation, the scalar field starts to oscillate on the bottom of the potential

(frequency m)

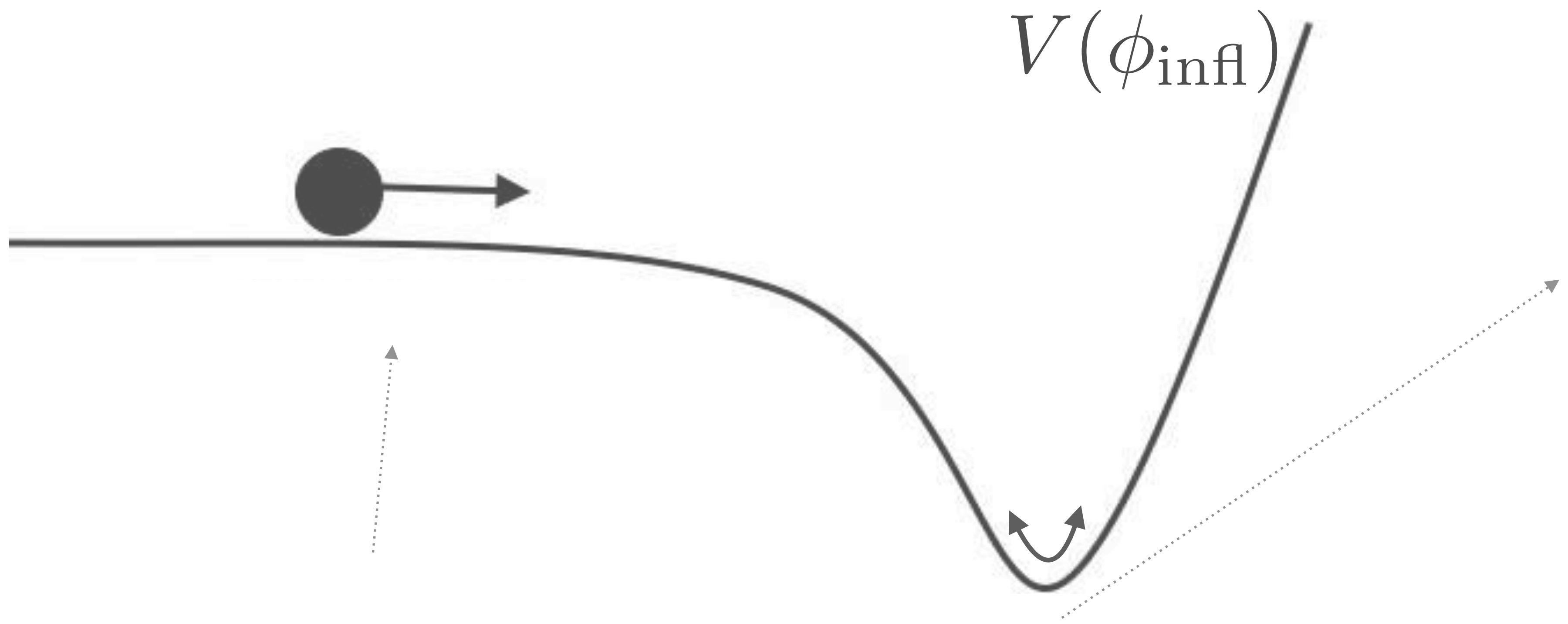
$$\ddot{\phi} + 3\cancel{H}\dot{\phi} = -m^2\phi$$

$$H^{-1} \gg m^{-1}$$

Efficient transfer of energy from the inflaton to scalar fields

Not fermions!
Pauli exclusion principle

After inflation - (p)reheating



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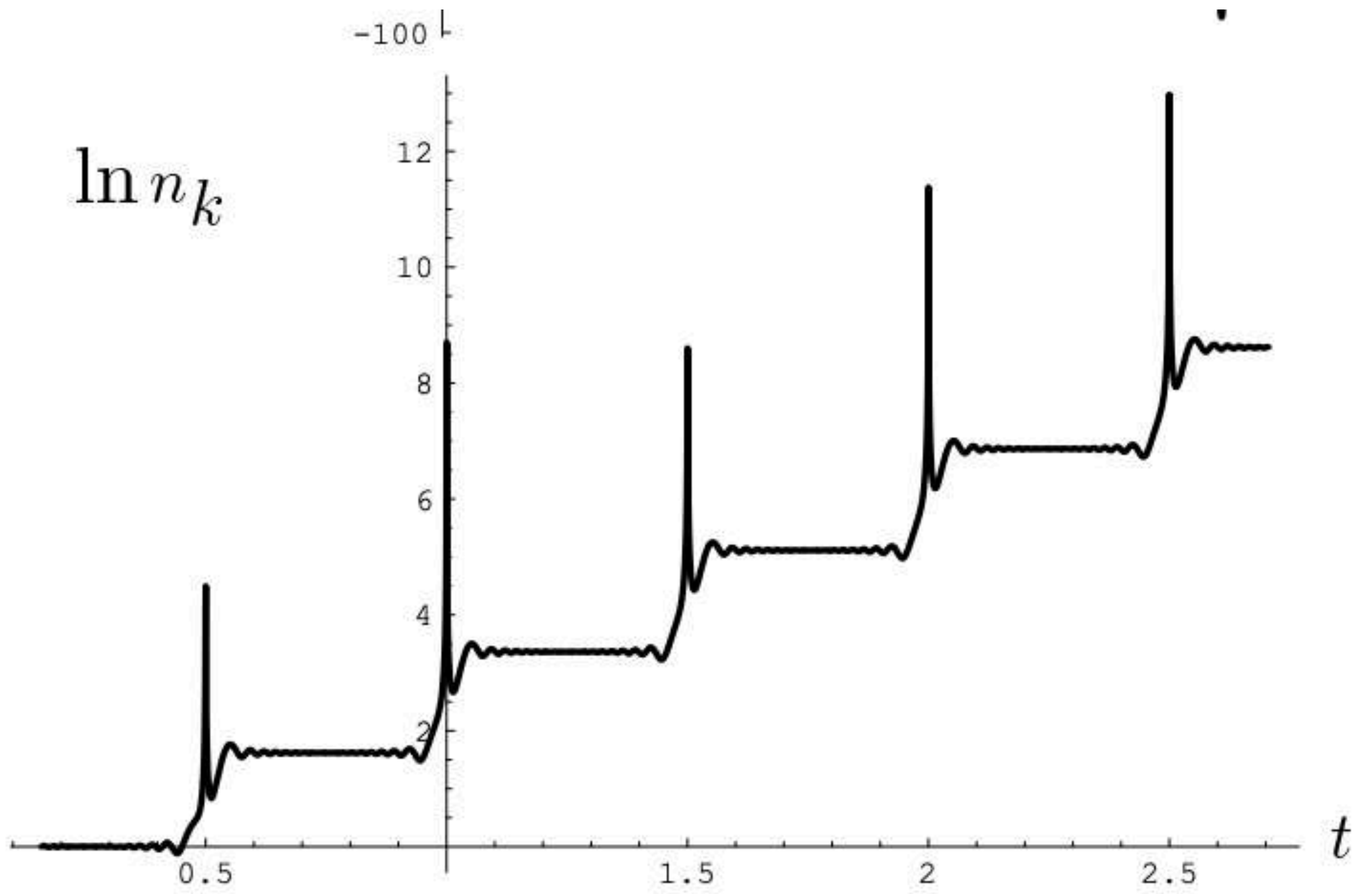


Copious production of particles

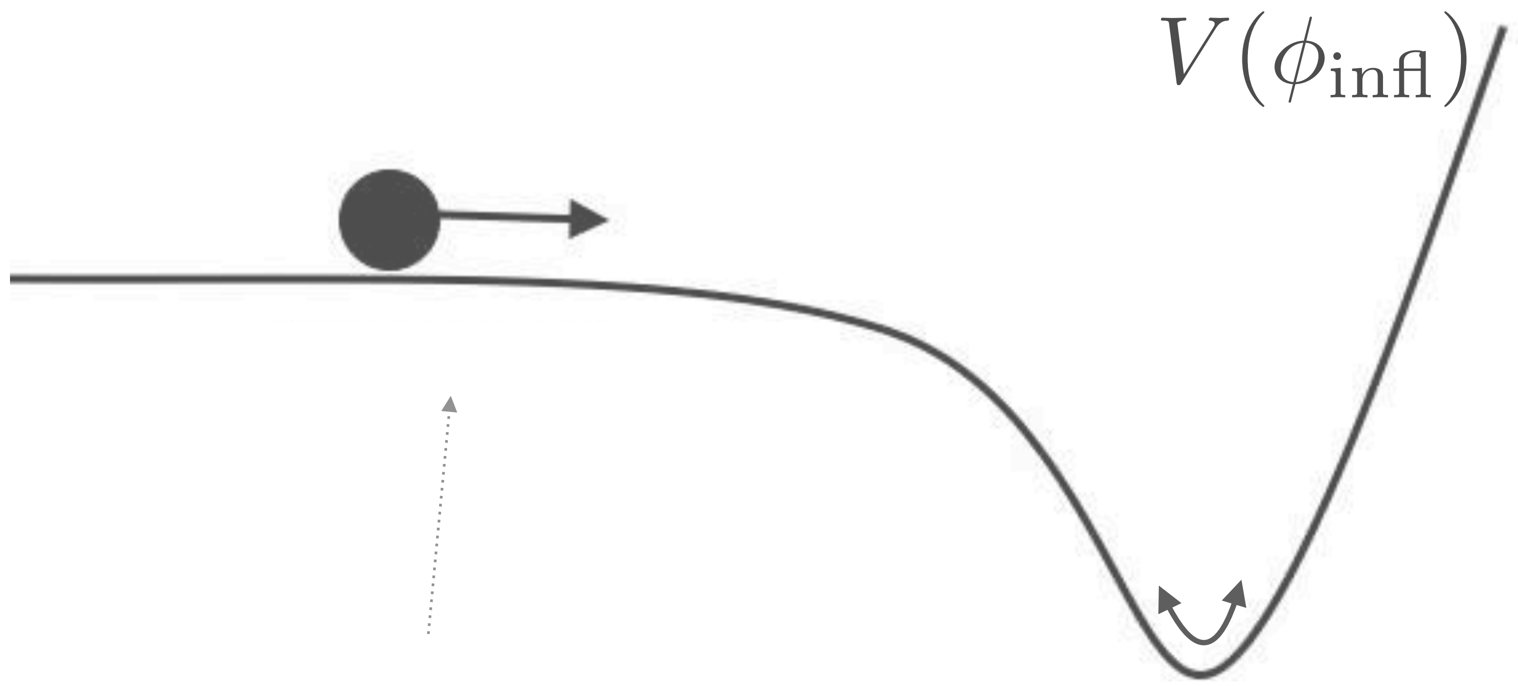
Oscillating inflation coupled to a quantum scalar field:

$$\ddot{\chi}_k + \left(k^2 + g^2\sigma^2 + 2g^2\sigma\Phi\,\sin mt\right)\chi_k = 0$$

$\phi(t)$

Parametric resonance!!

After inflation - (p)reheating



Causes the accelerated expansion - quasi-exponential

REHEATING

Perturbative

To avoid that the universe ends up empty, the inflaton has to couple to Standard Model field

parametrizes the inflaton decay rate

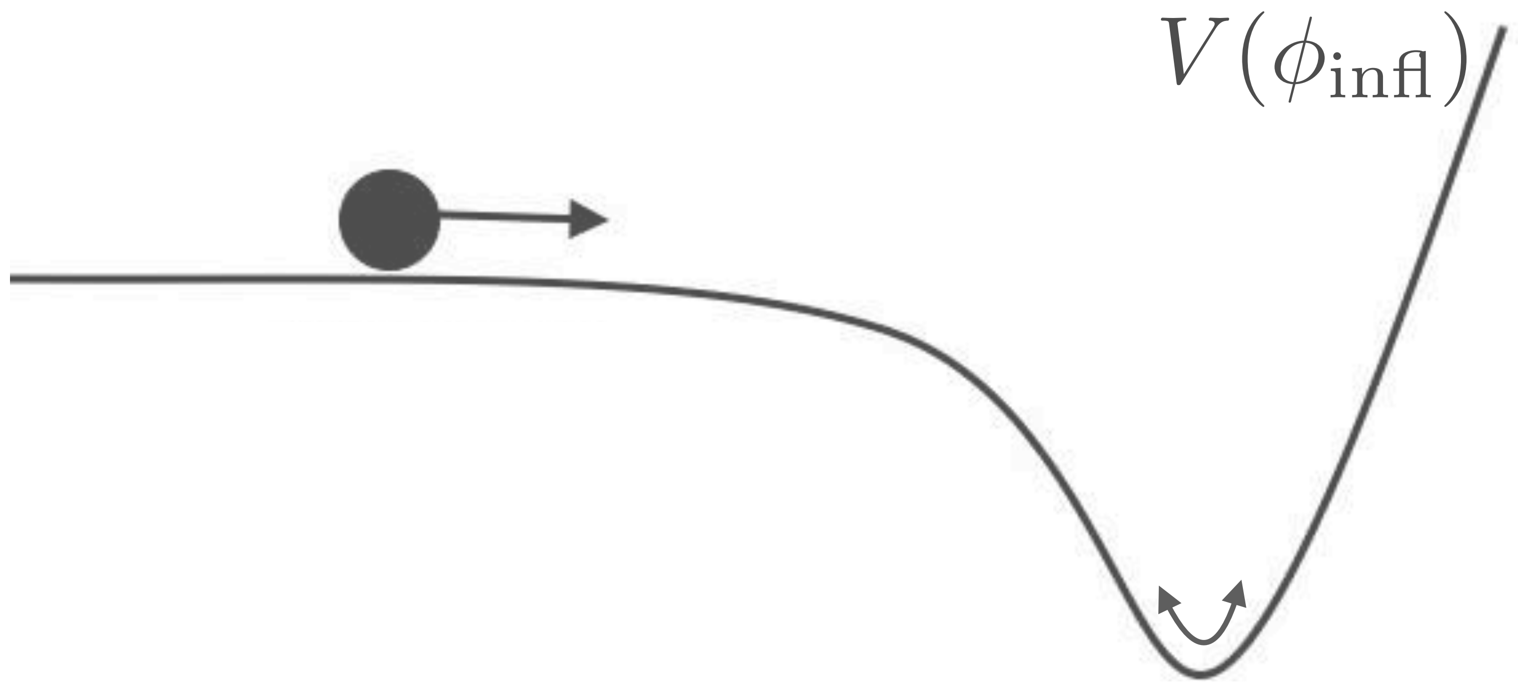
$$\dot{\rho}_\phi + 3H\rho_\phi = -\Gamma_\phi \rho_\phi$$

The energy stored in the inflaton field will then be transferred into ordinary particles

Slow-decay - like the case where the ase if the *inflaton* can only decay into fermions

Γ_ϕ??

After inflation - (p)reheating



REHEATING

Perturbative

To avoid that the universe ends up empty, the inflaton has to couple to Standard Model fields

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \varphi\right)^2 - V(\varphi) + \frac{1}{2}\sum_i \left[\left(\partial_\mu \chi_i\right)^2 - \underbrace{\left(m^2_{\chi_i}(0) + g_i^2\varphi^2\right)}_{m^2_{\chi_i}}\chi_i^2\right] - \sum_i g_i^2 \sigma \varphi \chi_i^2 \,.$$

Decaying rate:

$$\Gamma_{\varphi\to\chi\chi} = \frac{g_\chi^2\sigma^2}{8\pi m}\,,$$

$$\Gamma_{\varphi\to\psi\bar\psi} = \frac{g_\psi^2 m}{8\pi}\,. \qquad \text{(fermions)}$$

Particle number density: $n_\varphi = \frac{\rho}{m} \propto e^{-(3H+\Gamma_{total})t}$

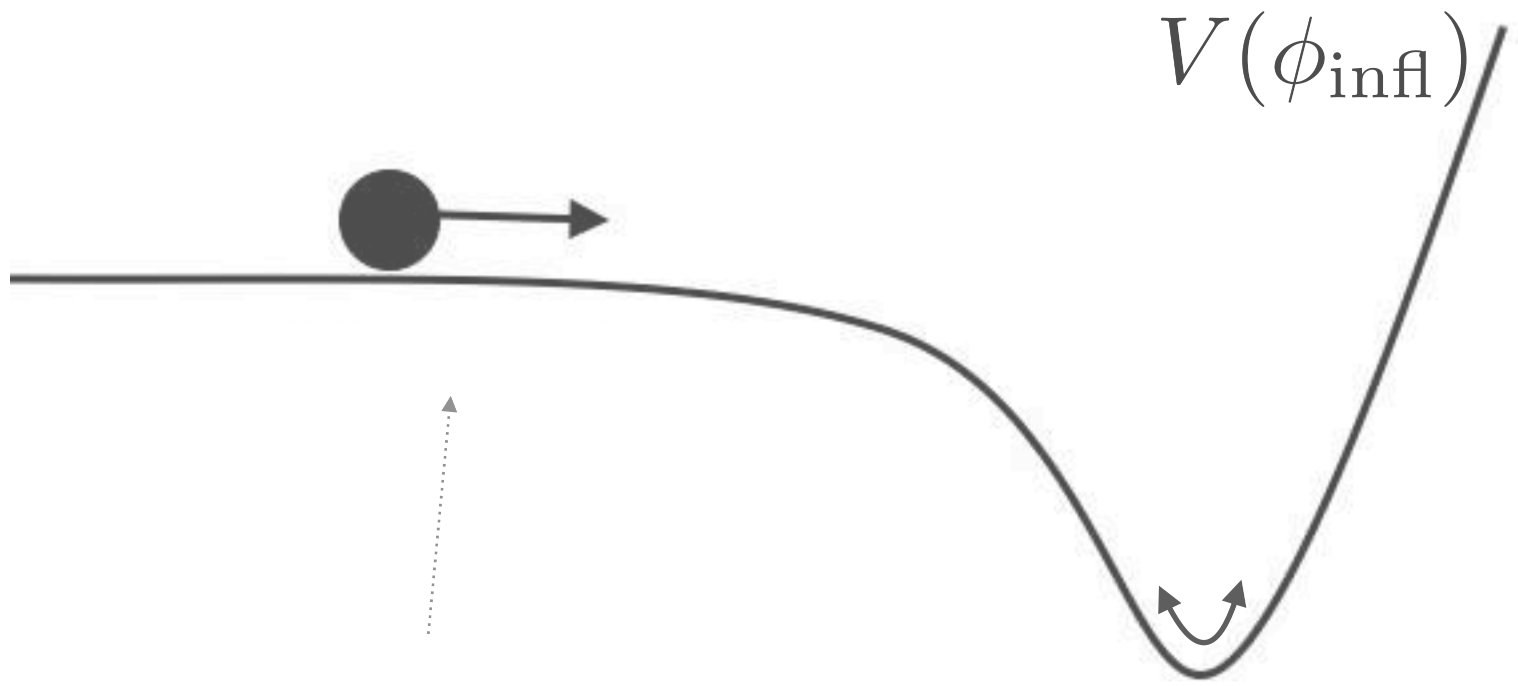
End of reheating: $\Gamma_{total} \sim H$

Assuming thermal equilibrium happens right after reheating

$$T_{reaq} \simeq 0,2\left(\frac{100}{g^*}\right)^{1/4}\sqrt{\Gamma_{total}m_{pl}}$$

Observations, CMB, put a limit - $T_{\text{reheat}} < 10^6$ GeV

After inflation - (p)reheating



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THERMALIZATION

Needs to lead into the SCM universe - in thermal equilibrium

HOW?

Problems with inflation

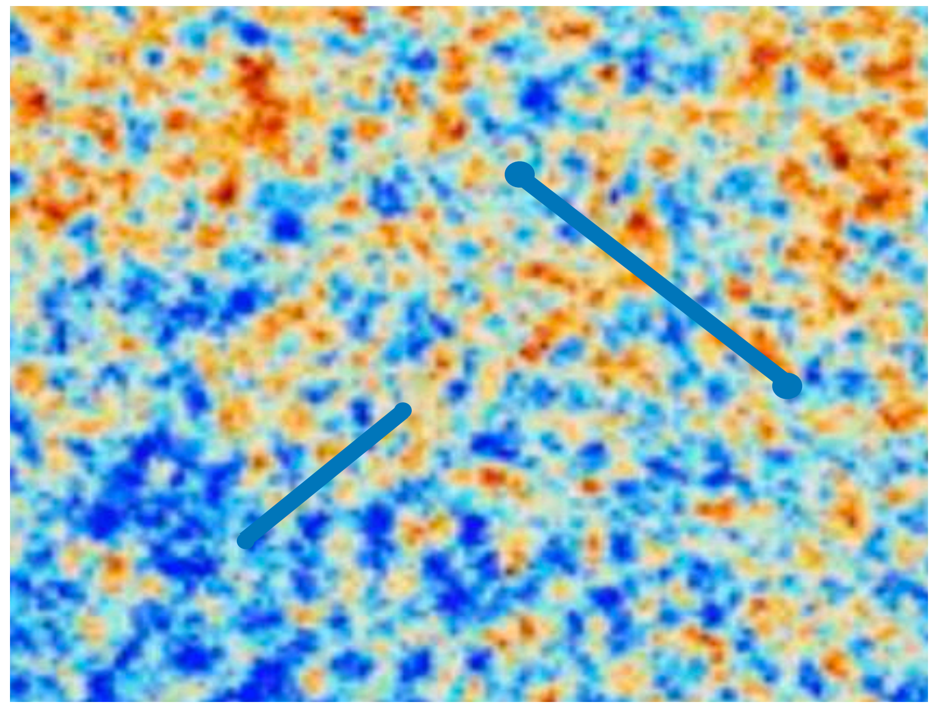
- Initial singularity
- Transplanckian problem
- Measure problem
- Hierarchy problem
- …

We will return to this in lesson 5!

* still highly debated in the literature

Spectrum of the initial perturbations

The initial fluctuations created in inflation, because of the inflationary dynamics, lead to a almost scale invariant spectrum



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